

Strange Light Nuclei

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Abstract. “Strange” means 1) unusual or surprising, especially in a way that is difficult to explain or understand or 2) having *strangeness* degree of freedom. Light nuclear systems with strangeness, light hypernuclei, are perfect playground to study baryon force which would be a bridge between well established nuclear force in low energy region and QCD, the first principle of the strong interaction. Overview of study of light hypernuclei is given and recent experimental findings are reviewed.

1 Introduction

Nuclear physics aims to understand many-body system interacting by the nuclear force. Based on rich nucleon-nucleon scattering data, nuclear force is well modeled and the nuclear structure can be discussed with the established nuclear force. QCD is the first principle of the strong interaction but direct deduction of nuclear force from it is not yet achieved though recent development of lattice QCD calculation would hopefully bridge between QCD and nuclear force in near future. Extension of the nuclear force to the baryon force by introducing quark degree of freedom other than up and down quarks will help to fill the gap between QCD and the nuclear force. However due to limited data on hyperon-nucleon scattering data, understanding of the YN interaction is much less satisfactory when compared to the NN interaction. Contrary to the NN interaction, the YN interaction has been studied mainly through the investigation of hypernuclear structure. Hypernuclear structure is calculated with the assumed YN interaction models and the results are compared with the experimental data to feed back the information to the YN interaction models. Data of hypernuclei measured with the state-of-art experimental techniques and modern theoretical techniques well tested for normal nuclei, have been serving to develop the YN interaction models. Especially light hypernuclei are important to extract information about the YN interaction while heavier hypernuclear system provides ideal test ground for baryon behavior in nuclear matter.

2 The s-shell hypernuclei in the 20th century

The established lightest hypernucleus is ${}^3_{\Lambda}\text{H}$ and Lambda's binding energies (B_{Λ}) of s-shell hypernuclei (${}^3_{\Lambda}\text{H}$, ${}^4_{\Lambda}\text{H}$ and ${}^5_{\Lambda}\text{He}$) were measured by emulsion [1, 2] in 1960s. At that time, it was already recognized that a simple ΛN potential model cannot explain the experimental data consistently. The measured B_{Λ} 's of s-shell hypernuclei with the limited YN scattering data were analyzed in terms of a

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model which employed phenomenological ΛN and ΛNN potentials [3] but it resulted in over-bound of ${}^5_{\Lambda}\text{He}$ (Fig. 1). The importance of Σ component mixing was pointed out by Refs. [4, 5] to solve the over-bound problem of ${}^5_{\Lambda}\text{He}$. The wavefunction of ${}^4_{\Lambda}\text{He}$ could be written with an explicit inclusion of Σ as:

$$|{}^4_{\Lambda}\text{He}\rangle = \phi_{\Lambda}(r)|{}^3\text{He}\rangle + \sqrt{\frac{2}{3}}\phi_{\Sigma^+}(r)|{}^3\text{H}\rangle - \sqrt{\frac{1}{3}}\phi_{\Sigma^0}(r)|{}^3\text{He}\rangle. \quad (1)$$

The explicit inclusion of Σ component in wavefunctions of Λ hypernuclei ($\Lambda N - \Sigma N$ coupling) solved the ${}^5_{\Lambda}\text{He}$ over-bound problem, but caused under-bound of $A = 4$ systems (${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$) in turn. Akaishi et al. introduced an idea of coherent $\Lambda - \Sigma$ coupling (effective ΛNN three-body force) which is dominant for systems with unbalanced isospin [6] and successfully explained B_{Λ} for $A = 3 - 5$ hypernuclear systems including first excited states of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$ (Fig. 2).

In the 20th century, it was believed that s-shell hypernuclei were reasonably understood.

