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Pairing and Blocking in High-K Isomers: Variation of the Collective Parameter g_R.

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Abstract. Using the principle of additivity, the quasi-particle contribution to magnetism in high-K isomers of Lu – Re has been estimated. Based on these estimates band structure branching ratio data is used to explore the behavior of the collective contribution as the number and neutron/proton nature (N_p, N_n), of the quasi-particle excitations, change. A striking systematic variation of the collective g-factor g_R with the difference, $N_p - N_n$, is revealed. Basic ideas of pairing, its quenching by quasi-particle excitation and the consequent changes to moment of inertia and collective magnetism are discussed. The new found systematic behaviour of g_R opens a fresh window on these effects amenable to detailed theoretical investigation.

1. Introduction

Nuclei of elements between Z = 71 (Lu) and Z = 75 (Re) show strong, approximately constant, deformation and have been well described by the axially symmetric deformation model. In this model quasi-particle motion is described as giving rise to a quantum number K, the projection of the quasi-particle angular momentum **J** on the axis of deformation, collective motion being associated with quantum number R which describes rotation about an axis perpendicular to the deformation axis. The spin of each nuclear state is the vector sum, **I** = **J** + **R**. The region typically exhibits rotational bands built upon each quasi-particle state, which forms the band head. The band heads are either single quasi-particles in the deformed potential, or multi-quasi-particle states made up of combinations of single-quasi- particle states. Frequently such multiple states are isomeric, having total K values differing from neighboring states. These states, known as high-K isomers, which have well defined quasi-particle make-up, and the bands built upon them, have been the subject of intense investigation since the first was found in the early 1960's [1]. Both quasi-particle and collective components of the state wavefunction contribute to the magnetic moment of the states, through g-factors g_K and g_R, the total moment being given by the expression (for general state of spin I in a band built upon a band-head of intrinsic spin K)

$$\mu = g_R I + (g_K - g_R)\frac{K^2}{I+1} \quad (1)$$

which for the bandhead, I = K, becomes

$$\mu = g_K \frac{I^2}{I+1} + g_R \frac{I}{I+1} \quad (2)$$

and for band states built on the ground state (K = 0) of even-even nuclei

$$\mu = g_R I \quad (3)$$

Two properties of the band states, the E2/M1 multipole mixing ratio δ in transitions between states of a band having spins differing by one, and the branching ratio λ in the decay of a band state of spin I to lower states in the band having spin I - 1 and spin I - 2 also depend upon both g_K and g_R, through the expressions

$$\frac{\delta^2}{1+\delta^2} = \frac{2K^2(2I-1)}{(I+1)(I-1+K)(I-1-K)}\frac{E^5_{\gamma,\Delta I=1}}{E^5_{\gamma,\Delta I=2}}\lambda \quad (4)$$

and

$$\frac{g_K - g_R}{Q_0} = \frac{0.933 E_{\gamma,\Delta I=1}}{\delta\sqrt{I^2-1}}\ , \quad (5)$$

where Q_0 is the intrinsic quadrupole moment in eb and E_γ in MeV. It has been a widespread feature of analysis of high-K isomers that their g_K factors are used to assist identification of the quasi-particle make-up of the state. To this end, empirical g_{Ki} values of individual quasi-particles of spin K_i are adopted based on magnetic moment measurements on single-quasi-particle band head ($I = K_i$) states of similar deformation in the same region of nuclei. The adopted empirical g_{Ki} values then give estimates for g_K of multi-quasi-particle states of total spin $K = \Sigma K_i$ using the additivity relation

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$$g_K = \frac{1}{K}\sum_i K_i g_{K_i} \ . \qquad (6)$$

There are few measurements of magnetic moments of high-K isomers and relatively few of the required E2/M1 mixing ratios so that usually the estimated g_K values are used, with assumed values of g_R and estimates of Q_0, to compare with measured branching ratios λ as an aid to identification of the quasi-particle configuration. The values taken for g_R have been set within a small range, between 0.25 and 0.35. Variation in g_R, as the number or identity of the quasi-particles involved in the state under consideration changes within a single isotope, has been little discussed [2]. Thus to date, although the valuable physics information contained in the parameter g_R has been recognized, in the field of high K-isomers no systematic investigation has been made (see e.g. [3],[4] and, in particular, [5]).

