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Formation of nonlinear holographic images in a system of periodically located nonlinear mediums

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Abstract. The formation of nonlinear holographic images in a system of periodically located nonlinear mediums is studied. Analytical expressions which describe the magnitudes and locations of intensity maximums depending on the corresponding image number are derived. Comparison with numerical calculation results is presented.

Dependence of a refraction index on intensity leads to beam self-focusing [1], transverse instability [2] and to formation of nonlinear holographic images (NHI) of obstacles in the optical path of powerful laser systems [3]. In Ref. [4] it has been shown that at value of B-integral of only 0.2 rad a round opaque obscuration increases the intensity peak which exceeds the average level by almost 1.5 times, and for a phase obstacle the peak excess is 2.5. One can present a disk amplifier as a system of periodically located (SPL) nonlinear mediums (NM). Formation of NHI in SPLNM differs from the NHI formation in continuous media with the same value of B-integral.

For the first time this problem has been considered in Ref. [3]. However, these authors [3] focused only on the analysis of the intensity dependence upon the size of an obscuration at a single point of space. Accidentally for SPLNM configuration considered in Ref. [3], the intensity in the chosen point is not a peak one. The locations of the NHI and their intensity were not explored. In Refs [5, 6] the results of peak intensity calculations for the various elements of the NIF laser chain with account for NHI formation were published. In Refs [7–9] the process of the NHI formation in a SPLNM was viewed in more details and it was shown that for K NM there are K NHI. However the chosen model parameters (number of NM, increment of B-integral for one NM, thickness of NM, size of the beam) are far from the parameters of typical disk amplifiers.

In this paper, we are considering the simplified system of infinitely thin NM without energy gain and losses. According to expression (39) in Ref. [3], when the powerful plane wave and spherical scattered wave incident on the single NM, the new wave is generated. For the infinitely thin NM the complex amplitude A_1 of this wave in the plane of NHI is equal to

$$A_1 = -iBA_S^* \tag{1}$$

where A_s is the complex amplitude of the scattered wave in the plane of obstacle.

After passage of a NM the incident scattered wave changes to

$$A_S^{out} = (1 - iB)\,A_S. \tag{2}$$

Let us note that upon the derivation of Eq. (39) in Ref. [3], it was not assumed that the incident scattered wave is diverging. Therefore expressions (1) and (2) remain valid for a converging incident wave. In this case the generated wave is diverging and the complex wave amplitude of perturbation as a result of interaction with a powerful wave in NM is multiplied by (*1-iB*).

Let us use Eqs (1, 2) to consider the NHI formation in a system of several (K) NM when the distance from an obstacle to the first NM is greater than the distance between the first and the last NM. For a typical powerful disk amplifier the increment of B-integral per slab is noticeably less than 1 rad. So as a first approach we will consider only the converging waves generated by single transformation from the initial scattered wave, neglecting all the waves generated by several transformations. To get the complex amplitude $A_j^{(1)}$ of the wave forming the j-th NHI it is necessary:
1. To apply to an scattered wave ($j{-}1$) times the functional of passage, Eq. (2)
2. Then to apply once the functional of a birth, Eq. (1)
3. Then to apply ($K - j$) times the functional of passage, Eq. (2)
As a result we get

$$A_j^{(1)} = (1 - iB)^{K-j}\,(-iB)\left((1 - iB)^{j-1}\,A_s\right)^{*} = -iB\,(1 - iB)^{K-j}\,(1 + iB)^{j-1}\,A_s^{*}. \tag{3}$$

Accordingly the module of amplitude and the phase of this wave are:

$$\left|A_j^{(1)}\right| = |A_s|\,B\left(1 + B^2\right)^{(K-1)/2} \tag{4}$$

$$\psi_j = -(\psi_s + \pi/2) + (2j - K - 1)\,arctg(B)\,. \tag{5}$$

The upper index $^{(1)}$ shows that the first approach is considered. Amplitudes of all waves forming NHI are equal, but phases relative to the powerful wave are various.

