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Unified description of shell and cluster coexistence in ^{16}O with a five-body model

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Abstract. A famous and long-standing problem so-called the "mysterious" 0^+ state in ^{16}O is discussed with a single scheme, a ^{12}C plus four-nucleon five-body model. The wave function is expressed in terms of explicitly correlated Gaussian functions. Preliminary results show that the energy levels are in good agreement with experiment. A well-developed ^{12}C+α cluster structure appears in the first excited 0^+ state, whereas the ground state has shell model like structure.

1 Introduction

A cluster structure is one of the most interesting phenomena in the nuclear many-body system and often appears in the excited spectrum around doubly closed shell nuclei. A long-standing problem so-called the "mysterious" 0^+ state in ^{16}O is a famous example of exhibiting both cluster and shell structure in its spectrum. The ground state of ^{16}O has been recognized as having a doubly closed shell model configuration, and thus the first excited state is expected to have the negative parity state in accordance with the particle-hole excitation but actually observed as 0^+ state. Many theoretical works are devoted to understand the mysterious state, however, the state has not been reproduced microscopically even in modern large scale calculations [1]. Only ^{12}C+α cluster model succeeds to describe both the ground and excited states with 0^+ as well as the parity inverted partner of these [2]. It is quite challenging problem to describe such systems in a single scheme, however, the unified description is desired for its deep understanding. Since a fully microscopic calculation is not feasible at present, it is interesting to study ^{16}O with a ^{12}C core plus four-nucleon model, that is without assuming the alpha cluster.

In this contribution, we present our attempt at describing such coexistence with the ^{12}C+four-nucleon five-body model. In the next section, we briefly explain our approach and present preliminary results for ^{16}O as well as ^{16}C in Section 3. Summary and future prospects are drawn in Section 4.

2 Method: A core plus four-nucleon model

A five-body system we consider here is characterized by the Hamiltonian which consists of a nucleon-nucleon ($N-N$) potential for valence nucleons and a ^{12}C-nucleon (C–N) potential. A central Minnesota (MN) potential [3] is employed as the $N-N$ potential. Symmetrized Woods-Saxon and its

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derivative forms are assumed for the C−N potential, and their parameters are determined to reproduce the low-lying spectrum of ^{13}C with $1/2^-$, $1/2^+$, and $5/2^+$. Our C−N potential is deep enough to accommodate some redundant or Pauli-forbidden states. To eliminate such states we impose the orthogonality constraint for the relative motion of the valence nucleons, which is practically achieved by adding a pseudo potential to the Hamiltonian [4]. Here we assume the harmonic oscillator wave functions of $0s_{1/2}$ and $0p_{3/2}$ as the occupied orbits of ^{12}C.

We express the five-body wave function with total angular momentum J in terms of a linear combination of many basis states of the following form

$$\Psi_{JM} = \mathcal{A}\left\{\exp\left[-\tilde{\boldsymbol{x}}A\boldsymbol{x}\right]\left[\left[\mathcal{Y}_{L_1}(\tilde{u}_1\boldsymbol{x})\mathcal{Y}_{L_2}(\tilde{u}_2\boldsymbol{x})\right]_L\chi_S\right]_{JM}\eta_{TM_T}\right\}, \tag{1}$$

where $\mathcal{A}$ is an antisymmetrizer, $\mathcal{Y}_L(\boldsymbol{r})$ is a solid spherical harmonic. The χ_{SM_S} (η_{TM_T}) is the spin (isospin) function of the valence nucleons. The 4×4 matrix A and 4-dimensional vectors u_1 and u_2 are variational parameters to be optimized and a tilde denotes a transpose of a matrix. It is noted that all coordinates are explicitly correlated, which enables us to obtain a precise solution of a many-body Schrödinger equation [5]. An advantage of this method is that its functional form does not change under any coordinate transformation, and thus both cluster- and shell-model like configurations can be expressed in a single scheme. Because of this flexibility the method have been applied to many quantum mechanical systems (See recent review [6]). In particular, the application to the spectrum of ^{4}He is interesting as an analog of the ^{16}O spectrum. Similarly to ^{16}O the first excited state of ^{4}He has $J^\pi = 0^+$ that is shown to have a cluster state of $3N + N$ (^{3}H+p and ^{3}He+n) configuration [7, 8].

Since we have to determine a large number of the variational parameters an efficient algorithm is needed to get the converged solution with limited computational resources. One of the methods is the stochastic variational method [9, 10]. Assuming that we already have some basis states, parameters of the next basis are determined by the following procedure: First we generate several candidates with randomly chosen parameters and select the basis which gives the lowest energy among them, and then include it to the set of bases. We repeat this procedure until the energy is converged.

3 Preliminary results and discussion

Figure 1 displays the calculated energy curves of ^{16}C and the ground and first excited states of ^{16}O as functions of the number of basis functions. The energy of ^{16}C is converged at 7000 basis states and the energy appears very closely to the experiment. The energy curves of ^{16}O are drawn in the right panel and exhibit very slow convergence compared to that of ^{16}C due to the difficulty of eliminating the forbidden states. Figure 2 plots the components of the forbidden states in the $A = 16$ wave functions. The components gradually decrease as increasing the basis dimension. The values are around 10^{-4} at 4000 basis dimension which corresponds to the ground state energy of ^{16}O crosses the ^{12}C+α threshold. Most of basis states are used for removing the forbidden states. The case of ^{16}C is easier than that of ^{16}O because the valence neutrons do not form a strongly bound cluster. This is because, when the valence nucleons form a cluster, the basis states may have a large overlap with the forbidden states, and thus more basis states are needed to get a converged solution. In fact, the components of the forbidden states decreases monotonically beyond 4000 basis in the case of ^{16}C, whereas the two 0^+ states in ^{16}O show some oscillatory behaviors. However, the preliminary results for ^{16}O are promising. The ground state appears below the ^{12}C+α threshold and the first excited state appears just at a few MeV from the ground state, consistently with experiment. The calculation is still being in progress.

