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A Conformal Geometric Approach to Quantum Entanglement for Spin-1/2 Particles

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Abstract. The problem of quantum entanglement of two spin-1/2 particles is faced in a conformally invariant geometric framework. The configuration space of the two particles is extended by adding orientational degrees of freedom and quantum effects, including entanglement, are derived from the conformal curvature of this space. A mechanism is proposed where the space curvature and the particle motion are in mutual interaction and it is proved that this feedback between geometry and dynamics reproduces all quantum features of the two-particle system. Entanglement, in particular, originates from the residual nonlocal interaction among the orientational degrees of freedom of the two spinning particles.

1 Introduction

Since the 1935 publication of the famous paper by Einstein – Podolsky – Rosen [1], entanglement became one of the most striking and puzzling features of quantum mechanics. The generation of entangled states for two particles is not only fundamental for demonstrating quantum nonlocality [2], but also useful in quantum information processing, such as quantum cryptography [3] and teleportation [4]. Recently, two-particle entangled states have been realized in both cavity QED [5] and ion traps [6]. However, in spite of the success of quantum theory in predicting all subtle features of quantum entanglement, the dichotomy raised by the EPR paper between the requirement of reality and of completeness of the theory and the failure of any hidden variables simulation in violating Bell's inequalities imply the existence of quite "mysterious" nonlocal correlations linking the outcomes of the measurements carried out over two spatially distant particles so that "...non-locality is deeply rooted in quantum mechanics itself and will persist in any completion" [7]. Deeper investigation about the true nature of quantum nonlocality is therefore desirable.

We present here an alternative approach to quantum entanglement where the Bell inequalities violation is originated by the geometric curvature of the two-particle configuration space. We postulate the existence of a very fundamental interplay between dynamics and geometry similar to the one prevailing in the General Relativity: the particle motion is affected by the space curvature which, in turn,

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depends on the particle motion in such a way to reproduce what we call "quantum phenomena". The main difference with the General Relativity is that here the curvature is not originated from the metric, but comes from suitable affine connections for parallel transport of vectors in spite of the Euclidean metric structure of the underlying 3D physical space. We assume the quantum particle point-like but endowed with additional internal degrees of freedom related to its “orientation” in space. The relevant configuration space of a single sinning particle is therefore spanned by six generalized coordinates $q^\mu = (\boldsymbol{r}, \theta) = (x, y, z, \alpha, \beta, \gamma)$, where $\boldsymbol{r} = (x, y, z)$ is the particle position and $\theta = (\alpha, \beta, \gamma)$ are three Euler angles for particle orientation at position $\boldsymbol{r}$. The choice of the affine connections is dictated by the very fundamental requirement of invariance of the theory with respect to the conformal change $g_{\mu\nu} \to \lambda(q) g_{\mu\nu}$ of the metric tensor $g_{\mu\nu}$ of the system *configuration space* spanned by position coordinates and Euler angles. Conformal invariance was considered first by Weyl in his unified approach to gravity and electromagnetism [8]. Here, we apply Weyl’s geometrical approach to build up a theory equivalent to quantum mechanics for spin, thus generalizing to spin previous approaches to spinless particle [9, 10].

2 Weyl’s conformal geometry

Weyl’s geometry in n-dimensional manifold spanned by coordinates q^μ $\mu = 1, \ldots, n)$ is based on two fields: the metric tensor $g_{\mu\nu}(q)$ fixing the length ℓ of any vector at point P and a vector field $\phi_\mu(q)$ defining the law for parallel transport $\delta\ell$ of lengths, viz. $\delta\ell = -2\ell\phi_\mu dq^\mu$. In arbitrary change of length calibration $\ell \to \lambda(q)\ell$, the transport law remains invariant if ϕ_μ changes as $\phi_\mu \to \phi_\mu - 2\partial_\mu(\log\lambda)$. Weyl’s parallel transport law has a unique affine connection given by

$$\Gamma^\sigma_{\mu\nu} = -\begin{Bmatrix} \sigma \\ \mu\,\nu \end{Bmatrix} + \delta^\sigma_\mu \phi_\nu + \delta^\sigma_\nu \phi_\mu + g_{\mu\nu}\phi^\sigma \tag{1}$$

where $\left\{ {\sigma \atop \mu\nu} \right\}$ are the Christoffel symbols built from the metric tensor $g_{\mu\nu}$. A space with connection (1) is curved even if the metric is Euclidean. Indeed, in the general case, the scalar curvature R_W in the Weyl’s space is given by

$$R_W = R + (n-1)\left[\frac{2}{\sqrt{g}} \partial_\mu(\sqrt{g} g^{\mu\nu}\phi_\nu) - (n-2) g^{\mu\nu}\phi_\mu\phi_\nu \right] \tag{2}$$

where n is the space dimension, $g = |\det(g_{\mu\nu})|$ and R is the Riemann curvature built up from the metric tensor $g^{\mu\nu}$. In the case of Euclidean metric, $R = 0$ but R_W is still nonzero. Here we made the further assumptions that the Weyl connection is integrable, i.e. that $\phi_\mu = \partial_\mu\phi$ is the gradient of some scalar field ϕ. In a change of space calibration most physical and geometrical quantities either remain invariant, as is for the connections (1), or they change simply as $G \to -> \lambda^{w(G)} G$ for some number $w(G)$, which is known as the Weyl’s type of G. An example is the metric tensor, which changes as $g_{\mu\nu} \to \lambda g_{\mu n^{\mathrm{o}}}$, so that its Weyl type is $w(g_{\mu\nu}) = 1$. Similarly we have $w(g^{\mu\nu}) = -1$, since $g^{\mu\nu} g_{\nu\rho} = \delta^\nu_\rho$. Weyl-invariant quantities have $w(G) = 0$. Weyl’s curvature has $w(R_W) = -1$.