Classically g_R relates directly to the degree to which the rotating body carries charge, so that, for a nucleus made up of Z protons and N neutrons rotating as a uniform solid body, the simple expectation is $g_R = Z/(Z + N)$ or Z/A. However the fact that nuclear rotation is more complex was recognized from the 1960's when early studies of rotation bands in even-even nuclei yielded moments of inertia (MoI) much reduced from classical body predictions. These reductions were understood as being associated with the phenomenon of pairing, whereby nucleon pairs, treated as bosons, fall into a superfluid state and do not contribute to the MoI. The number of nucleons involved in the superfluid state is a function of the pairing strength. As the rotational frequency increases in higher members of the band, nucleon pairs are broken and the MoI rises towards the classical value [6].

In relation to g_R we note that pairing affects protons and neutrons separately. Furthermore the effect of breaking a pair, as in odd-A nuclei, should block that state from participation in the pairing effect and so weaken the effect of pairing for the nucleon type involved. Thus pairing affects the contributions of protons and neutrons to the MoI and to g_R. Bohr and Mottelson [7] in their landmark text remark that g_R values for single quasi-particle states having a broken proton/neutron pair should be changed compared to the even-even neighbor states. They tabulate evidence, based on data then available, that odd proton single quasi-particle states have higher g_R values and odd neutron single particle states lower g_R values than their even-even neighbors (see [7] Table 5-14, Vol II). Further direct discussion of the variation in g_R has been limited, despite the great activity in study of high-K isomers in the intervening years. Stuchbery et al. [8], focusing on properties of low-spin single-quasiparticle bands, concluded that Coriolis mixing was more important than pairing effects. The present study, by selecting high-spin single and multiple quasiparticle bands, where Coriolis effects are much reduced, allows a systematic, up to date, examination of the importance of pairing.

2. Data Analysis and Results

In the analysis reported in this contribution a different approach has been adopted. High-K isomers involve states having single, multiple and combinations of quasi-neutron and quasi-proton excitations, frequently in the same nucleus and certainly in close neighbors with very similar nuclear deformation. Thus, if good estimates can be made of the g_K parameters of the band heads in these nuclei, g_R values can be extracted from branching ratios and the influence of specific quasi-particle excitations on pairing and the superfluid state explored with a new perspective.

To establish the potential of this approach it is necessary to test the quality of estimates of the g_K parameter. For this we have taken the assumption of additivity as expressed in (6). Comparison is made between estimates

Table 1. Adopted g_{Ki} values for individual quasi-particle states. Experimental moments taken from [10] with g_R = 0.29(5).

Quasi-particle state	Adopted g_{Ki}	Basis of adopted value
Protons		
$5/2^+[402]$	1.67(6)	moments of $5/2^+$ 482 keV state in ^{181}Ta and ground states of ^{181,183,185,187}Re
$7/2^+[404]$	0.765(25)	moments of $7/2^+$ ground states in ^{175,177,179,181}Ta
$9/2^-[514]$	1.37(3)	moment of $9/2^-$ 6 keV state in ^{181}Ta
$11/2^-[505]$	1.281(14)	moment of $11/2^-$ 434 keV state in ^{187}Ir
Neutrons		
$5/2^-[512]$	-0.48(2)	moment of $5/2^-$ ground state of ^{175}Hf
$7/2^-[514]$	0.206(14)	moment of $7/2^-$ ground state of ^{177}Hf
$7/2^-[503]$	-0.319(15)	moment of $7/2^-$ ground state of ^{175}Yb
$7/2^+[633]$	-0.323(18)	moment of $7/2^+$ ground state of ^{175}W
$9/2^+[624]$	-0.239(11)	moment of $9/2^+$ ground state of ^{179}Hf

based on the adopted g_{Ki} values and experimental g-factors measured in multi-quasi-particle states in this region. The chosen g_{Ki} values are given in Table 1, with the basis for their choice. All are derived from measured moments of states in the region. Table 2 compares the estimates derived from additivity with experimental measurements.

