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Stellar electron-capture rates on nuclei based on Skyrme functionals

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Abstract. In this work, electron-capture rates on nuclei for stellar conditions are calculated for Ni isotopes, using a self-consistent microscopic model based on the finite-temperature Skyrme Hartree-Fock plus finite-temperature charge-exchange random-phase approximation approach. The results of the calculations show that electron-capture rates obtained either with different Skyrme sets or with different available models can differ by up to a few orders of magnitude.

1 Introduction

Weak interaction processes play a crucial role in the life of a star, especially during the late evolutionary stages of massive stars [1, 2]. During the pre-supernova phase, β decays, $(Z,N) \longrightarrow (Z+1,N-1) + e^- + \bar{\nu}_e$, and electron captures (ECs), $(Z,N) + e^- \longrightarrow (Z-1,N+1) + \nu_e$, determine the core entropy and the electron fraction Y_e, which defines the mass of the inner core. When such a core exceeds the Chandrasekhar mass, $M_{\rm Ch} \propto Y_e^2$, it undergoes a gravitational collapse followed by a bounce. This leads to the formation of a shock wave that expels the outer layers of the star and triggers the explosion. During the supernova core collapse, EC on free protons and on exotic nuclei controls the neutronisation phase, until the formation of an almost deleptonised central compact object, the neutron star. While the EC on free protons is reasonably well known, computing this on nuclei requires knowledge of the nuclear structure, and therefore it is not straightforward. ECs and β decays are dominated by Gamow-Teller (GT) and Fermi transitions. For initial $Y_e \approx 0.5$, the EC dominates over β decay, and mainly occurs on nuclei with mass number $A \lesssim 60$ for densities below few 10^{10} g cm^{-3} and temperatures between 300 and 800 keV. Under these conditions, the electron chemical potential μ_e is of the same order of magnitude as the nuclear Q-value, and EC cross sections are sensitive to the details of the GT strength distribution. With increasing density and temperature during collapse, the EC occurs on heavier nuclei and the electron chemical potential is higher than the nuclear Q-value. The EC rates are therefore mainly determined by the centroid and the total GT strength [2, 3].

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Because of their pivotal role in astrophysical applications, weak interactions were extensively investigated within various approaches (see e.g. Refs. [2, 3] and Refs. therein for a review). Recently, mean-field based models have also been used to predict EC cross sections and rates (see e.g. Refs. [4–8]). In the present work, we compute EC rates using the mean-field self-consistent model introduced in Ref. [4], based on the finite-temperature Skyrme Hartree-Fock (FTSHF) and finite-temperature charge-exchange random-phase approximation (FTRPA) approach. We will summarise the framework and the formalism of the FTSHF + FTRPA in Sect. 2, we discuss the results on nickel isotopes in Sect. 3, and we will give our conclusions in Sect. 4.

2 Formalism

We recall here the formalism to calculate the EC cross sections and rates within the FTSHF+FTRPA approach (see Refs. [4, 7] and Refs. therein for details). The FTSHF model is employed to calculate the single-nucleon spectra, occupation probabilities and wave functions of the initial state of target nuclei, and the chemical potential is calculated from the particle conservation equation. To describe the transitions relevant in EC on nuclei, we use the charge-exchange FTRPA in the matrix form. Both in the ground state calculations and in the equations of FTRPA, the same Skyrme functional has been employed. Therefore, the FTSHF + FTRPA model is completely determined by the choice of the nuclear interaction, which is the only input in the calculation. This is the specificity and the advantage of this approach, which can be extended over arbitrary mass regions of the nuclide chart without additional assumptions or adjustment of the parameters.

Iron group nuclei contribute to stellar EC rates in the temperature interval $0.3 \lesssim T \lesssim 2$ MeV. The expression for the total cross section for EC on a nucleus (Z, N) at temperature T is:

$$\sigma(E_e, T) = \frac{G_F^2}{2\pi} \sum_i F(Z, E_e) \frac{(2J_i + 1)e^{-E_i/(kT)}}{G(Z, A, T)} \sum_{f,J} (E_e - Q + E_i - E_f)^2 \frac{|\langle i|\hat{O}_J|f\rangle|^2}{(2J_i + 1)} , \tag{1}$$

where $G_F = G_F/(\hbar c)^2$ is the Fermi coupling constant, E_e is the energy of the incoming electron, Q is the Q-value of the reaction (computed from the experimental masses [9]: $Q = M_f - M_i$, $M_{i,f}$ being the masses of the parent and daughter nucleus respectively), J is the total angular momentum, $\hat{O}_J$ is the generic notation for the multipole operators, $G(Z, A, T)$ is the partition function accounting for the thermal average of levels, and $F(Z, E_e)$ is the Fermi function correcting the cross section for the distortion of the electron wave function by the Coulomb field of the nucleus (see Ref. [4] for details). The finite temperature also induces the thermal population of excited states in the parent nucleus (labelled as "i"), connected by the multipole operators to many levels in the daughter nucleus (labelled as "f"). Since the calculation of all possible transitions is computationally prohibitive, the Brink hypothesis is often adopted (see e.g. Refs. [10–12]). Using this approximation, Eq. (1) becomes [4]:

$$\sigma(E_e, T) = \frac{G_F^2}{2\pi} F(Z, E_e) \sum_f (E_e - Q - \omega_f)^2 \sum_J S_J(\omega_f, T) , \tag{2}$$

where ω_f is the excitation energy in the daughter nucleus, and S_J is the discrete finite-temperature RPA response for the multipole operator $\hat{O}_J$. The EC rate is then computed from the EC cross section [12]:

$$\lambda_{\rm ec}(T) \left[{\rm s}^{-1}\right] = \frac{V_{ud}^2\, g_V^2\, c}{\pi^2(\hbar c)^3} \int_{E_{\rm min}}^{\infty} \sigma(E_e, T)\, E_e\, p_e c\, f_e(E_e)\, {\rm d}E_e , \tag{3}$$

where V_{ud} is the up-down element in the CKM quark mixing matrix, $g_V = 1$ is the weak vector coupling constant, $E_{\rm min}$ is the threshold energy for EC, and $p_e c = (E_e^2 - m_e^2 c^4)^{1/2}$ is the electron momentum. Under stellar conditions encountered in core-collapse supernova, the electron distribution function f_e is well represented by a Fermi-Dirac distribution, $f_e = \left[1 + e^{\frac{E_e - \mu_e}{k_B T}}\right]^{-1}$. The electron chemical potential μ_e is determined from the baryon density ρ by inverting the relation:

$$\rho Y_e = \frac{1}{\pi^2 N_A} \frac{1}{(\hbar c)^3} \int_0^\infty [f_e(E_e) - f_{e^+}(E_e)] \, (p_e c)^2 \, \mathrm{d}(p_e c) \, , \tag{4}$$

where N_A is the Avogadro's number. The positron distribution function, f_{e^+}, is also given by a Fermi-Dirac distribution, with $\mu_{e^+} = -\mu_e$. Since neutrinos are expected to escape from the star at least in the first stage of core collapse, we assume $f_\nu = 0$.

3 Results on Ni isotopes

In this section, we present the results on the EC rates for even-even $^{58-62}$Ni for typical stellar conditions encountered during the pre-supernova and initial phase of core-collapse supernova. In Fig. 1, the FTSHF+FTRPA rates are displayed as a function of temperature for the Skyrme forces SLy4 [13] and BSk17 [14]. For comparison, the results obtained by different theoretical models (shell model with the GXPF1J interaction [15] and deformed Skyrme Hartree-Fock plus quasi-particle RPA [8] taken from Figs. 19, 21, 22 in Ref. [8]), as well as the rates obtained from the experimental GT distributions extracted from (n, p) reactions [16–18] (taken from Figs. 16, 20, 22 in Ref. [19]), are also shown. The trends of the results agree, i.e. the EC rates increase with temperature and electron density. The discrepancy among the various approaches decreases with increasing temperature, as already noticed in Ref. [7] for Fe and Ge isotopes. In particular, for ^{60}Ni, the rates calculated with BSk17 agree reasonably well with those obtained from experimental data and from other calculations. For ^{58}Ni (^{62}Ni), the FTSHF+FTRPA method predicts larger (smaller) rates with respect to those obtained from other approaches or from (n, p) experiments. The inclusion in our calculations of the multipole transitions $J^\pi = 0^\pm, 1^\pm, 2^\pm$ might account for larger rates. Indeed, only the GT transitions have been taken into account in the other theoretical works considered here. Moreover, the EC cross sections, and thus the rates, are very sensitive to the single particle spectra and the GT strength. In our approach, correlations beyond mean field other than 1 particle-1 hole excitations treated at the RPA level, as well as pairing correlations, are neglected. On the other hand, in the calculations by Sarriguren [8], pairing has been included. Indeed, these correlations may have an important impact on the nuclear spectra, especially at low temperatures, where the discrepancies among different methods are more pronounced.

Regarding the rates extracted from the experimental GT, we notice that these are not necessarely the rates encountered in the stellar environment at high ρ and T, as pointed out in Ref. [8]. Moreover, apart from uncertainties inherent to the extraction of the experimental GT strength, the latter is measured in laboratories up to some excitation energies, and thus does not include contributions coming from transitions beyond the measured energy range, which might be important in stellar environments.

Finally, the results obtained with different approaches or different Skyrme interactions deviate by up to a few orders or magnitude. This is in agreement with the conclusions of Ref. [7].

4 Conclusions

We have presented EC rates on even-even $^{58-62}$Ni isotopes, carried out in the microscopic FTSHF+FTRPA approach. The only input is the Skyrme interaction employed; therefore, the method is self-consistent. Comparing the calculations performed with different Skyrme interactions or with various approaches leads to a few orders of magnitude difference on the EC rates.

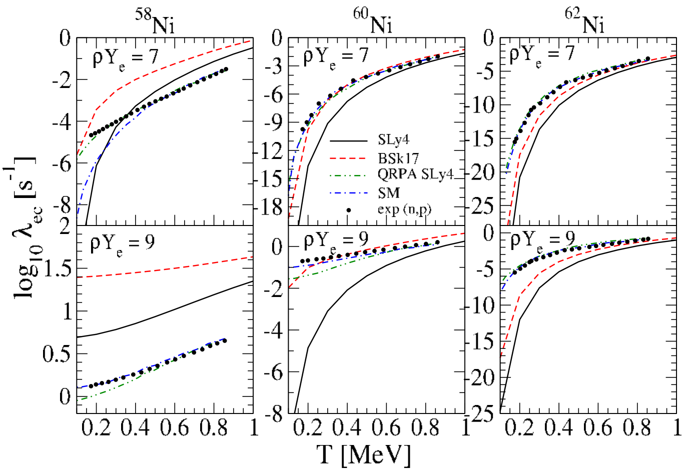

Figure 1. Electron-capture rates as a function of temperature for ^{58,60,62}Ni and for different stellar conditions.

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