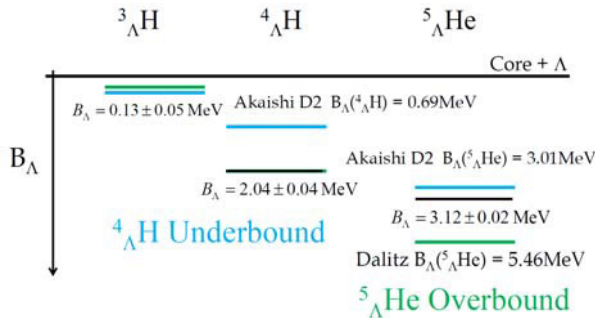


Figure 1. Lambda binding energies of s-shell hypernuclei, ${}^3_{\Lambda}\text{H}$, ${}^4_{\Lambda}\text{H}$ and ${}^5_{\Lambda}\text{He}$.

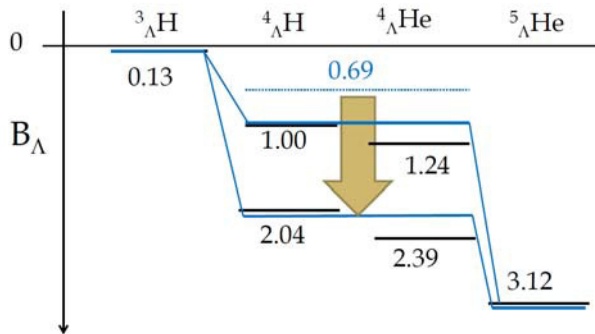


Figure 2. B_{Λ} of s-shell hypernuclei with coherent $\Lambda - \Sigma$ coupling [6]. Unit of numbers is MeV.

3 Charge symmetry breaking effect of the ΛN interaction

Though B_{Λ} 's of s-shell hypernuclei were reasonably understood, there exist small energy differences between ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$. Based on charge symmetry of the nuclear force, the binding energies for them

should be the same. Figure 3 shows closer look of the ground and the first excited states of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$. There exist 350 keV and 240 keV energy differences for 0^+ and 1^+ states of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$. The Coulomb correction due to core nucleus compression by the existence of Λ is expected to be less than 50 keV [7] and thus the difference is not negligible. Binding energy difference between ${}^3\text{H}$ and ${}^3\text{He}$ after the Coulomb correction is $764 - 693 = 71$ keV [8] and thus the charge symmetry breaking effect (CSB) of the ΛN interaction is much larger than it of the NN interaction.

Modern calculation techniques were used to understand the CSB for $A = 4$ hypernuclei, such as the Faddeev-Yakubovsky calculation with explicit admixture of Σ and chiral perturbation theory up to NLO, but comprehensive understanding of 0^+ , 1^+ states of $A = 4$ hypernuclear iso-doublet has not been obtained [9]. The B_{Λ} differences between ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$ were represented by phenomenologically introduced CSB terms in the ΛN potential [7]. Recent progress of cluster calculation enabled to calculate $A = 7$ hypernuclear iso-triplet (${}^7_{\Lambda}\text{He}$, ${}^7_{\Lambda}\text{Li}^*$ and ${}^7_{\Lambda}\text{Be}$) [10]. Since the $A = 7$ hypernuclear iso-triplet was calculated with the same calculation technique used for $A = 4$ hypernuclear iso-doublet therefore consistency between $A = 4$ and $A = 7$ systems can be studied by switching the CSB effect on and off in the calculation.

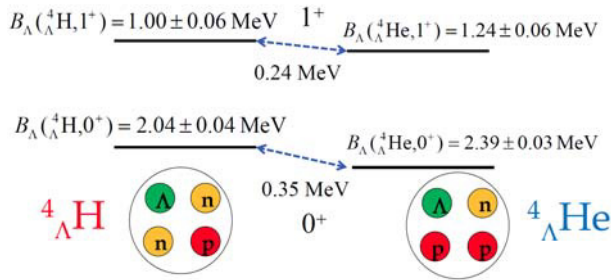


Figure 3. Energy levels for the ground and first excited states of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$.