Although making this comparison requires use of an estimate of g_R (see Eq. 2), which could give rise to a circular argument, an important feature of the two expressions (2) and (5), allows this procedure to be followed without problems. The weight of the g_R term in the moment expression (2) is less, by a factor K of the state, than the weight of the g_K term, whereas, in (5) the g_R and g_K terms have equal weight. Since K has values of 6 and above in this study, an approximate value of g_R can be used in (2) to estimate the moment and give rise to only a small additional uncertainty, never more than a few %, in the resulting moment prediction.

Table 2 Check for additivity in magnetic moments of multi-quasi-particle states. Kg_K are from Eq. 6 and μ_{est} from Eq. 2 with g_R = 0.29(5). μ_{exp} are taken from [10]. Nilsson labeling: proton states: $5/2^+$[402}, $7/2^+$[404], $9/2^-$[514], $11/2^-$[514]; neutron states $5/2^-$[512], $7/2^-$[503] (1), $7/2^-$[514] (2), $7/2^+$[633], $9/2^+$[624].

Isotope	I^π	q-p configuration		Kg_K	$\mu[\mu_B]$		(est/exp-1)
		protons	neutrons		est	exp	%
^{182}Os	25^+	$9/2^-$, $11/2^-$	$7/2^-$(1), $7/2^-$(2) $7/2^+$, $9/2^+$	10.56(19)	10.43(19)	10.62(20)	- 2(3)
^{182}Re	16^-	$9/2^-$	$9/2^+$, $7/2^-$(1), $7/2^-$(2)	4.74(15)	4.72(15)	3.82(13)	24(4)
^{177}Hf	$37/2^-$	$7/2^+$, $9/2^-$	$5/2^-$, $7/2^-$(2), $9/2^+$	7.29(18)	7.20(18)	7.33(9)	-2(3)
^{177}Lu	$23/2^+$	$7/2^+$	$7/2^-$(2), $9/2^+$	2.32(11)	2.40(11)	2.32(10)	3(6)
^{179}W	$35/2^-$	$5/2^+$, $9/2^-$	$5/2^-$, $7/2^-$(2), $9/2^+$	8.79(22)	8.58(22)	7.5(15)	14(20)
^{176}W	14^+	$7/2^+$, $9/2^-$	$7/2^+$, $5/2^-$	6.51(18)	6.35(18)	6.7(2)	-5(4)
^{178}Hf	16^+	$7/2^+$, $9/2^-$	$7/2^-$(2), $9/2^+$	8.49(18)	8.26(17)	8.16(5)	1(2)
^{182}Re	7^+	$5/2^+$	$9/2^+$	3.09(16)	2.96(15)	2.79(6)	6(6)
^{179}Hf	$25/2^-$	$7/2^+$, $9/2^-$	$9/2^+$	7.68(17)	7.46(17)	7.43(34)	0(5)
^{178}Hf	6^+	$7/2^+$, $5/2^+$		6.85(16)	6.12(15)	5.84(5)	5(3)
^{180}Hf	8^-	$7/2^+$, $9/2^-$		8.84(16)	8.12(15)	8.6(10)	-6(12)
^{172}Hf	8^-	$7/2^+$, $9/2^-$		8.84(16)	8.12(15)	7.96(7)	2(3)
^{172}Hf	6^+	$7/2^+$, $5/2^+$		6.85(16)	6.12(15)	5.5(6)	11(11)
^{174}Hf	6^+	$7/2^+$, $5/2^+$		6.85(16)	6.12(15)	5.35(4)	14(3)
^{176}Hf	6^+	$7/2^+$, $5/2^+$		6.85(16)	6.12(15)	5.75(5)	6(3)

Table 2 shows that the assumption of additivity produces magnetic moment values which deviate from experiment by an average of less than 5%. For the remainder of this work we have used the adopted g_{Ki} values from Table 1 in all estimates of g_K. No previous attempt to use this finding to explore the behavior of g_R, as reported here, has been made.

Having established a reliable method for estimating g_K for multi-quasi-particle states we can use published values of the parameter $|(g_K - g_R)/Q_0|$, which are available from branching ratio measurements in many bands built upon high-K isomeric states, to obtain values of g_R. The necessary choice of sign of this parameter has been made in most cases to eliminate unrealistically high positive or negative values of g_R.

