Abstract. The differential cross-sections of the elastic and inelastic $^{13}$C + $\alpha$ scattering were measured at $E(\alpha) = 65$ MeV. The radii of the states: 8.86 (1/2$^-$), 3.09 (1/2$^+$) and 9.90 (3/2$^-$) MeV were determined by the Modified diffraction model (MDM). The radii of the first two levels are enhanced relatively that of the ground state of $^{13}$C, confirming the suggestion that the 8.86 MeV state could be an analogue of the Hoyle state in $^{12}$C and the 3.09 MeV state has a neutron halo. No enhancement of the radius of the 9.90 MeV state was observed.

1 Introduction

$^{13}$C is a good example of a “normal” nucleus well described by the shell model. Its level scheme is reliably determined up to the excitation energies ~ 10 MeV (see e.g. [1]). However, some new approaches such as the hypothesis [2] of the $\alpha$-particle condensation suggests that cluster states with an enhanced radius can appear. The famous Hoyle state (0$^+_2$, $E^* = 7.65$ MeV) in $^{12}$C was considered as the most probable candidate for having such structure. It was also expected [3] that the analogues of the Hoyle state would reveal themselves in some neighboring nuclei, e.g., the 1/2$^-$ ($E^* = 8.86$MeV) state in $^{13}$C. Our analysis [4] of the $^{13}$C + $\alpha$ scattering data measured at $E(\alpha) = 388$ MeV [5] really demonstrated a considerable enhancement of the radius of this particular state. However, the method of extracting the radii used in Ref. [4] (the Modified diffraction model, MDM [6]) may not quite adequate at high energies (~ 100 MeV) when nuclei are too transparent. Besides, the existence in $^{13}$C of some states with enhanced dimensions but of different structure was discussed as well. Thus, a neutron halo was identified in the first excited state 3.09 MeV (1/2$^+$) by two independent and complementary methods [7, 8]. There are predictions [9] that the 3/2$^-$, 9.90 MeV state and the
members of the rotational band based on it also are diluted. Consequently, new measurements especially at lower energies are highly desirable.

In this paper we studied the inelastic and elastic $^{13}$C + $\alpha$ scattering at $E(\alpha) = 65$ MeV with the aim of measuring the radii of $^{13}$C excited states. In addition to our standard approach using MDM we also analyzed the shifts of the rainbow (Airy) minima in the angular distribution according to the idea proposed in Ref. 10. The main attention was devoted to three $^{13}$C levels: 8.86 MeV (1/2$^-$), 3.09 MeV (1/2$^+$) and 9.90 MeV (3/2$^-$) state.

2 Results and discussion

The experiment was done at the JYFL cyclotron, Finland. A set of the $\Delta E - E$ telescopes was used for detection of the alpha-particles. A self-supporting $^{13}$C target (0.3 mg/cm$^2$) with the 98% enrichment was used. It contained some impurities of $^{12}$C and $^{16}$O. A system of reducing the energy spread of the cyclotron beam down to 0.3% was used. A sample spectrum is shown in Fig. 1(left). The differential cross-sections of $^{13}$C + $\alpha$ elastic scattering are presented in Fig.1 (right) with the results of optical model calculations.

![Figure 1](Image)

**Figure 1.** Left: A sample spectrum ($\theta = 38.5^o$) for the $^{13}$C + $\alpha$ scattering at $E(\alpha) = 65$ MeV. Right: Differential cross-sections of the $^{13}$C + $\alpha$ elastic scattering in ratio to the Rutherford one at 65 MeV together with the results of optical model (OM) calculations (solid curve). The dashed curve corresponds to the far component of the cross-section at zero absorption ($W = 0$). The position of the rainbow minimum is denoted by an arrow.

![Figure 2](Image)

**Figure 2.** Left: Differential cross-sections of the inelastic $^{13}$C + $\alpha$ scattering leading to the 8.86 MeV state in $^{13}$C. The results of DWBA calculations ($L = 0$) are shown by the solid curve. Right: Comparison of the inelastic cross-sections with the excitation of the 8.86 MeV (1/2$^-$) state in $^{13}$C (red points) and 0$^+_2$, 7.65 MeV (Hoyle) state in $^{12}$C (black points). The positions of the rainbow minima are denoted by the arrows.

The measured differential scattering cross-section leading to the excitation of the 1/2$^-$, 8.86 MeV state is presented together with the DWBA calculations in Fig. 2(the left part). The abovementioned
angular distribution is presented in comparison with that for the Hoyle state at the same energy in Fig.2 (the right part). Similar diffraction patterns corresponding to the angular momentum transfer $L = 0$ were observed. The rainbow (Airy) minima were also identified. For the 8.86 MeV state the minimum is located at the angle larger than that in the case of the elastic scattering and smaller than for the Hoyle state. According to Ref. [10] the observed shifts of the Airy minima positions to the larger angles in the inelastic differential cross-sections relative to that of the elastic one indicate an enhancement in the radius of the excited state.