For the next-order approximation it is necessary to consider additional waves which arise at transformation of a converging wave to a diverging one, and then again to a converging one. In comparison with considered waves with the first order the amplitude of these waves are multiplied by factor $B^2/(1 + B^2)$, which is much less than 1. However, for NHI with numbers close to $K/2$, when $K = 9$ to 11, the number of such waves can be large enough. Let's consider one of the third order waves which is focalized in the j-th NHI. Let us designate through p, q and r the NM numbers where the new waves have been generated. Thus $2 \le j \le K - 1;\ \ 1 \le p < j;\ \ \ j < r \le K;\ \ q = r + p - j$. Simple analysis demonstrates that the complex amplitude of any third-order wave does not depend upon p, q, r and the phase is precisely equal to the phase of the first order wave focalized in the corresponding NHI. The number of waves of the third order for the j-th NHI is equal to $(j - 1)\cdot(K - j)$. Hence, taking into account waves of the first and the third order the module of amplitude of the total of waves forming the j-th NHI, is equal to

$$\left|A_j^{(3)}\right| = |A_s|\,B\left(1 + B^2\right)^{(K-1)/2}\left[1 + \left(B^2/\left(1 + B^2\right)\right)(j - 1)(K - j)\right]. \tag{6}$$

One can see that the amplitude of the waves forming the j-th NHI depends on j and is maximal for NHI with numbers close to $K/2$.

Having conducted similar reasoning for the higher orders waves, we will get the final formula for amplitude magnitude of the summary wave forming the j-th NHI

$$\left|A_j\right| = |A_s|\,B\left(1 + B^2\right)^{(K-1)/2}\left[1 + \sum_{l=1}^{[(K-1)/2]}\left(\left(B^{2l}/\left(1 + B^2\right)^l\right)\frac{\prod_{m=1}^{l}(j - m)}{l!}\,\frac{\prod_{m=1}^{l}(K + 1 - j - m)}{l!}\right)\right] \tag{7}$$

where $[(K - 1)/2]$ is the integer part of $(K - 1)/2$.

The phase of the summary wave focalized in the j-th NHI is described by the expression (5).

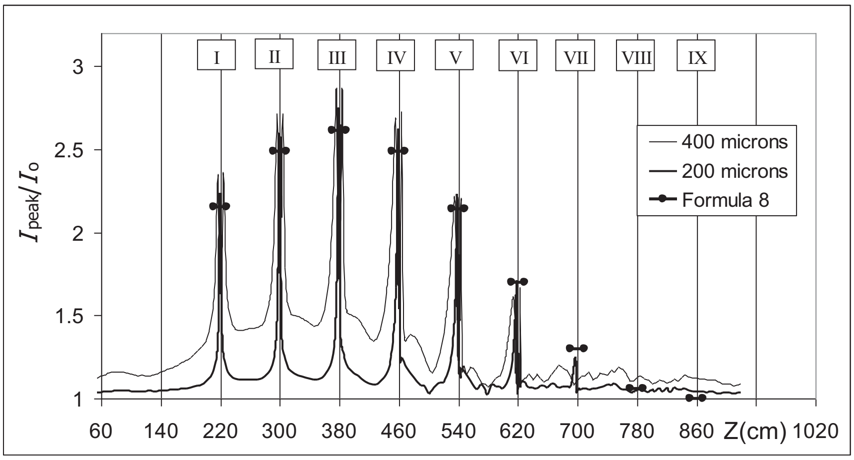

Figure 1. Peak intensity evolution for the system of 9 NM and opaque disk obstacle. Reference is on the last (the 9-th) NM.

