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Pre- and post- scission particle emission in 3D Langevin calculations with various macroscopic potentials

K. Mazurek[1,a], P.N. Nadtochy[2], C. Schmitt[3], P. Wasiak[1], M. Kmiecik[1], A. Maj[1], E. Bonnet[3], A. Chbihi[3], J. Frankland[3], D. Gruyer[3], and J.-P. Wieleczko[3]

[1] The Niewodniczański Institute of Nuclear Physics - PAN, ul. Radzikowskiego 152, 31-342 Kraków, Poland
[2]Omsk State University, Department of Theoretical Physics, Mira Prospect 55-A, 644077 Omsk, Russia
[3]Grand Accelerateur National d'Ions Lourds, CEA/DSM-CNRS/IN2P3, Bvd. Henri Becquerel, 14076 Caen, France

Abstract. The fission dynamics described by solving differential equations of the Langevin type in three dimensional space of the deformation parameters is very sensitive on the choice of the macroscopic components such as potential energy models. The mass or charge distribution or total kinetic energy has been already shown to be different when one uses the Finite Range Liquid Drop Model or Lublin - Strasbourg Drop model. Also the shape-dependent congruence or shape-dependent Wigner energy and A^0 terms are important especially for the fission of medium mass nuclei. We would like to make step forward and answer the question about the varying of the post-scission multiplicity by including different PES. Up to now there are only few experimental data for the medium mass nuclei where the pre- and post- scission emission has been estimated and isotopic distributions have been shown. The isotopic distributions of the fission products for light compound nucleus such as ^{111}In with two beam energies ($E_{beam} = 10.6$ AMeV and 5.9 AMeV) and two heavy systems: ^{229}Np with $E_{beam} = 7.4$ AMeV and ^{260}No ($E_{beam} = 6$ AMeV and 7.5 AMeV) have been studied theoretically. The agreement with the experimental data is discussed.

1. Introduction

The emission of the particles during the fission process can be divided in two stages: firstly the particle can be emitted by the hot compound nucleus during its way to the scission point and secondly, each fission fragment can cool down by evaporation of particles. Nowadays most of the calculation has focused on the correct description of the time evolution of the nucleus in the potential energy surfaces (PES) as it gives the information about the mass and charge distributions of the fission fragments, its mean total kinetic energy (TKE), fission probability and prescission particle multiplicity [1, 2]. The post-scission emission has been discussed for heavy systems [3]. Up to now there are only few experimental

[a]e-mail: Katarzyna.Mazurek@ifj.edu.pl

data for the medium mass nuclei where the pre- and post- scission emission has been estimated and the isotopic distribution of products has been shown.

The hot, rotating system is described within the stochastic approach based on solving the differential coupled Langevin equations in three dimensional space of the deformation parameters. The time evolution of the system can be described within formulas:

$$\frac{dq_i}{dt} = \sum_j \mu_{ij}(\vec{q})p_j$$

$$\frac{dp_i}{dt} = -\frac{1}{2}\sum_{j,k}\frac{d\mu_{ij}(\vec{q})}{dq_i}p_jp_k - \frac{dF(\vec{q})}{dq_i} - \sum_{j,k}\gamma_{ij}(\vec{q})\mu_{ij}(\vec{q})p_k + \sum_j \theta_{ij}(\vec{q})\Gamma_j(t) \quad (1)$$

where $\tilde{\mathbf{q}} = (q_1, q_2, q_3)$ are collective coordinates and $\tilde{\mathbf{p}}$ - the corresponding conjugate momentum. The collective coordinates are derived [1] from the nuclear shape parametrization so-called Funny Hills (c, h, α) [4], which describes the elongation, mass asymmetry and neck parameters. The potential energy enters in the driving potential $F(\vec{q}) = V(\vec{q}) - a(\vec{q})T^2$ which is the Helmholtz free energy for the given shape and temperature $T = \sqrt{E_{int}/a(\vec{q})}$, obtained within a Fermi-gas model according to $T = \sqrt{E_{int}/a(\vec{q})}$ where E_{int} and $a(\vec{q})$ are the intrinsic excitation energy and level-density parameter, respectively. In this work the level-density parameter is taken as a constant but in general it can depend on the nuclear shape as it is proposed by Ignatyuk [5]. The mass tensor $m_{ij}(\vec{q})$ $(\|\mu_{ij}\| = \|m_{ij}\|^{-1})$ is calculated based on the Werner-Wheeler approximation for incompressible irrotational flow [6]. The friction tensor $\gamma_{ij}(\vec{q})$ is derived within the so-called *wall-and-window* one-body dissipation mechanism [7] with the possibility of reducing the strength of the contribution from the *wall* by means of a factor denoted k_s [1, 2]. The diffusion tensor $D_{ij}(\vec{q}) = \theta_{ik}\theta_{kj}$ is derived from Einstein's relation: $D_{ij}(\vec{q}) = \gamma_{ij}(\vec{q})T$, where θ_{ik} is the random force strength tensor. The stochastic nature of the diffusion process is accounted for by the normalized Gaussian white noise term $\Gamma_j(t)$.