The ground state energies of ${}^7_{\Lambda}\text{Li}$ and ${}^7_{\Lambda}\text{Be}$ were already measured by emulsion [1] and γ -ray data gives the excitation energy of ${}^7_{\Lambda}\text{Li}^*$ ($T = 1$) [11]. However, B_{Λ} of ${}^7_{\Lambda}\text{He}$ was not yet reported due to limited statistics and scattered data of emulsion [1, 2]. Since ${}^7\text{He}$ target is unavailable, ${}^7_{\Lambda}\text{He}$ cannot be produced by the conventional (K^-, π^-) and (π^+, K^+) reactions which convert neutron to Λ . The $(e, e'K^+)$ reaction which converts proton to Λ produces ${}^7_{\Lambda}\text{He}$ using ${}^7\text{Li}$ target.

In 20th century, hypernuclei were spectroscopically studied extensively at BNL-AGS and KEK-PS by using (K^-, π^-) and (π^+, K^+) reactions. Though the importance of $(e, e'K^+)$ hypernuclear spectroscopy had been recognized, the first experiment had not been performed until 2000. The $(e, e'K^+)$ hypernuclear spectroscopy has advantages such as 1) high energy resolution due to high quality of primary electron beams, 2) absolute energy calibration with $p(e, e'K^+)\Lambda/\Sigma^0$ reaction. However electromagnetic production cross section is 2-3 orders of magnitude smaller than hadronic cross sections, and electron backgrounds due to Bremsstrahlung and Møller scattering are very severe. Coincidence measurement between e' and K^+ limits statistics and continuous beam is essential. In 2000, there was only JLab CEBAF which provided high quality electron beam with an energy of over 1.5 GeV. The pilot experiment E89-009 (HNSS) was performed at JLab Hall-C in 2000 [12, 13]. Then, improved experiments E01-011, E05-115 with newly developed spectrometers HKS and HES in Hall C [14, 15] and E94-107 in Hall-A [16, 17] were carried out. After a decade of efforts at JLab, the $(e, e'K^+)$ hypernuclear spectroscopy from $A = 1 \sim 52$ was established.

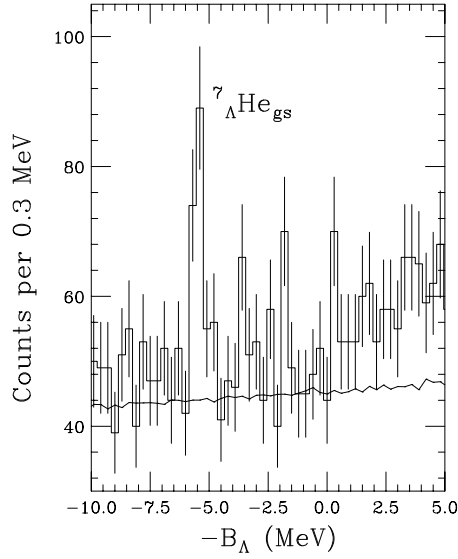


Figure 4. Binding energy spectrum of ${}^7_{\Lambda}\text{He}$ [15]. Ground state peak was clearly observed. There is structure which might correspond to excited states, but statistical significance is not enough to be discussed. Data with more statistics were obtained by the E05-115 experiment and the analysis is in progress.

The last missing B_{Λ} information of $A = 7$ hypernuclear iso-triplet (${}^7_{\Lambda}\text{He}$, ${}^7_{\Lambda}\text{Li}^*$ and ${}^7_{\Lambda}\text{Be}$) was obtained by the ${}^7\text{He}(e, e'K^+){}^7_{\Lambda}\text{He}$ reaction in JLab E01-011 experiment [15]. As shown in Fig. 4, a clear peak corresponding to ${}^7_{\Lambda}\text{He}$ ground state was observed and B_{Λ} was measured as:

$$B_{\Lambda} = 5.68 \pm 0.03(\text{stat}) \pm 0.25(\text{sys}) \text{ MeV}. \quad (2)$$

Figure 5 shows energy levels of $A = 7$ hypernuclear iso-triplet measured by experiments and theoretical calculations with and without the CSB effect. Though the phenomenological CSB potential is necessary to explain energy levels of 0^+ and 1^+ states of the $A = 4$ hypernuclear iso-doublet, inclusion of CSB term makes agreement of experimental results and calculated results worse for the $A = 7$ hypernuclear iso-triplet. It indicates that the assumed CSB potential is too naive or re-measurement of $A = 4$ hypernuclear doublet, ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$, with modern experimental techniques is necessary.