The measured differential cross-sections of the inelastic $^{13}$C + α (65 MeV) scattering leading to the 9.90 MeV and 3.09 MeV states are presented in Fig.3 in the left and right parts correspondingly.

![Figure 3](image.png)

**Figure 3.** Left: Differential cross-sections of the inelastic $^{13}$C + α (65 MeV) scattering to the 9.90 MeV (3/2⁻) state. The results of DWBA calculations are shown by a solid curve for angular momentum transfer $L = 2$. The position of the Airy minimum is denoted by an arrow. Right: The same for the 3.09 MeV (1/2⁺) state with $L = 1$.

The diffraction structure of the measured cross-sections was analysed by MDM, and the radii of all three $^{13}$C excited states were determined (Table 1). The RMS radius of the 8.86 MeV state (2.68±0.10 fm) is a little smaller than the radius of the Hoyle state ($R_{\text{rms}} = 2.89 ± 0.04$ fm [6]). This result is in agreement with previous estimates [4] obtained from the analysis of the literature data. The observed shift of the positions of the Airy minima in $^{12}$C and $^{13}$C confirms the conclusion reached by the MDM analysis. Thus, the obtained results clearly indicate that the 1/2⁻, 8.86 MeV state and the Hoyle state (Fig.2, right) have much in common and probably could be named analogues. On the other hand, the Hoyle state is the head of a rotational band [11] but there is no indication of the existence of the analogue band based on the 8.86 MeV state. This difference may reflect important difference in the structure of the both states.

The radius of the 3.09 MeV state was determined to be $R_{\text{rms}} = 2.92 ± 0.07$ fm. Our previous result obtained from the analysis of some published data at lower energies gave $R_{\text{rms}} = 2.74 ± 0.06$ fm [8]. The position of the Airy minimum was not determined in the present work. However in any case it is located in the range (~ 55° − 70°) sufficiently larger than the corresponding minimum in the elastic scattering to indicate the radius enhancement. A neutron halo radius $R_h$ of the 3.09 MeV state can be determined [12] from the asymptotic normalization coefficients (ANC) extracted by the analysis of the $^{12}$C (d, p) $^{13}$C* (3.09 MeV) reaction. Then $R_h$ can be transformed to the corresponding RMS radius of the state. The details of the method are given in our paper [13]. The RMS radius obtained there

<table>
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<tr>
<th>$E^*$, MeV, I^⁻</th>
<th>$R_{\text{diff}}, \text{fm}$</th>
<th>$R_{\text{rms}}, \text{fm}$</th>
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<tbody>
<tr>
<td>0.00, 1/2⁻</td>
<td>(5.31±0.07)</td>
<td>2.31</td>
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<tr>
<td>3.09, 1/2⁺</td>
<td>(5.96±0.06)</td>
<td>2.92±0.07</td>
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<tr>
<td>8.86, 1/2⁻</td>
<td>(5.66±0.10)</td>
<td>2.68±0.10</td>
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<tr>
<td>9.90, 3/2⁻</td>
<td>(5.00±0.12)</td>
<td>2.00±0.14</td>
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from the analysis of the (d, p)-reactions at different energies is \( R_{\text{rms}} = 2.68 \pm 0.26 \) fm. Thus, all the used methods, MDM, ANC and nuclear rainbow gave (qualitatively) similar results. This finding increases the reliability both of the halo observation in the 3.09 MeV state and the application of the MDM and ANC models for radii determination.

The MDM analysis of the differential cross-section corresponding to the formation of the 9.90 MeV state showed that the predicted radius enhancement for the 9.90 MeV state in \(^{12}\text{C}\) [9] does not take place. As one can see from Fig. 3 (left) the position of the Airy minimum observed in the inelastic cross-section coincides with that in the elastic scattering angular distribution (Fig.1, right) confirming the result obtained by MDM. It is interesting to note that the 9.90 MeV state and the other members of its rotational band are strongly excited in the \( \alpha \)-cluster transfer reactions \(^6\text{Li},d\) and \(^7\text{Li},t\) on \(^9\text{Be}\) [14] while the 8.86 MeV state is not. This means that the \( \alpha \)-cluster structures of the 8.86 and 9.90 MeV states are different: the latter one has a strong \(^9\text{Be} + \alpha\) component which is absent in the 8.86 MeV state. If the 8.86 MeV state really does not form a rotational band this fact would provide evidence enabling its structure to be compared to that of the Hoyle state.

The obtained results demonstrate a co-existence of different structures in \(^{12}\text{C}\). A more elaborate theoretical study of structure of the states under discussion is required.

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References