The peak intensity which is reached in the vicinity of the corresponding image is equal to [4]:

$$I_j^{\max} = \left(\left|A_0 + A_j\right| + \left|A_j\right|\right)^2. \tag{8}$$

Because of distinction ψ_j values of the peak intensity can essentially vary for different NHI. Figure 1 shows the modeling results and calculations according to Eq. (8) for a system of 9 NM. Eqs (5) and (7) were used to get the phase and amplitude of appropriate waves. The increment of B-integral in each NM is equal to 0.2 rad. An opaque disk obstacle with diameter (d) 0.2 or 0.4 mm is located 7 meters apart from the fifth (central) NM, i.e. 5.4 meters apart from the first NM (Z_0). The size of a powerful beam is equal $20 \times 20\,\text{cm}^2$. The distance ($L$) between adjacent NM is 40 cm, the wavelength (λ) is 1000 nm.

Roman numerals in Fig. 1 show the plane NHI number for the corresponding NM. The analysis of the presented results shows that Eq. (8) gives qualitatively correct description of peak intensity in the vicinity of each NHI. But there is some systematic discrepancy between modeling results and calculations according to (8). So far we have neglected the influence of waves focusing in other NHI, as well as the influence of diverging waves. Taking into account all converging waves, and the most powerful diverging wave, whose centre of curvature coincides with the obstacle, formula (8) will be converted into

$$I_{j\max} = \left(\left|A_0 + A_j + i\sum_{m=1,m\neq j}^{K} \frac{A_m \pi z_R}{2L\,(m-j)} + \frac{A_S\left(1+B^2\right)^{K/2}\pi z_R}{2Z_0 + 2\,(j-1)\,L} \times \exp\left(-iK\arctan(B) + i\pi/2\right)\right| + \left|A_j\right|\right)^2 \tag{9}$$

$$z_R = d^2/4\lambda.$$

Upon the deduction of these expressions it is assumed that $z_R \ll L$.

In Fig. 2, the part of Fig. 1 corresponding to the vicinity of the 1-st NHI is shown. Black rhombuses mark the points calculated using Eq. (9) where Eq. (5) and Eq. (8) were used for phase and amplitude computation. Values of abscissa for these points are defined by Eq. (5) in Ref. [4]. The corrected formula, Eq. (9), allows to achieve a good agreement with the modeling results.

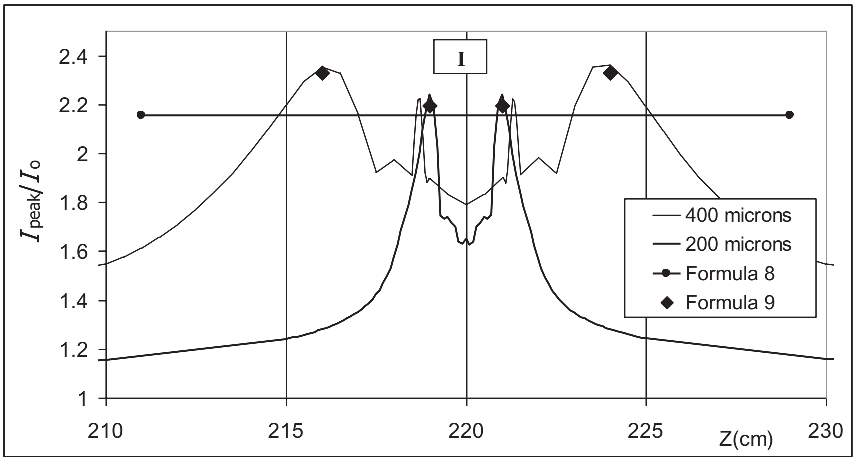

Figure 2. Peak intensity evolution in the vicinity of the 1-st NHI.

CONCLUSION

It is shown that in the case of a system of 9 NM (B = 0.2 for each NM), the amplitude of the wave forming the central image is 65% more than the amplitude of the wave forming the first and the last images. The phase of the wave forming the corresponding image is crucial for the peak intensity magnitude reached in the vicinity of each of the NHI. The derived analytical expressions for the intensity peak magnitudes and locations depending on the image number and obstacle type are in good agreement with numerical calculations. It is shown that generally the influence of a diverging wave scattered on the obstacle and the waves focusing in the adjacent images should be taken into account.

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