The results are plotted in Fig. 1. Details of the data shown are available on request [9]. The errors in g_R are a compound of uncertainty in g_K, and in the branching ratio. For the intrinsic quadrupole moment Q_0 we have adopted the value 7.2 eb. Other Q_0 values in the literature vary by, at most, 5% from this for all states considered, so giving a relatively small contribution to the final uncertainty in g_R. The results for g_R are grouped and plotted as a function of the variable N_p - N_n the difference between the numbers of proton and neutron quasi-particles involved in the isomer. This variable was chosen since, to first order, the effects of breaking proton

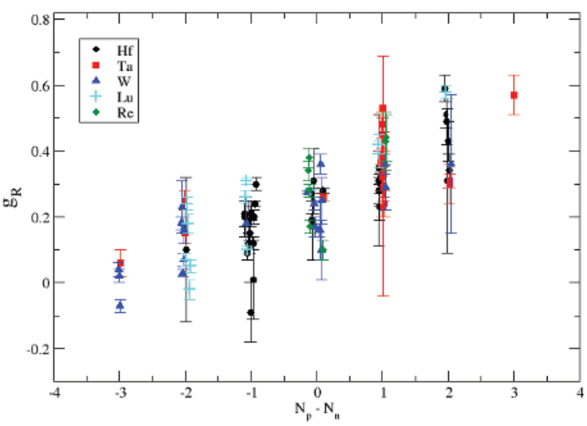

Figure 1 Values of g_R for multi-quasi-particle isomers plotted as a function of $N_p - N_n$.

and neutron pairs are expected to cancel. Fig. 1 shows a clear systematic behavior in g_R, with variation from close to zero to 0.6. As expected the lowest values are found for the greatest excess in numbers of broken neutron pairs and the highest for the most broken proton pairs. The fact that, for example at N_p - N_n = 0, there is considerable variation in g_R, is understood since we do not expect exact cancellation between the effects of breaking different proton and neutron pairs.

3 Discussion

It is not the objective of this paper to offer a quantitative explanation for the variation of g_R that has been revealed by the present approach. Rather we offer the results, and some comparisons with existing empirical and theoretical observations. The variation of the g_R parameter is a challenging new and very direct window into the collective behavior of nucleons in these isotopes, in particular how pairing and superfluidity are modified by the blocking effects of specific proton and neutron excitations.

The first comparison utilizes the basic assumption that the relation

$$g_R = \Im_p/(\Im_p + \Im_n), \qquad (7)$$

where $\Im_{p,n}$ is the proton, neutron contribution to the MoI, holds good. We examine the change found in g_R in states having a single quasi-particle excitation as compared to the quasi-particle vacuum ground state bands of even-even nuclei. Differentiation of g_R with respect to change in $\Im_p$ and $\Im_n$ yields the results

$$\Delta g_R = \frac{\Delta\Im_p(1-g_{R(e-e)})}{\Im_p+\Im_n} \quad \text{for proton excitation and}$$

$$\Delta g_R = g_{R(e-e)}\frac{-\Delta\Im_n}{\Im_p+\Im_n} \quad \text{for neutron excitation} \quad (8).$$

Here $\Delta\Im_p$ and $\Delta\Im_n$ are the changes produced in the MoI in the single quasi-particle nuclei as compared to their even-even (quasi-particle vacuum) neighbors in which $g_{R(e-e)}$ is the 2^+_1 state g-factor. Relevant $\Im_{p,n}$ values are found in Table 5.17 of [7] and the comparison between the predicted changes in g_R and the current analysis is given in Table 3. There is good order of magnitude agreement for both single proton and single neutron excitation, with quantitative agreement for specific single proton excitation.

The second point of discussion compares predictions for g_R derived from existing analyses of quasi-particle excitation energies with the experimental g_R values here extracted from the bands built upon them. Several models have been used to provide values of the pairing gaps for calculation of the MoI for high-K isomers in Hf, Ta and W isotopes, e.g blocked BCS [4] and Lipkin-Nogami [11,12]. Using $\Delta_{p,n}$ obtained from these analyses and the expression

$$\Im_{p,n} = \Im_{rig}\left(1-\frac{\ln[x_{p,n}+(1+x_{p,n}^2)^{1/2}}{x_{p,n}(1+x_{p,n}^2)^{1/2}}\right)$$

where

$$x_{p,n} = \frac{(\delta\hbar\omega_0)_{p,n}}{2\Delta_{p,n}}$$

values of $\Im_n$ and $\Im_p$ and hence prediction for g_R can be obtained.