3 The dynamics of the spinning particle

To include spin, we enlarge the configuration space of the particle by adding to the positional coordinates $\boldsymbol{r} = \{x, y, z\}$ also orientational degrees of freedom described by Euler’s angles $\theta = \{\alpha, \beta, \gamma\}$ [11]. The particle is seen as a small rotating object having kinetic energy $K = (1/2)mv^2 + (1/2)I_C\omega^2$, where

m is the particle mass, $I_C = ma^2$ the moment of inertia (with gyration radius a [1]), $\boldsymbol{v} = \{\dot{x}, \dot{y}, \dot{z}\}$ the velocity, and

$$\boldsymbol{\omega} = \{-\dot{\beta}\sin\alpha + \dot{\gamma}\cos\alpha\sin\beta, \dot{\beta}\cos\alpha + \dot{\gamma}\sin\alpha\sin\beta, \dot{\alpha} + \dot{\gamma}\cos\beta\} \tag{3}$$

the angular velocity. The dynamics is determined by assuming that Weyl's curvature R_W acts on the particle motion as an additional potential. In the absence of other external fields, the dynamics is found by the classical minimum action principle $\delta \int L(q, \dot{q}, t)dt = 0$ with Lagrangian

$$L(q, \dot{q}, t) = \left(\frac{1}{2}mv^2 + \frac{1}{2}I_C\omega^2 - \frac{\xi\hbar^2}{m}R_W\right) = \frac{1}{2m}g_{\mu\nu}\dot{q}^\mu\dot{q}^\nu - \frac{\xi\hbar^2}{m}R_W(q) \tag{4}$$

where $\xi = [(n-2)/8(n-1)] = 1/10$ is a numeric coupling constant and R_W is given by Eq. (2) with $\phi_\mu = \partial_\mu\phi$, ϕ being Weyl's scalar potential. The factor $\hbar^2/m$ was inserted in front of R_W in the Lagrangian for dimensional reasons and because, as we shall see, the curvature R_W accounts for quantum effects (including entanglement) and the classical limit should be recovered when $\hbar \to 0$. The right-hand side of Eq. (4) is obtained inserting expression (3) in the kinetic energy and it defines the metric tensor $g_{\mu\nu}$ in the configuration space. This tensor is given explicitly by

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^2 & 0 & a^2\cos\beta \\ 0 & 0 & 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^2\cos\beta & 0 & a^2 \end{pmatrix} \tag{5}$$

The addiction of external fields is straightforward, as shown elsewhere for the spinless particle both in the relativistic [10] and not relativistic [9] case. Very recently, we extended our conformal geometric approach to a charged relativistic spin-1/2 particle, obtaining a theory essentially equivalent to Dirac's equation [12]; but because in the present work we are mainly interested to single out the effects related to quantum entanglement, we will consider only the simpler case of two non relativistic spin-1/2 particles and ignore external electric and magnetic fields. The Riemann curvature associated to $g_{\mu\nu}$ has the constant value $R = 3/(2a^2)$ and the determinant of $g_{\mu\nu}$ is $g = a^6\sin^2\beta$.

Newton's equation of motion along single trajectories can be easily derived from Lagrangian (4), but we are particularly interested here to bundles of particles trajectories each one being solution of Newton's equation. Such bundles can be obtained by the solving of the Hamilton-Jacobi Equation (HJE) associated to L, viz.

$$-\partial_t S = \frac{1}{2m}g^{\mu\nu}\partial_\mu S\,\partial_\nu S + \frac{\xi\hbar^2}{m}R_W \tag{6}$$

where Weyl's curvature R_W plays the role of an applied potential. Equation (6) governs the dynamics of the system for fixed geometry and to each solution of this equation a corresponding newtonian bundle of trajectories is associated by solving the set of first-order ODEs $\dot{q}^\mu = v^\mu = (1/m)g^{\mu\nu}\partial_\nu S$. The metric being already given by the linetic energy, all geometric properties of the configuration space are fixed once the Weyl potential ϕ is prescribed. In place of ϕ it is convenient to introduce the potential $\rho = A\exp[-(n-2)\phi]$, where A is a constant, and write the equation for ϕ in the form of a continuity equation, viz.

$$\partial_t\rho + \frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}\rho g^{\mu\nu}\partial_\nu S) = 0. \tag{7}$$

[1]The radius a is not determined in the present theory however, when the theory is extended to the relativistic framework, the radius a is fixed by the theory itself to be of the order of the particle Compton wavelength [12].