Using the ${}^4\text{He}(e, e'K^+){}^4_{\Lambda}\text{H}$ reaction, the energy levels of 0^+ , 1^+ of ${}^4_{\Lambda}\text{H}$ can be measured and such experiment is now planned at JLab. By using the decay pion spectroscopy technique of electrons-produced hyper-fragments which will be explained later in this article, the ground state energy of ${}^4_{\Lambda}\text{H}$ will be measured with an accuracy of a few 10 keV. The excitation energy of ${}^4_{\Lambda}\text{He}$ (energy difference between 0^+ and 1^+) will be measured by Ge-array (Hyperball-J) with a resolution of a few keV at J-PARC [18]. JLab E05-115 experiment will provide data on ${}^7_{\Lambda}\text{He}$ with higher statistics and ${}^7_{\Lambda}\text{Be}$ which can be studied as $\alpha n \Lambda$ by the same calculation frame work [19, 20]. With these new experimental and theoretical efforts, the CSB effect of the ΛN interaction will be better understood in near future.

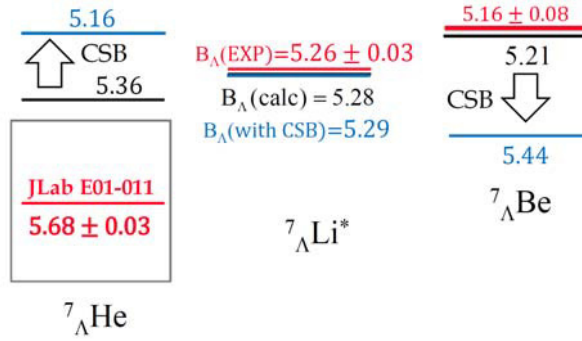


Figure 5. Energy levels for the ground states of $A = 7$ hypernuclear iso-triplet obtained by experiments (red) and theoretical calculations with (blue) and without CSB (black). Unit of numbers is MeV.

4 Two loosely bound neutrons with s-shell hypernucleus core

We have discussed s-shell hypernuclei ($A \leq 5$) which have been considered as already understood well and $A = 7$ iso-triplet which is important for the discussion of the CSB effect. For $A = 6$ system, recent study of ${}^6_{\Lambda}\text{H}$ by FINUDA [21] activates discussion again about light hypernuclei, a system with s-shell hypernucleus core plus two loosely bound neutrons.

FINUDA used ${}^6\text{Li}(\text{K}^-, \pi^+){}^6_{\Lambda}\text{H}$ double charge exchange reaction with stopped K^- from ϕ decay and measured three events which were assigned as ${}^6_{\Lambda}\text{H}$. FINUDA obtained ${}^6_{\Lambda}\text{H}$ mass for the events by two means. One is missing mass with π^+ from the ${}^6\text{Li}(\text{K}^-, \pi^+){}^6_{\Lambda}\text{H}$ and the other is invariant mass with π^- from the weak-decay of ${}^6_{\Lambda}\text{H} \rightarrow {}^6\text{He} + \pi^-$. As we discussed in section 2, the coherent $\Lambda N - \Sigma N$ coupling is important to explain masses of s-shell hypernuclei, additional 1.4 MeV binding energy is expected to be gained from it for ${}^6_{\Lambda}\text{H}$ since the coherent $\Lambda N - \Sigma N$ coupling is significant for neutron rich hypernuclei [23]. Though energies of three ${}^6_{\Lambda}\text{H}$ events measured by FINUDA are scattered and there exist systematic differences between masses obtained from production and decay, the averaged binding energy of ${}^6_{\Lambda}\text{H}$ is obtained as $B_{\Lambda} = 4.0 \pm 1.1$ MeV while Ref. [23, 24] predicted $B_{\Lambda} = 5.8$ MeV (Fig. 6). Reference [22] concluded the ${}^6_{\Lambda}\text{H}$ data disagree the coherent $\Lambda N - \Sigma N$ coupling model's prediction. Recently, similarity between ${}^6_{\Lambda}\text{H}$ and ${}^7_{\Lambda}\text{He}$ were discussed treating ${}^6_{\Lambda}\text{H}$ as ${}^4_{\Lambda}\text{H}$ plus $2n$ and ${}^7_{\Lambda}\text{He}$ as ${}^5_{\Lambda}\text{He}$ plus $2n$; those neutrons are in p-shell and loosely bound [25, 26]. Naively adding such $2n$ to ${}^4_{\Lambda}\text{H}$ and to ${}^5_{\Lambda}\text{He}$ would give similar B_{Λ} changes:

$$B_{\Lambda}({}^6_{\Lambda}\text{H}) \approx B_{\Lambda}({}^7_{\Lambda}\text{He}) - B_{\Lambda}({}^5_{\Lambda}\text{He}) + B_{\Lambda}({}^4_{\Lambda}\text{H}) = 5.68 - 3.12 + 2.04 = 4.60 \text{ MeV}. \quad (3)$$

It is deeper than FINUDA's result of 4.0 MeV, but still consistent within the large error (± 1.1 MeV). However, this kind of phenomenological discussion is very rough and there exists a four body cluster calculation which gives no bound ${}^6_{\Lambda}\text{H}$ [27]. Therefore, we need more experimental data as well as detailed theoretical discussions to draw conclusion about coherent $\Lambda N - \Sigma N$ coupling effects for the system of s-shell hypernucleus core + loosely bound p-shell neutrons. Recently a new experiment [28] with ${}^6\text{Li}(\pi^-, \text{K}^+){}^6_{\Lambda}\text{H}$ double charge exchange reaction was performed at J-PARC. The result did not confirm the FINUDA's result [29] and thus we need further study on ${}^6_{\Lambda}\text{H}$ to make the situation clearer.

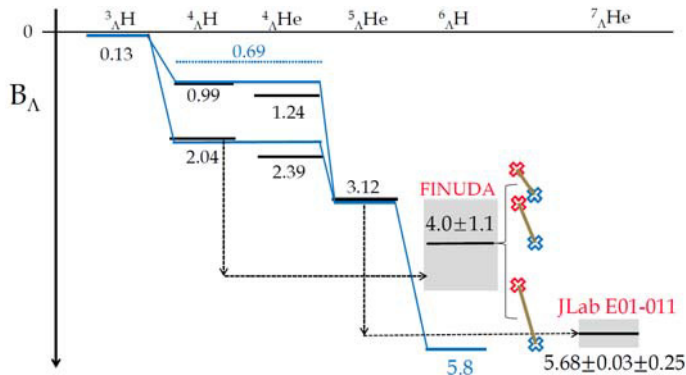


Figure 6. Energy levels for $A = 3 - 7$ hypernuclei including ${}^6_{\Lambda}\text{H}$ by FINUDA experiment at DAΦNE and ${}^7_{\Lambda}\text{He}$ by HKS-HES (E01-011) experiment at JLab. FINUDA obtained the ${}^6_{\Lambda}\text{H}$ mass by averaging of masses measured by production (red crosses) and decay (blue crosses). Unit of numbers is MeV.

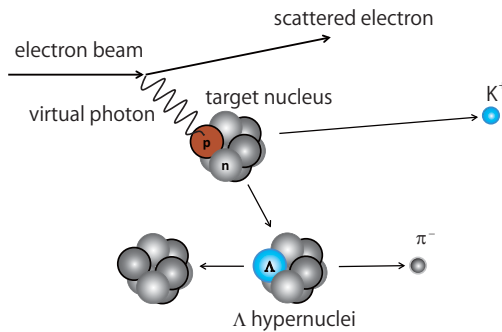


Figure 7. Concept of the decay π^- spectroscopy of electro-produced hypernuclei.

5 Decay π^- spectroscopy of electron-produced hypernuclei

We have discussed mainly on reaction spectroscopy which measures incoming beam and outgoing particles from hypernuclear production to deduce the missing mass of the hypernucleus. However, we can measure decay products from produced hypernuclei. For example, γ -ray spectroscopy of hypernuclei was successfully developed by measuring γ -rays from de-excitation of hypernuclei produced by meson beams [30].