Table 3. Comparison between changes in g_R, estimated from Eqs. 7, 8 and [7] (column 2) and experiment (this work) for single quasi-particle bands. Values of $g_{R(e-e)}$ are Hf [0.280(7)] and W [0.275(4)].

	Δg_R(MoI)	Δg_R(this work)	Bands analysed in
Odd Protons			
$[404]7/2^+$	+0.07	+0.09(1)	^{175,177}Lu , ^{175,177,179}Ta
$[402]5/2^+$	+0.09	+0.10(4)	^{175,177,179}Ta, ^{185,187}Re
$[514]9/2^-$	+0.13	+0.17(2)	^{175,177,179}Ta
Odd Neutrons			
$[512]5/2^-$	-0.04	-0.16(3)	^{173,175,179}Hf
$[514]7/2^-$	-0.03	-0.16(2)* or -0.03(1)	^{177,179}Hf
$[633]7/2^+$	-0.07	-0.18(2)	^{173,175}Hf
$[624]9/2^+$	-0.09	-0.075(15)	^{177,179}Hf

- depends on choice of sign of $[g_K - g_R]$

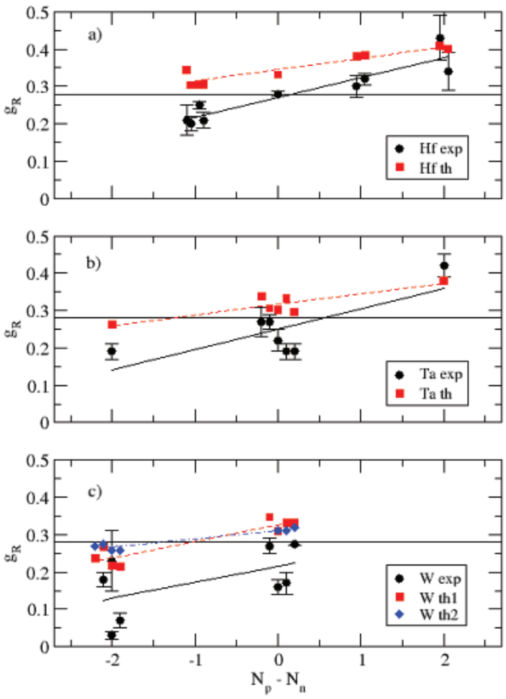

Figure 2 g_R obtained in this work as a function of the number of qps N_p, N_n, in comparison with model calculations [4], [11], [12].

Comparison with experimental g_R values produced by the present analysis in the same Hf, Ta and W isomers is shown in Fig 2. In each panel of the figure the horizontal line is at the quasi-particle vacuum g value. The round data points are this work and the squares and diamonds are obtained using the pairing gaps. The full and dashed sloping lines are least squares fits to the respective data to guide the eye and emphasize the differences between them. Although the trend of the g_R variation with $N_p - N_n$ is correct in all cases, detailed agreement is seen to be poor, the pairing gap data showing too little slope, that is too small a variation of g_R , by about a factor of two.

Summary

This work has used the assumption of additivity in estimation of the intrinsic quasi-particle contribution to magnetism in high-K isomers to extract information regarding the behavior of the collective contribution parameter g_R. A striking systematic behaviour of g_R has been found, showing wider variation than previously appreciated, for large excess of either proton or neutron quasi-particles. The behavior of g_R is closely connected to the question of pairing and the superfluid state of protons and neutrons in nuclei, offering a sensitive new window on this phenomenon previously known in association with reduced moments of inertia. Simple ideas have been shown to give broadly satisfactory qualitative consistency between effects found in the g_R parameter as in the moment of inertia for single quasi-particle excitation. Broader comparison with more detailed theoretical analyses based on various pairing models has been found not to reproduce the g_R results well. It is hoped that this work will stimulate new activity in several directions. It is clear that in many cases better precision in experimental spectroscopic studies would improve our knowledge of the variation of g_R. More and precise direct magnetic moment measurements of high-K isomers would be of great value. However it would appear that existing data should be enough to encourage more theoretical work to reconcile the g_R and moment of inertia results.

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