Equations (6) and (7) are the main equation of the theory. It is worth noting that ρ and ϕ are equivalent from the geometric point of view: for example, Weyl's curvature can be written also as

$$R_W = R + \left(\frac{n-1}{n-2}\right)\left[\frac{g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho}{\rho^2} - \frac{2\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu\rho)}{\rho\sqrt{g}}\right]. \tag{8}$$

However, Eqs. (6) and (7) must be solved together, thus establishing a feedback between dynamics [Eq. (6)] and geometry [Eq. (7)]. There is nothing really quantum in Eqs. (6) and (7) and they have been derived using classical concepts only. In the next Section a more direct connection between (6) and (7) and standard quantum mechanics is established by introducing a suitable wavefunction. This is made possible, because the form of Eq. (7) allows to establish a connection with the familiar statistical interpretation of quantum mechanics [9] and with Madelung-Bohm hydrodynamic model [13, 14]. Before concluding this Section we notice that Eqs. (6) and (7) are Weyl-gauge invariant, provided $w(S) = 0$, $w(m) = -1$ and $w(\rho) = (2-n)/2$.

4 The spin wavefunction

In spite of their apparent awkward nonlinear form, Eqs. (6) and (7) can be transformed into a linear one by encoding geometry and dynamics in the modulus and phase of a complex function

$$\psi = \sqrt{\rho}\exp(iS/\hbar). \tag{9}$$

In fact, introducing this *ansatz* into Eqs. (6) and (7) leads to the wave equation

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m\sqrt{g}}\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu\psi) + \frac{\xi\hbar^2}{m}R\psi. \tag{10}$$

It is worth noting that any reference to the Weyl's geometry has been cancelled out in Eq. (10), which is purely Riemannian in character. Therefore, something fundamental is lost when the wave equation alone is considered, namely the evidence of the underlying Weyl's geometry and the overall invariance under conformal gauge changes of the metric. The particle mass in Eq. (9) has $w(m) = -1$ and is not gauge invariant; but the mass ratio is gauge invariant, which is all we need to have well defined mass values [2]. It is also worth noting that the velocities v^μ (and, hence the particle trajectories), the mass density $m\rho\sqrt{g}$ and the mass current density $j^\mu = m\rho\sqrt{g}v^\mu = \rho\sqrt{g}g^{\mu\nu}\partial_\nu S$ all are gauge invariant. Finally, the "square modulus" Born's rule, on which quantum interference effects are based, comes out in a very natural way, because $\rho = |\psi|^2$. We notice also that the wavefunction (9) and its wave equation (10) play not a so fundamental role as they have in the standard quantum mechanics: the fundamental equations of the theory are Eqs. (6) and (7) and the wavefunction (9) is introduced as a mathematical *ansatz* to solve Eqs. (6) and (7). This implies, among other things, that the particle motion is the motion of a frame in space and the particle orientation is described by an element of SO(3) group. The wavefunction, being just an ansatz, can evolve in the universal covering group SU(2), instead, which is parametrized by same Euler's angles. In this way, the spin-1/2 operators $\hat{s}_i = (\hbar/2)\hat{\sigma}_i$ $(i = 1,2,3)$ along the coordinate axes ($\hat{\sigma}_i$ are Pauli's matrices) are introduced as the derivatives along the arcs s^i of a set of three orthogonal congruences in the Euler's angle subspace,

[2] The mass enters as a parameter in the present non relativistic approach, which is carried out in the gauge where m = const.; however, when the theory is extended to the relativistic framework, the particle mass is no longer an external parameter, but it becomes a consequence of the theory itself [12].

viz. $\hat{s}_i = -i\hbar\partial/\partial s^i$. As an example, we report here the solution $\psi_\uparrow(q,t)$ of Eq. (10) corresponding to the state $\Psi_\uparrow(\boldsymbol{r},t)$ of a spin 1/2 particle with spin aligned up along the fixed z-axis [3]

$$\psi_\uparrow(q,t) = e^{-i\Omega t}\left(e^{\frac{i}{2}(\gamma+\alpha)}\cos\frac{\beta}{2}\right)\Psi_\uparrow(\boldsymbol{r},t) = e^{-i\Omega t}D_\uparrow(\alpha,\beta,\gamma)\Psi_\uparrow(\boldsymbol{r},t) \tag{11}$$

where $\Omega = 21\hbar/(40ma^2)$ and $\Psi_\uparrow(\boldsymbol{r},t)$ obeys the time-dependent Schrödinger equation of the *free* particle with mass m. It is also worth noting that the scalar character of the wavefunction $\psi_\uparrow(q,t)$ implies that in a rotation of the fixed reference frame the field $\Psi_\uparrow(\boldsymbol{r},t)$ changes as the first component of the spinor $\tilde{\Psi} = \binom{\Psi_\uparrow(\boldsymbol{r},t)}{0}$, thus making a bridge between our "bosonic" wave equation (10) and the usual fermionic description based on two-component spinors. The action S, the Weyl potential ρ and the Weyl curvature R_W are easily evaluated from Eq. 11:

$$S(q,t) = S_0(\boldsymbol{r},t) + \frac{\hbar}{2}(\gamma+\alpha) - \hbar\Omega t \tag{12}$$