Decay π^- spectroscopy of hypernuclei is a new experimental technique which measures π^- from weak-decay of Λ [31]. Some fraction of produced hypernuclei by any means stops in the target, and then decays by the weak interaction in two-body to π^- and a normal nucleus like:

${}^A_{\Lambda}(Z) \rightarrow {}^A(Z+1) + \pi^-$. Since masses of normal nucleus ${}^A(Z+1)$ and π^- are well known, π^- momentum is enough to deduce the mass of ${}^A_{\Lambda}(Z)$. In principle any reaction can be used to produce the hypernuclei, but target thickness should be optimized; it should be thick enough to stop produced hypernuclei in the target and thin enough to make momentum straggling of π^- in the target small to achieve high resolution. Since electron beams are much more intense than meson beams and a thin target can be

used, it is a good strategy to establish the decay π^- spectroscopy combined with electro-production of hypernuclei (Fig. 7). Important point of this method is that the produced hypernucleus stopped in the target and decay in two-body, so it is not necessary to be directly produced. Electro-produced hypernuclei can make fragmentation and lighter hyper-fragments stop in the target and decay in two-body, then this technique works.

Single monochromatic π^- measurement from weak-decay of hypernuclei would suffer from background associating with decay of normal nuclei, so K^+ will be tagged to guarantee that strangeness is produced. Therefore, this experiment is a coincidence measurement of K^+ and π^- , however, final mass resolution is only determined by the momentum resolution of π^- .

The pilot experiment started already at MAMI-C which can deliver 1.5 GeV high-quality electron beams after recent upgrade. We have quite promising results of decay π^- spectrum with K^- tag from ${}^9\text{Be}$ target and analysis is in progress. We believe that the ground state peak of ${}^4_{\Lambda}\text{H}$ which is important experimental input for the ΛN CSB discussion will be observed in near future.

6 Summary

The s-shell hypernuclei were studied by emulsion and their B_{Λ} 's show importance of the $\Lambda N - \Sigma$ coupling. Introduction of coherent $\Lambda - \Sigma$ coupling (effective ΛNN force) solved long-standing over-bound/under-bound problems of s-shell hypernuclei. However recent experimental result on ${}^6_{\Lambda}\text{H}$ at DAΦNE indicates that coherent $\Lambda - \Sigma$ coupling might not be simply applied to a system of s-shell hypernuclear core with two loosely bound neutrons though the observed three ${}^6_{\Lambda}\text{H}$ events with large errors cannot draw definite conclusion. New experimental result from J-PARC is eagerly awaited.

The charge symmetry breaking effect of the ΛN potential was phenomenologically introduced to explain B_{Λ} differences of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$. With a decade of efforts at JLab, the $(e, e'K^+)$ hypernuclear spectroscopy was established and the binding energy of ${}^7_{\Lambda}\text{He}$ was measured. The $B_{\Lambda}({}^7_{\Lambda}\text{He})$ is the last missing part of $A = 7, T = 1$ hypernuclear iso-triplet of which binding energies can be calculated by recent four-body cluster calculation. Careful comparison of experimental results and theoretical calculation indicates that inclusion of naive CSB term which is necessary for ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$, deteriorates the agreements between experiments and calculation for $A = 7, T = 1$ hypernuclei. Further studies on CSB of ΛN interaction are necessary experimentally as well as theoretically. Study of ${}^4_{\Lambda}\text{H}$ has started at MAMI-C with a new experimental technique, the decay π^- spectroscopy of electro-produced hypernuclei.

At JLab, a new comprehensive program of hypernuclear study from elementary to heavy hypernuclei with the $(e, e'K^+)$ reaction is planned [32]. At J-PARC, a new experiment on ${}^6_{\Lambda}\text{H}$ was carried out and γ -ray spectroscopy of ${}^4_{\Lambda}\text{He}$ is ready to run. Many experimental and theoretical works are on going since light hypernuclei are still strange.

Acknowledgments

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