Note that the present theoretical approach is not suited to describe neither low-energy fission (as, e.g. induced by neutrons), nor quasi- and fast-fission processes.

2. Congruence/Wigner energy

The potential energy can be obtained in a fully microscopic way by using the mean field theory with for example Skyrme forces or by more phenomenological macroscopic-microscopic method. In the regime of high spins and high temperatures the shell effects are washed out and the interaction between pairs of nucleons is very weak. It allows to employ the approximation to the macroscopic-microscopic approach by avoiding the microscopic corrections and focusing on the main part of the energy of the nucleus, which comes from the macroscopic part. Two macroscopic models have been tested: the Finite Range Liquid Drop Model [8, 9] and Lublin – Strasbourg Drop model [10, 11]. Both of them have parameters fitted to the newest set of experimental masses, the FRLDM has been additionally fitted to fission barriers and the LSD reproduces it very well without fitting. The fission barrier reproduction has been obtained by including a shape-dependence in the Wigner/congruence energy. The congruence energy has been introduced by Myers [12] to reproduce the binding energies for nuclei close to the N=Z. It can have two forms which are derived from common assumptions, but give different results. The isospin dependent exponential expression used with the Liquid Drop models is called *congruence energy* and the linear expansion of this exponential formula employed in FRLDM is traditionally called *Wigner energy*. The deformation dependence assures the doubling of the congruence/Wigner energy in the scission point because the fission fragments isospin is about equal to that the compound nucleus.

The results obtained with LSD model contains the deformation-dependent congruence energy term parameterized according to [13, 14]:

$$E_{cong}^{LSD}(\vec{q}) = W_0^{LSD}(I) B_{cong}(\vec{q}) \ ; \quad W_0^{LSD}(I) = -C_1 \exp(-W_1 |I| / C_1) \tag{2}$$

$$B_{cong}(\vec{q}) = \left(2 - \frac{R_{neck}(\vec{q})}{R_{frag}(\vec{q})} \right) \text{, necked-in shapes, otherwise 1} \tag{3}$$

where $C_1 = 10\,\text{MeV}$ and $W_1 = 42\,\text{MeV}$. The radius $R_{frag}(\vec{q})$ was originally defined [13] as the mean of the effective transverse radii of the two nascent fragments.

The deformation-dependence of the Wigner energy within FRDLM is the following update prescription [15]:

$$E_{Wign}^{FRLDM}(\vec{q}) = (W_0^{FRLDM}(I) + a_0 A^0) B_{Wign}(\vec{q}) \tag{4}$$

$$B_{Wign}(\vec{q}) = \left(1 - \left(\frac{R_{neck}(\vec{q})}{R_{frag}(\vec{q})} \right)^2 \right)^2 + 1 \text{, necked-in shapes, otherwise 1} \tag{5}$$

$$W_0^{FRLDM}(I) = \begin{cases} W_2 \cdot |I| \cdot \left(1 - \frac{C_2}{C_3} |I| \right), |I| \leq 0.35 \\ \\ W_2 \cdot C_2 \cdot C_3, \ |I| > 0.35 \end{cases} \tag{6}$$

where $I = (N - Z)/A$, $a_0 = 2.615\,\text{MeV}$, $W_2 = 38.38\,\text{MeV}$, $C_2 = 0.5$ and $C_3 = 0.35$. The deformation-dependence is determined by the radius $R_{neck}(\vec{q})$ of the neck and the radius $R_{frag}(\vec{q})$ of the nascent light fission fragment. The calculations are performed with the newest set of the FRLDM parameters and the shape dependence of the A^0 term is also taken into account.