$$\rho(q,t) = A|\psi_\uparrow(\boldsymbol{r},t)|^2\cos^2\frac{\beta}{2} \tag{13}$$

$$R_W(q,t) = \frac{21}{4a^2} - \frac{5}{2a^2(1+\cos\beta)} \tag{14}$$

where A is a normalization constant and $S_0(\boldsymbol{r},t) = \arg(\Psi_\uparrow(\boldsymbol{r},t))$. The first of Eqs. (12) shows that the motion of the particle frame in this state is a uniform precession of its axis ζ around the fixed z-axis with constant angle β. The last of Eqs. (12) shows that a self-force acts however on the particle due to the nonzero Weyl's curvature. We notice once again that in the standard spin 1/2 quantum mechanics based on $\Psi_\uparrow(\boldsymbol{r},t)$ alone, the existence of self-action is lost, because $\Psi_\uparrow(\boldsymbol{r},t)$ obeys the Schrödinger equation for the free particle. In the next Section we will show that in the case of two spins, Weyl's curvature produces a geometric interaction between the two particles and that this interaction, which is hidden in the standard approach to quantum mechanics, originates all quantum effects due to entanglement.

5 The two-spin wavefunction

The dynamics of two particles with spin is obtained by simply extending the configuration space to the product space spanned by the twelve coordinates associated to the particle positions $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ and orientations θ_1 and θ_2. However, as we know from quantum mechanics, the behavior of the two-particle system is different when product states or entangled states are considered. Here we show that this different behavior reflects into different Weyl curvatures of space and, hence, into different forces acting on the particles and that these forces have mechanical effects in full agreement with predictions of standard quantum mechanics and experiments, including the EPR paradox.

5.1 The product state case

Let us consider first the case of a product state of two identical spin 1/2 particles having opposite spins along the z-axis. The wavefunction $\psi_{\uparrow\downarrow}(q,t)$ associated to this state is

$$\psi_{\uparrow\downarrow}(q,t) = \left(e^{i\left(\frac{\gamma_A+\gamma_B}{2}-2\Omega t\right)}e^{-i\frac{\Delta\alpha}{2}}\cos\frac{\beta_A}{2}\sin\frac{\beta_B}{2}\right)\Psi_\uparrow^{(A)}(\boldsymbol{r}_A,t)\Psi_\downarrow^{(B)}(\boldsymbol{r}_B,t), \tag{15}$$

[3] In the case of spin down, the function $D_\uparrow(\alpha,\beta,\gamma)$ is replaced by $D_\downarrow(\alpha,\beta,\gamma) = D_\uparrow(-\alpha,\pi-\beta,\gamma)$.

where $\Delta\alpha = \alpha_B - \alpha_A$ and suffixes A and B denote the two particles, respectively [4]. In a space rotation the wavefunction $\psi_{\uparrow\downarrow}(q,t)$ changes as a scaler field and $\Psi^{(A)}_{\uparrow}(\boldsymbol{r}_A,t)$, $\Psi^{(B)}_{\downarrow}(\boldsymbol{r}_B,t)$ change as the components of the spinors $\tilde{\Psi}^{(A)} = \binom{\Psi^{(A)}_{\uparrow}(\boldsymbol{r}_A,t)}{0}$ and $\tilde{\Psi}^{(B)} = \binom{0}{\Psi^{(B)}_{\uparrow}(\boldsymbol{r}_B,t)}$, respectively. Thus, the connection with the familiar spinorial formulation is established. It can be easily proved that $\psi_{\uparrow\downarrow}(q,t)$ is a solution of Eq. (10), provided $\Psi^{(A)}_{\uparrow}(\boldsymbol{r},t)$ and $\Psi^{(B)}_{\downarrow}(\boldsymbol{r},t)$ obey the time-dependent Schrödinger equation of the *free* particle with mass m. We are led here to Schrödinger's equation of the free particle because, as said above, the information about the underlying Weyl's curvature and associated forces on the particles is lost at the wavefunction level. To recover this information we must have recourse to Eqs. (6) and (7). In the case of the $|\uparrow,\downarrow\rangle$ state, we find

$$S(q,t) = S^{(A)}(\boldsymbol{r}_A,\theta_A) + S^{(B)}(\boldsymbol{r}_B,\theta_B) \tag{16}$$

$$\rho(q,t) = \rho^{(A)}(\boldsymbol{r}_A,\theta_A)\rho^{(B)}(\boldsymbol{r}_B,\theta_B) \tag{17}$$

$$R_W(q,t) = R^{(A)}_W(\boldsymbol{r}_A,\theta_A) + R^{(B)}_W(\boldsymbol{r}_B,\theta_B) \tag{18}$$

so that each particle moves as if the other were absent. This independence of motions reflects the absence of entanglement in the considered state. However, each particle is not free, because it is affected by the self-force arising from the Weyl curvature. The angular part of R_W can be calculated explicitly to be $-\frac{11}{5a^2}\left(\frac{1}{1+\cos\beta_A}+\frac{1}{1-\cos\beta_B}\right)$.

