The existence of triaxial nuclei has been the subject of a long standing debate. The possibility of soft and rigid triaxiality has been proposed very early, and many theoretical and experimental studies have been devoted to this intriguing phenomenon since then. More recently, two unique fingerprints of triaxiality in nuclei have been intensively studied: the wobbling motion [1] and the dynamic chirality [2]. These exotic types of motion were observed in specific regions of the nuclear chart: the wobbling motion in the odd-even Lu nuclei with $A \sim 160$ [3], and the chirality primarily in the odd-odd and odd-even nuclei with $A \sim 130$ [4, 5]. We have recently studied the Nd nuclei up to very high spins and identified several bands, which were interpreted as the manifestation of various types of collective motion: tilted axis rotation, principal axis rotation along the short and long axes, wobbling motion, and chiral bands [7]. Another phenomenon revealed by our recent results on the Nd nuclei with neutron numbers just below the N=82 shell closure, is the shape coexistence. It is induced by the existence of some high-spin seniority isomers which are built on a spherical shape and are surrounded by triaxial bands. This is the case for $^{140}$Nd which has a $^1F^e = 20^+$ isomer at $E_x = 7.43$ MeV with $T_{1/2} = 1.23$ µs, which is surrounded by states based on triaxial shapes [6]. The shape coexistence phenomenon is well described by Cranked Nilsson Strutinsky calculations.

The triaxial shapes are expected at high spins, where aligned high-$j$ orbitals with different driving forces can stabilize a non-axial nuclear shape, characterized by the triaxial parameter $\gamma$. Triaxial bands extending up to very high spins were observed in $^{158}$Er, which were interpreted using Cranked Nilsson-Strutinsky (CNS) calculations as based on the three triaxial strongly deformed minima (TSD) in the potential energy surface, with positive and negative $\gamma$ [8]. However, the calculated transitional quadrupole moment of the lowest minimum with positive $\gamma$ is $Q_2 \approx 7.5$ eb, which underestimate the measured value of $\sim 11$ eb for the yrast band TSD1. This discrepancy could be solved by assigning the band TSD1 to the other minimum with similar quadrupole deformation but negative $\gamma$, or to a third minimum with larger triaxial deformation [8]. However, recent calculations for $^{158}$Er using the shell-correction tilted axis cranking method (SCTAC) [9] and the three-dimensional self-consistent Skyrme-Hartree-Fock version of tilted axis cranking (SHFTAC) [10], which allows the rotational axis to change direction, show that the higher-energy minimum with negative $\gamma$ predicted by CNS calculations is a saddle point. This seems to give an answer to the long-standing question of the physical interpretation of the two triaxial minima with positive and negative $\gamma$ at a very similar quadrupole shape obtained in the principal axis cranking approach.

Bands with stable triaxiality were also observed in the $^{138,139,140}$Nd nuclei [7, 11, 12], and were interpreted using CNS calculations as based on a minimum with a smaller quadrupole deformation than that of the TSD bands and positive $\gamma (\epsilon_2 = 0.25$ and $\gamma \approx +35^\circ$). Very recently, a couple of bands were identified in $^{136}$Nd, presenting a crossing at high spins, which has been consistently interpreted using CNS calculations as due to a switch of the rotational axis from the short to the intermediate axis, or alternatively, a change of triaxiality from positive to negative $\gamma$ [13]. This result aliments the debate on the existence of triaxial minima with positive and negative $\gamma$ predicted by principal axis cranking models, which are not supported by the more sophisticated tilted axis cranking models.

One of the most complete set of bands revealing the triaxiality at medium and high spins has been recently published for $^{135}$Nd [7]. A rich level scheme was constructed including four bands of negative parity at low spins, eight bands of dipole transitions, and eight bands of quadrupole transitions at medium spins.
A partial level scheme is shown in Figure 1. The cranked shell model and the tilted-axis cranking model were used to assign configurations to the observed bands, in calculations without pairing. For selected configurations the case of finite pairing was also considered. The observed bands were interpreted as rotation around the short and long principal axes (quadrupole bands), as well as around a tilted axis (dipole bands). An example of the comparison between calculated and experimental Routhians and single-particle alignments for the comparison between calculated and experimental Routhians around a tilted axis (dipole bands). An example of the short and long principal axes (quadrupole bands), as well observed bands were interpreted as rotation around the tions the case of finite pairing was also considered. The observed bands were interpreted as rotation around the short and long principal axes (quadrupole bands), as well as around a tilted axis (dipole bands). An example of the comparison between calculated and experimental Routhians and single-particle alignments for the ΔI = 2 bands is given in Figure 2, taken from Ref. [7]. The bands L2-L5 and N2-N4 are drawn with respect to the reference band L1 whose configuration involves two aligned \( h_{11/2} \) protons. For these bands the rotation is around the short axis. A similar figure was been drawn for the ΔI = 2 bands constructed above the \( F' = 10^+ \) isomer whose configuration involves two \( h_{11/2} \) neutrons (see Fig. 1). For these bands the rotation is around the long axis. A special attention was given to the interpretation of the yrast \( ΔI = 2 \) band with odd-spins built above the \( F' = 10^+ \) isomer, band L7, which after a careful analysis of all the possible single-particle configurations and extensive RPA calculations, resulted to be a manifestation of the wobbling motion. In the configurations assigned to the 8 dipole bands identified in \(^{138}\)Nd, out of which 6 are present in the partial level scheme shown in Fig. 1, the nucleus rotates about a tilted axis and have an intermediate character, between magnetic and collective electric. A pair of dipole bands, D2 and D3 in Fig. 1 was identified as candidates for chiral partners, for which the rotation is out of the principal planes.

The consistent interpretation of the multitude of observed bands strongly supports the existence of stable triaxial deformation at medium spins in \(^{138}\)Nd and neighboring nuclei. However, a confirmation of the interpretation in terms of triaxial deformation can be obtained by lifetime measurements and the systematic study of high-spin states in nuclei from this mass region.
Figure 2. Experimental and calculated Routhians and single-particle alignments relative to band L1 for the $\Delta I = 2$ bands L2-L5 and N2-N4 of $^{138}$Nd. Parity and signature ($\pi, \alpha$) are indicated by the line type: full (+,0), dash (+,1), dot (-,0), dash-dot (-, 1).

References

[13] C. M. Petrache et al., accepted to Phys. Rev. C.