3. Results

The discussion of the influence of the congruence/Wigner energy on several physics observables are presented in [16]. The mass and charge distribution, the isotopic distribution of the evaporation residues and the fission fragments for ^{84}Kr + ^{27}Al at 5.9 MeV/nucleon [17] and 10.6 MeV/nucleon [18] have been compared as obtained with LSD model in two cases: when the shape-dependent congruence was taken into account (LSD+C) and without (LSD), and with the FRLDM with (FRLDM+C) and without (FRLDM) the shape-dependent Wigner energy. There exist many pre and postscission neutron data, which are almost mostly been used to study the fission dynamics and e.g. extract the value of viscosity. Yet, in general, only prescission are investigated. The multiplicities of pre and post-scission charged particle emitted from medium mass compound nuclei are investigated very rarely, because of the lack of the experimental data but still they are assumed to be sensitive on the fission barrier heights and the PES details. The biggest influence of the shape-dependent congruence/Wigner energy is expected in the medium-mass nuclei where the saddle point is close to scission and the geometrical factor $B_{Wign/cong} \approx 2$. In the LSD model the congruence energy lowers the fission barriers, and in the FRLDM approach, the Wigner energy increases the fission barrier height. In a static picture this mechanism allows to expect that the mean multiplicity of the particles emitted before fission and obtained with LSD+C. and FRLDM+C. would be similar as far as the barrier heights are approximately equal. This one-dimensional static assumption is not confirmed in the calculations presented below. The results for the evaporation residue and fission channel shows that it is not nescessarily true since it depends on the whole dynamics and not only on fission barriers.

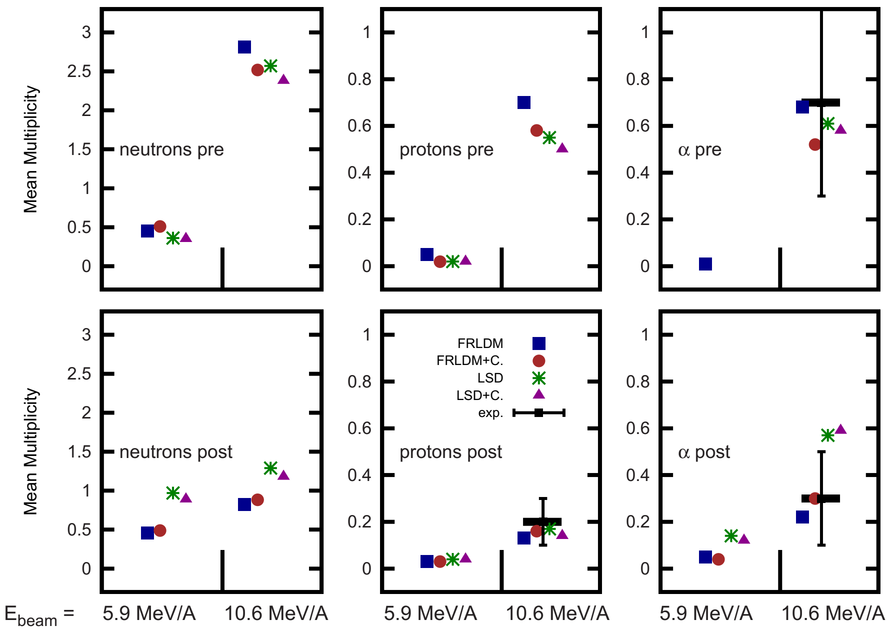

Figure 1. Mean multiplicities for neutrons, protons and α particles emitted pre and post-scission in reaction ^{84}Kr $+^{27}$Al $\rightarrow^{111}$ In for two beam energies: E_{beam} = 5.9 AMeV [17] and E_{beam} = 10.6 AMeV [18], compared with experimental data (black squares) obtained with the potential energy surfaces: FRLDM, FRLDM+C, LSD and LSD+C.

Table 1. The impact of the congruence/Wigner energy on the pre and post-scission particle multiplicities obtained for reactions leading to medium-mass ^{111} In and heavy ^{229}Np nucleus.

	^{84}Kr + ^{27}Al 5.9AMeV		^{84}Kr + ^{27}Al 10.6AMeV		^{20}Ne + ^{209}Bi 7.4AMeV	
	n_{pre}	n_{post}	n_{pre}	n_{post}	n_{pre}	n_{post}
FRLDM	2.81	0.82	0.45	0.46	2.63	3.26
FRLDM+C	2.95	0.88	0.51	0.49	2.83	3.22
LSD	2.91	1.29	0.36	0.97	2.35	3.56
LSD+C	2.77	1.18	0.35	0.89	2.35	3.57