5.2 The EPR state

Now we pass to consider the maximally entangled EPR state $|\psi_{EPR}\rangle = (1/\sqrt{2})(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle)$. The corresponding wavefunction is

$$\begin{aligned}\psi_{EPR}(q,t) \;&=\; \frac{1}{\sqrt{2}}e^{i(\frac{\gamma_A+\gamma_B}{2}-2\Omega t)}\left(e^{-i\frac{\Delta\alpha}{2}}\cos\frac{\beta_A}{2}\sin\frac{\beta_B}{2} - e^{i\frac{\Delta\alpha}{2}}\cos\frac{\beta_B}{2}\sin\frac{\beta_A}{2}\right)\times \\ &\quad\times\Psi^{(A)}_{\uparrow}(\boldsymbol{r}_A,t)\Psi^{(B)}_{\downarrow}(\boldsymbol{r}_B,t),\end{aligned} \tag{19}$$

where once more we do not consider Pauli's exclusion principle, because the *external-space* wavefunctions $\Psi^{(A)}_{\uparrow}(\boldsymbol{r}_A,t)$ and $\Psi^{(B)}_{\downarrow}(\boldsymbol{r}_B,t)$ are supposed to be associated to two well separated wavepackets centered at positions $\bar{\boldsymbol{r}_A}$ and $\bar{\boldsymbol{r}_B}$ respectively (see footnote 4). The action and the density associated to $\psi_{EPR}(q,t)$ are

$$\begin{aligned}S(q,t) \;&=\; \hbar[-2\Omega + \frac{\gamma_A+\gamma_B}{2} + \arctan\left(\csc\frac{\beta_A-\beta_B}{2}\sin\frac{\beta_A+\beta_B}{2}\tan\frac{\alpha_B-\alpha_A}{2}\right) + \\ &\quad + \arg(\Psi^{(A)}_{\uparrow}(\boldsymbol{r}_A,t)) + \arg(\Psi^{(B)}_{\downarrow}(\boldsymbol{r}_B,t))]\end{aligned} \tag{20}$$

and

$$\rho(q^\mu,t) = \frac{1}{4}\left|\Psi^{(A)}_{\uparrow}(\boldsymbol{r}_A,t)\right|^2\left|\Psi^{(B)}_{\downarrow}(\boldsymbol{r}_B,t)\right|^2[1-\cos\beta_A\cos\beta_B - \cos(\Delta\alpha)\sin\beta_A\sin\beta_B]. \tag{21}$$

The differential equations of motion $\dot{q}^\mu = (1/m)g^{\mu\nu}\partial_\nu S$ derived from (20) splits into three decoupled sets: one involving the center of mass coordinates of particle A only, one involving the center of mass coordinates of particle B only, and a third set involving the Euler's angles of both A and B. The last set

[4] We assume very little superposition between the wavepackets $\Psi^{(A)}_{\uparrow}(\boldsymbol{r}_A,t)$ and $\Psi^{(B)}_{\downarrow}(\boldsymbol{r}_B,t)$ so that Pauli's antisymmetrization is unnecessary.

of equations cannot be decoupled because of quantum entanglement. The presence of entanglement is also unveiled by the expression of the Weyl curvature R_W derived from Eq. (21):

$$R_W = \frac{48}{5a^2} + \frac{22}{5a^2(1-\cos\beta_A\cos\beta_B - \cos\Delta\alpha\sin\beta_A\sin\beta_B)} + R_W^{(A)}(\boldsymbol{r}_A,t) + R_W^{(B)}(\boldsymbol{r}_B,t) \qquad (22)$$

where $R_W^{(A)}$ and $R_W^{(B)}$ are the Weyl spacetime curvatures associated with the fields $\Psi_\uparrow^{(A)}(\boldsymbol{r}_A,t)$ and $\Psi_\downarrow^{(B)}(\boldsymbol{r}_B,t)$, respectively. We see again that the total Weyl curvature is equally splitted into three terms: the curvatures $R_W^{(A)}$ and $R_W^{(B)}$, depending on the space-time "external" coordinates and yielding the self-action of each particle on itself, and the coupling term which depends on the Euler angles only. Unlike the spacetime terms, this last term cannot be splitted into the sum of two independent potentials acting on each particle and it is the responsible of all phenomena related to quantum entanglement. In this way, the very nature of entanglement is explained as originating from the residual coupling of the orientational degrees of freedom of the two spins due to the presence of the Weyl's curvature R_W (22). This one is the origin of a inter-particle coupling consisting of a real orientational force that one particle exerts on the other. As said, this force originates from R_W, which in turn originates from Weyl's potential ϕ, which ultimately arises from the system's wavefunction ψ_{EPR}. This last one then looses its meaning of a purely mathematical entity in favor of the more pregnant concept of a physical field. To summarize, in the presence of entanglement, the *internal coordinates* θ_A and θ_B, viz. the Euler angles of the particle orientation, cannot be separated *irrespective* of the mutual spatial distance between the two travelling particles. Even if these ones are space-like separated by a large distance d, an inter-particle coupling independent of d arises that cannot be eliminated and is responsible for the *nonlocal EPR correlations*. Furthermore, we conjecture from the present nonrelativistic standpoint that the space-time superluminality of the nonlocal correlations comes from the geometrical independency, i.e. disconnectedness, of the internal and external manifolds: $\{\boldsymbol{r},t\}$ and $\{\theta\}$. This is the key result of the present work. As shown in Eqs. (16) the dynamical Euler's angle coupling disappears in the *absence of entanglement*, i.e. in the case of a "*product-state*".