It is interesting to see how the cluster structure develops with looking at the expectation values of some Hamiltonian pieces. Figure 3 displays the expectation values of C−N and $N-N$ potentials

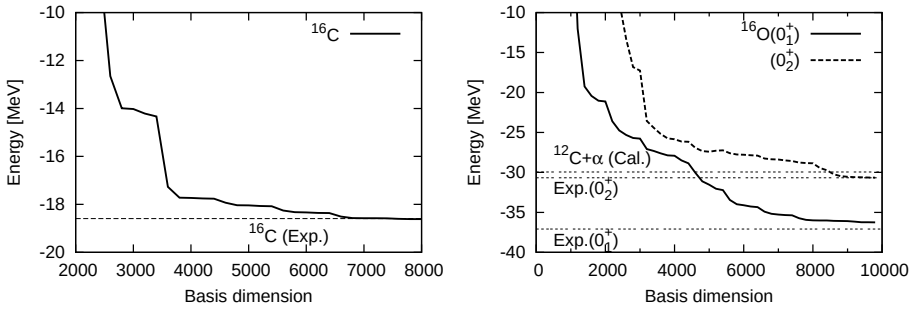

Figure 1. Energy curves for the ground state of ^{16}C (left) and the ground and excited states of ^{16}O (right). The calculated ^{12}C(0^+_1)+α threshold and experimental energies are shown by thin lines.

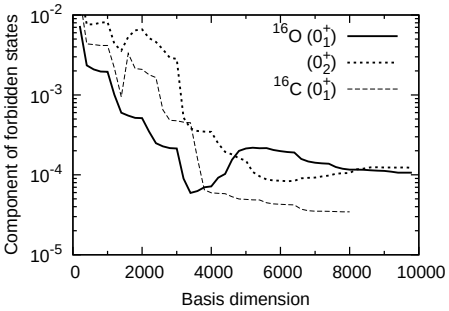

Figure 2. Components of the forbidden states in the $A = 16$ wave functions as a function of basis dimension.

acting among the valence nucleons as a function of the number of basis dimensions. For ^{16}C, the contribution of the C$-N$ potential is very large compared to the $N-N$ contribution, and thus the state can be regarded as having a shell-model like state in which the valence nucleons move almost independently around the core. Reflecting above simple structure, two energy curves of ^{16}C shows a constant behavior with the increasing number of basis. In contrast to this, the curves for the ground state of ^{16}O changes drastically at 4000 to 6000 basis states where the energy crosses over the ^{12}C+α threshold (See Fig. 1). We expect the structure change from the cluster structure to shell-model like structure at this basis dimension. Beyond 8000 basis states where the energies are close to the convergence, the C$-N$ contribution obtained is similar to that of ^{16}C, whereas the $N-N$ contribution is larger. The $N-N$ energy is found approximately a half of the $N-N$ contribution of the free alpha particle (-88.03 MeV) obtained with the MN potential [5]. In the ground state of ^{16}O, the α particle is strongly distorted by attraction of the core nucleus as well as influence of the forbidden states. In the case of the first excited state of ^{16}O, the $N-N$ contribution is dominant and is very close to that of the free alpha particle. This implies that the first excited state has a well developed ^{12}C+α structure as predicted by the cluster model [2]. It should be noted that the expectation value of the C$-N$ potential in the ground states of ^{16}O is almost the same value as the $N-N$ contribution in the first excited state

of ^{16}O. Two different aspects coexist in the spectrum of ^{16}O due to the balance of $N-N$ and C$-N$ contributions.

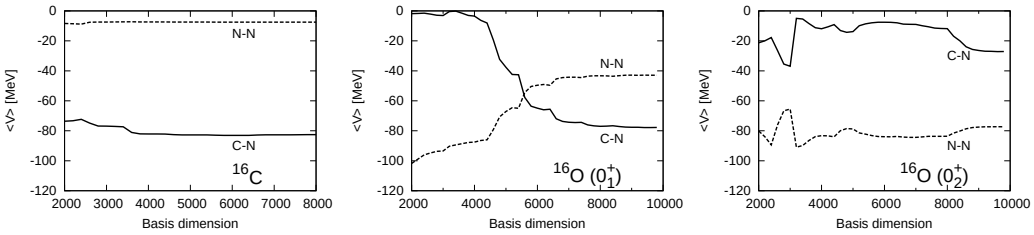

Figure 3. Expectation values of C$-N$ and $N-N$ potentials for the ground state of ^{16}C, the ground state of ^{16}O (0^+_1), and the first excited state of ^{16}O (0^+_2) from left to right, respectively, as functions of the number of basis.

4 Summary and future works

We describe the low-lying 0^+ levels of ^{16}O in a single scheme, a ^{12}C+four-nucleon model. Solving the five-body equation is very hard mainly because we face the difficulty of removing the forbidden states. However, our preliminary results for ^{16}O are promising: The calculated energies for the ground and first excited 0^+ states appear at very close to the experiment. We show the expectation values of the potential terms to see the correlated structure of the systems and find that the ground states of ^{16}C and ^{16}O have a shell-model like state, whereas the first excited state has well-developed ^{12}C+α structure. To make that discussion more quantitative, we will calculate a spectroscopic amplitude (factor) of ^{12}C+α to investigate the cluster degree in the spectrum. The calculations are almost done and will be reported elsewhere soon.

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