The mean multiplicities for protons and α particles have been measured in the reaction: ^{84}Kr + ^{27}Al $\rightarrow$ ^{111}In for beam energy E_{beam} = 10.6 AMeV. Figure 1 shows the mean multiplicities for neutrons, protons and α particles obtained with our four macroscopic potentials: FRLDM (squares), FRLDM+C (circles), LSD (stairs), LSD+C (triangle) and compared to the experimental data (black squares). The data have been available only for E_{beam} = 10.6 AMeV [18] while for lower energy the isobaric distributions of the evaporation residue has been measured in [16]. The difference between the results obtained with various potentials are visible mainly for α particles and neutrons emitted from fission fragments. The main distinction is however for the LSD and FRLDM itself and the shape-dependent congruence/Wigner energy has a minor influence. Table 1 contains the comparison between the pre and post-scission multiplicities for reaction ^{84}Kr + ^{27}Al $\rightarrow$ ^{111}In with E_{beam} = 5.9 AMeV and E_{beam} = 10.6 AMeV and also the reaction leading to a heavy nucleus ^{20}Ne(7.4 MeV) + ^{209}Bi $\rightarrow$ ^{229}Np, where the pre and post-scission emission of the neutrons has been measured. This data compilation exhibits clearly that the medium mass nuclei are more sensitive to the PES details than

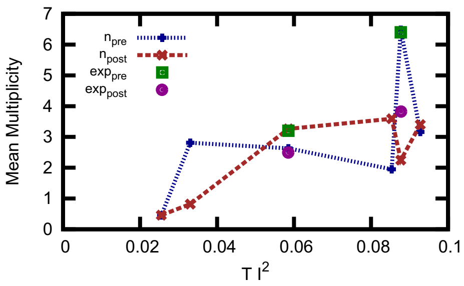

Figure 2. The mean pre and post-scission neutron multiplicities for the reactions leading to ^{111}In, ^{229}Np and ^{260}No compared with experimental data (square- pre and circles – post-scission emission).

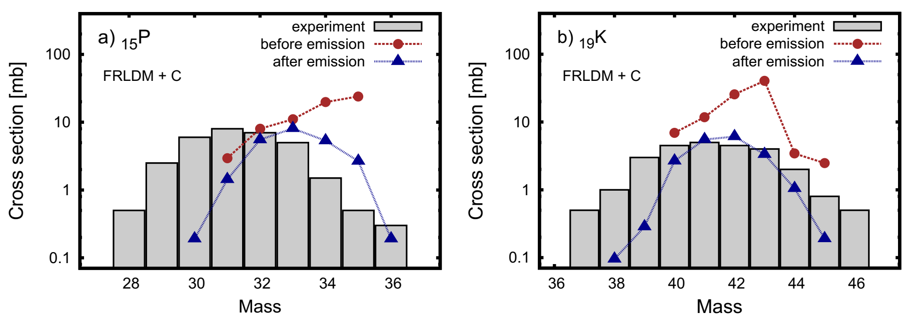

Figure 3. The isotopic distribution of the fission fragments before (red circles) and after (blue triangles) particle evaporation from daughter nuclei of scission ^{111} In produced in the reaction ^{84}Kr + ^{27}Al (10.6 AMeV) compared with experimental data (bars) [18].

heavy ones. It is due to the fact that the PES calculated with LSD and FRLDM are very similar for heavy systems as it was shown in [19] and from the side of congruence/Wigner energy there is not much difference. Heavy nuclei have the saddles far away from the scission, what implies that the values of the shape-dependent factor in congruence/Wigner energy is close to 1. There exist several works reporting on the measurement of the pre and post-scission neutron multiplicities for heavy systems coming from fusion-fission reactions such as ^{22}Ne ($E_{lab} = 6$ and 7.5 AMeV) + ^{238}U → ^{260}No [20] and ^{20}Ne($E_{lab} = 7.4$ AMeV) + ^{209}Bi → ^{229}Np [21]. The comparison between neutron multiplicities experimental and estimated theoretically shown in Figure 2 confirms that not only charged particles are well reproduced within Langevin approach but also neutrons. The calculations for one model (FRLDM+C.), only, are presented in Figure 2. Neutron multiplicities for heavy systems are almost insensitive to PES details (Table 1). For lighter systems, the sensitivity is slightly larger, but the difference between various models would hardly be visible according to the range of the y-scale in the figure. The correct estimation of the mean particle multiplicities evaporated from fission fragments allows to ameliorate the reproduction of the isotopic distribution. In the Figure 3 the change of the isotopic distribution due to emission of particles from the fission fragments is presented for isotopic chains of phosphorus (Z = 15) and potassium (Z = 19) and compared with the measurement [18].

4. Summary

The congruence/Wigner energy should double at the scission point and it helps reproducing the fission barrier heights since it changes the potential energy surfaces at large elongation. However, this does not change fission fragment charge distributions for heavy nuclei. The pre- and post-scission particles depend mainly on the potentials: FRLDM or LSD. The prescission particles multiplicity are more sensitive to congruence/Wigner energy than postscission ones. The inclusion of the post-scission emission allows to well reproduce the isotopic distributions.

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