6 The meaning of quantum measurement

To better understand why the internal variables are not directly accessible to experiments, we need a closer view of how experiments are carried out in the quantum world. In essence, any experimental apparatus designed to measure some physical property of a quantum particle is made of two parts: a “filtering” device which addresses the particle to the appropriate detector channel according the possible values of the quantity to be measured (e.g. a spin component) and one (or more) detectors able to register the arrival of the particle. To fix the ideas, we consider here the particular case of the measure of a spin 1/2 particle by a Stern-Gerlach (SGA) apparatus. The spin component along the SGA axis can have two values, so we need two detectors D_u and D_d coupled to the "up" and "down" output channels of the orientable SGA. Each detector measures the flux Φ of particles entering its acceptance area A. Let's assume single particle detection. Then this flux is given by $\Phi = \int_A j^\mu n_\mu dA = \int_\Sigma \rho\sqrt{g}g^{\mu\nu}\partial_\nu S\, n_\mu d\Sigma$ extended to the hypersurface Σ in the particle configuration space with normal unit vector $n_\mu = n_\mu = \{\boldsymbol{n},0,0,0\}$ where $\boldsymbol{n}$ is the usual 3D-normal to the detector area A. Let us assume that the scalar wavefunction of the particle at the detector location has its spacetime and angular parts factorized [5], i.e. $\psi = \psi_1(\boldsymbol{r},t)\psi_2(\alpha,\beta,\gamma)$. Then $\rho = \rho_1(\boldsymbol{r},t)\rho_2(\alpha,\beta,\gamma)$, $S = S_1(\boldsymbol{r},t) + S_2(\alpha,\beta,\gamma)$ and $\Phi = \int_A \mathbf{j}\cdot\mathbf{n}dA\int\rho_2(\alpha,\beta,\gamma)d\mu(\alpha,\beta,\gamma)$, where $\boldsymbol{j} = \rho_1(\boldsymbol{r},t)\nabla S_1$ and $d\mu(\alpha,\beta,\gamma)$ is the invariant measure in the Euler angle subspace. The particle flux Φ is the only

[5] All experiments, including the SGA are arranged to met this condition.

quantity directly accessible to the detector and depends only on the spacetime part $\psi_1(x, y, z, t)$ of the wavefunction. The Euler's angles are integrated away for the simple reason that the detector is located in the physical space-time. It is worth noting that the current density j^μ and, hence, the flux Φ is Weyl-gauge invariant as it must be for any quantity having a measurable value.

Let us consider now the role played by the filtering apparatus. Unlike the detector, whose role is just to count particles, the filtering stage of the experimental setup must be tailored on the quantity to be measured. In the case of the SGA the filtering device is the spatial orientation of the inhomogeneous magnetic field crossed by the particle's beam. In an ideal filtering apparatus no particle is lost, so its action on the particle's wavefunction is unitary. The role of the filter is to correlate the spacetime path of the particle with the quantity to be measured (the spin component, in our case) so to extract from the incident beam all particles with a given value of the quantity (spin "up", for example) by addressing them to the appropriate detector. The filter acts on the particle motion in space-time only. But, as said before, there is a feedback between the particle motion and the geometric curvature of the space, so that the insertion of the filter changes not only the particle path in spacetime, but also the overall geometry of the particle configuration space, because it modifies its Weyl's curvature R_W through the environmental *World vector potential* ϕ_μ [15].

In our present approach, both particle motion and space geometry are encoded in the scalar wavefunction, which indeed changes under the action of the "unitary", i.e. lossless, action of the SGA filter. Solving the full dynamical and geometric problem inside the SGA is a difficult problem, but the asymptotic behavior of the scalar wavefunction far from the SGA may be easily found. In this "far-field scattering approximation", a uniformly polarized particle beam is transformed by a SGA rotated at angle θ with respect to the z-axis as follows,

$$\begin{aligned}
&[aD_\uparrow(\alpha,\beta,\gamma) + bD_\downarrow(\alpha,\beta,\gamma)]\psi(x,y,z,t) \xrightarrow{SGA} \\
&\left(a\cos\frac{\theta}{2} + b\sin\frac{\theta}{2}\right)\left(D_\uparrow(\alpha,\beta,\gamma)\cos\frac{\theta}{2} + D_\downarrow(\alpha,\beta,\gamma)\sin\frac{\theta}{2}\right)\psi(x_u,y_u,z_u,t) + \\
&+\left(a\sin\frac{\theta}{2} - b\cos\frac{\theta}{2}\right)\left(D_\uparrow(\alpha,\beta,\gamma)\sin\frac{\theta}{2} - D_\downarrow(\alpha,\beta,\gamma)\cos\frac{\theta}{2}\right)\psi(x_d,y_d,z_d,t)
\end{aligned} \tag{23}$$

where a, b are arbitrary complex constants with $|a|^2 + |b|^2 = 1$, and labels u and d refer to the positions of the detectors located to the up and down exit channels of the θ-oriented SGA. The experimental apparatus is arranged so that the wave packets $\psi(x_u, y_u, z_u, t)$ and $\psi(x_d, y_d, z_d, t)$ have negligible superposition so that each detector sees a wavefunction with space and angular parts factorized. Thus, for example, the particle flux detected in the "up" channel of the SGA is given by $\Phi_u P_u(\theta)$, where Φ_u is the particle flux on the detector and $P_u(\theta) = \left|a\cos\frac{\theta}{2} + b\sin\frac{\theta}{2}\right|^2$ is usually interpreted as the probability that the particle in the input wavepacket is found with its spin along the "up" direction of the SGA. What the filter really does is to correlate the particle space-time trajectory with the quantity to be measured. In the standard quantum mechanical language, we may say that the filter introduces a controlled entanglement among the quantity to be measured and the particle spacetime path (in the SGA case, the spacetime degrees of freedom become entangled with the orientational ones). However, the filter is configured so that the wavepackets arriving on each detector (D_u and D_d, in our case) are not superimposed, and the (approximate) wavefunction seen by each detector is of the product form as considered above. The last requirement ensures that the detected particle flux Φ provides a correct measure (in the quantum sense) of the measured quantity[6].

[6] It is precisely the lack of this condition which prevents to use the SGA to measure the spin of electrons. A way to overcome this fundamental limitation was proposed very recently [16].

7 The EPR state and Bell inequalities

Let's now turn our attention to the joint spin measurements of the EPR entangled particles A and B described by Eq. (19). After leaving the source, particles A and B travel towards two Stern Gerlach setups, SGA_A and SGA_B, respectively, located at Alice's and Bob's stations on two distant sites along $\overrightarrow{y}$. As said before, each SGA acts *locally*, by a *unitary* transformation, on the particle spatial, i.e. *external*, degrees of freedom by correlating its exit direction of motion with the direction of its spin respect to the SGA axis, rotated around the y-axis at angle θ, taken respect to the z-axis. Since we are dealing with (1/2)-spins, there are only two exit directions, either "up" or "down" available to each particle which will be then finally registered by a corresponding detector. Let's refer to the Alice's and Bob's detectors as D_{Au}, D_{Ad}, D_{Bu}, D_{Bd} and let θ_A and θ_B the angles of SGA_A and SGA_B, respectively. As said above, the presence of the two SGA changes not only the trajectories of the two particles, but also the Weyl curvature of their configuration space. These changes are both encoded in the change of the wavefunction ψ_{EPR} in Eq. (19). Near the source that wavefunction remains approximately unchanged, but far beyond the spatial positions of the two SGA's the paths of the particles acquire different direction according to their spin so that near the locations of the detectors the input wavefunction is transformed according to

$$\psi_{EPR} \overset{SGAs}{\longrightarrow} A_{u,u}\psi_A(\mathbf{r}_{Au}t)\psi_B(\mathbf{r}_{Bu},t) + A_{u,d}\psi_A(\mathbf{r}_{Au},t)\psi_B(\mathbf{r}_{Bd},t) + \\ + A_{d,u}\psi_A(\mathbf{r}_{Ad},t)\psi_B(\mathbf{r}_{Bu},t) + A_{d,d}\psi_A(\mathbf{r}_{Ad},t)\psi_B(\mathbf{r}_{Bd},t) \tag{24}$$

where $\boldsymbol{r}_{Au}$, $\boldsymbol{r}_{Ad}$, $\boldsymbol{r}_{Bu}$, $\boldsymbol{r}_{Bd}$ are the positions of the detectors and $A_{u,u}$, $A_{u,d}$, $A_{d,u}$, $A_{d,d}$ are coefficients depending on the two particle Euler's angles and on the angles θ_A and θ_B of SGA_A and SGA $_B$, respectively. The coefficients A can be easily calculated by applying Eq. (23):

$$A_{u,u} = \chi(t)\left(D_\uparrow(\alpha_1,\beta_1,\gamma_1)\cos\frac{\theta_A}{2} + D_\downarrow(\alpha_1,\beta_1,\gamma_1)\sin\frac{\theta_A}{2}\right)\times \\ \times\left(D_\uparrow(\alpha_2,\beta_2,\gamma_2)\cos\frac{\theta_B}{2} + D_\downarrow(\alpha_2,\beta_2,\gamma_2)\sin\frac{\theta_B}{2}\right)\sin\Delta\vartheta \tag{25a}$$

$$A_{u,d} = \chi(t)\left(D_\uparrow(\alpha_1,\beta_1,\gamma_1)\cos\frac{\theta_A}{2} + D_\downarrow(\alpha_1,\beta_1,\gamma_1)\sin\frac{\theta_A}{2}\right)\times \\ \times\left(-D_\uparrow(\alpha_2,\beta_2,\gamma_2)\sin\frac{\theta_B}{2} + D_\downarrow(\alpha_2,\beta_2,\gamma_2)\cos\frac{\theta_B}{2}\right)\cos\Delta\vartheta \tag{25b}$$

$$A_{d,u} = \chi(t)\left(-D_\uparrow(\alpha_1,\beta_1,\gamma_1)\sin\frac{\theta_A}{2} + D_\downarrow(\alpha_1,\beta_1,\gamma_1)\cos\frac{\theta_A}{2}\right)\times \\ \times\left(D_\uparrow(\alpha_2,\beta_2,\gamma_2)\cos\frac{\theta_B}{2} + D_\downarrow(\alpha_2,\beta_2,\gamma_2)\sin\frac{\theta_B}{2}\right)\cos\Delta\vartheta \tag{25c}$$

$$A_{d,d} = \chi(t)\left(-D_\uparrow(\alpha_1,\beta_1,\gamma_1)\sin\frac{\theta_A}{2} + D_\downarrow(\alpha_1,\beta_1,\gamma_1)\cos\frac{\theta_A}{2}\right)\times \\ \times\left(-D_\uparrow(\alpha_2,\beta_2,\gamma_2)\sin\frac{\theta_B}{2} + D_\downarrow(\alpha_2,\beta_2,\gamma_2)\cos\frac{\theta_B}{2}\right)\sin\Delta\vartheta \tag{25d}$$

where $\chi(t) = \frac{1}{\sqrt{2}}e^{-2i\Omega t}$ and: $(\Delta\vartheta) = (\theta_B - \theta_A)/2$. The coincidence rate are given by the joint particle fluxes intercepted by the detectors, viz. $\Phi_{i,j}(\theta_A,\theta_B) = \iint \left|A_{ij}(\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2;\theta_A,\theta_B)\right|^2 d\mu(\alpha_1,\beta_1,\gamma_1)d\mu(\alpha_2,\beta_2,\gamma_2)\int \mathbf{j}_i\cdot\mathbf{n}_i dA_i \int \boldsymbol{j}_j\cdot\boldsymbol{n}_j dA_j$, where $i,j = u,d$ and $\boldsymbol{j}_i = |\psi_A(\mathbf{r}_i,t)|^2\,\nabla S_A(\boldsymbol{r}_i,t)$, $\boldsymbol{j}_j = \left|\psi_B(\boldsymbol{r}_j,t)\right|^2\nabla S_B(\boldsymbol{r}_j,t)$ are the particle current densities at the detectors. A simple calculation shows that if all particles falling into the detectors are counted, the coincidence fluxes are given by $\Phi_{u,u}(\theta_A,\theta_B) = \Phi_{d,d}(\theta_A,\theta_B) = \frac{1}{2}\sin^2(\Delta\vartheta)$ and $\Phi_{u,d}(\theta_A,\theta_B) =$

$\Phi_{d,u}(\theta_A, \theta_B) = \frac{1}{2}\cos^2(\Delta\vartheta)$. The coincidence fluxes Φ_{ij} are Weyl-invariant and can be experimentally measured. Moreover, they are equal to the joint probabilities $P_{i,j}(\theta_A, \theta_B)$ associated with the EPR state (19), in full agreement with the standard quantum theory and lead straightforwardly to the violation of the Bell's inequalities within all appropriate experiments consisting of statistical measurements over several choices of the angular quantity $(\Delta\vartheta)$, as shown by many modern texts [17–20]. For instance, Redhead considers the inequality: $F(\Delta\vartheta) \equiv |1 + 2\cos(2\Delta\vartheta) - \cos(4\Delta\vartheta)| \leq 2$, which is violated for all values of $(\Delta\vartheta)$ between 0°and 45°.

8 Conclusions

We have demonstrated that the quantum nonlocality enigma, epitomized by the violation of the Bell's inequalities, may be understood on the basis of a Weyl's conformal geometrodynamics. This result was reached through a theory that bears several appealing properties and may lead to far reaching consequences in modern physics. We summarize them as follows:

1. The linear structure of the standard first quantization theory is fully preserved, in any formal detail.
2. The quantum wavefunction acquires the precise meaning of a physical quantum "*Weyl's gauge field*" acting in a curved configurational space.
3. A proper theoretical analysis of any quantum *entanglement* condition must involve the entire configurational space of the system including the usual space-time of General Relativity as well as the "internal coordinates" of the system. If entanglement is present and if the internal coordinates are really "hidden", i.e. if they are absent in the theory – as they are in standard quantum theory – severe limitations may arise on the actual interpretation of any dynamical problem. There physics may even be an impossible task, in principle, and paradoxes may spring out. Indeed, in addition to "*quantum nonlocality*", many counterintuitive concepts of quantum mechanics, such as those related to several aspects of "*quantum indeterminism*" and of "*quantum counterfactuality*" may precisely arise from these theoretical limitations. Which are indeed limitations to the human knowledge and understanding.
4. The "sinister", "disconcerting" and "discomforting" aspects of entanglement were expressed right after the publication of the EPR paper by a surprised and highly concerned Erwin Schrödinger. Who also added: "I would not call that one but rather *the* characteristic trait of quantum mechanics, the one that enforces the departure from the classical lines of thought" [21].
5. By solving an utterly important enigma the present paper clarifies *the* – according to Schrödinger – characteristic trait of quantum mechanics. The adopted theory is based on a necessary significant aspect of the interplay between geometry and matter motion on which also rests the modern theory of gravitation, i.e. General Relativity. Consequently, our work may be considered to belong to a unifying theoretical scenario linking necessarily gravitation and quantum mechanics. This is indeed the long sought, paradigmatic "quantum gravity" scenario.

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