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Isospin Symmetry Violation in *sd*-Shell Nuclei

Yi Hua Lam[1,2,a], Nadezda A. Smirnova[1], and Etienne Caurier[3]

[1] *CEN Bordeaux-Gradignan (CNRS/IN2P3 — Université Bordeaux 1), France.*
[2] *Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany.*
[3] *IPHC (CNRS/IN2P3 — Université de Strasbourg), Strasbourg, France.*

Abstract. We construct a new set of isospin-nonconserving (INC) shell-model Hamiltonians and apply it (i) to study the isobaric-mass multiplet equation (IMME) beyond a quadratic form in the $A = 35$, $3/2^+$ quartet, (ii) to compute the isospin mixing corrections to superallowed Fermi branch of $1^+ \to 1^+$ β decay of ^{32}Cl, and (iii) to obtain the isospin-forbidden branching ratios of β-delayed proton emission from ^{25}Si.

1 Introduction

An accurate theoretical description of the isospin mixing in nuclear states has important consequences for various domains of nuclear physics. In nuclear structure, the INC Hamiltonian is important for understanding properties of proton-rich nuclei and it allows to describe the isospin-forbidden transition rates, such as isospin violating nucleon emission and isospin-forbidden electromagnetic transition rates [1]. Besides, theoretical corrections of isospin-symmetry-breaking (ISB) effects to β decay rates are crucial for the tests of the fundamental symmetries underlying the Standard Model [2]. Existing calculations within various microscopic approaches do not agree on the magnitude of isospin impurities and predict very different values for the corrections to nuclear β decay.

Recently, we have constructed a realistic INC shell-model Hamiltonian in *sd* shell. It includes a well-established isospin-conserving Hamiltonian, i.e. USD, or USDA, or USDB interactions [3], a two-body Coulomb interaction and a phenomenological charge-dependent part describing the ISB of the effective nucleon-nucleon interaction. Moreover, four short range correlation (SRC) schemes have been considered [4], i.e. (i) Jastrow-type function with three different sets of parametrizations as proposed in Ref. [5], (ii) unitary correlation operator method (UCOM) [6]. The charge-dependent strengths were determined by a least-squares fit to reproduce newly compiled experimental coefficients of the quadratic IMME [7] with very low root-mean-square (rms) deviation values, i.e. ~32 keV for *b* coefficients (81 data points) and ~9 keV for *c* coefficients (51 data points) [4]. In general, the method of construction follows the earlier work of Ormand and Brown [8], however, it incorporates new features and exceeds the latter by the accuracy in the whole *sd* shell. For the same data selection, the INC interaction of Ref. [8] produces rms deviation of ~76 keV for *b* coefficients and ~12 keV for *c* coefficients. In this contribution, we present a few new applications of the INC Hamiltonians [4].

[a] e-mail: LamYiHua@gmail.com

Table 1. Comparison of theoretical and experimental b, c, and d coefficients of $A = 35,\ 3/2^+$ quartet.

	Quadratic form of IMME b, c (keV) coeff. present work	exp. [7]	Cubic form of IMME b, c, d (keV) coeff. present work	exp. [7]
b	−5870.77	−5890.79 (17)	−5873.44	−5883.46 (89)
c	198.40	202.08 (14)	198.40	203.67 (24)
d	—	—	1.302	−3.39 (41)
χ^2/n	3.0518	69.9170	—	—

[a]Present calculations use V_{coul} (UCOM) and V_0 (USD) combination.
[b]All χ^2/n are calculated by assuming uncertainty ±1 keV for every theoretical mass excess.

2 Results of Application and Discussions

The breaking of the quadratic IMME in $A = 35$ ***quartet.*** Recently, mass measurement of $A = 35$ has been updated with unprecedented high precision [9]. Consequently, we can test whether the quadratic IMME is still valid, or should be extended to a cubic form as

$$M(\alpha, T, T_z) = a(\alpha, T) + b(\alpha, T)T_z + c(\alpha, T)T_z^2 + d(\alpha, T)T_z^3\,, \tag{1}$$

where $\alpha = (A, J^\pi, N_{exc}, \ldots)$ denotes all other quantum numbers, besides the isospin T, necessary to label an isobaric multiplet of states, whereas a, b, c and d are coefficients. The d coefficient is zero for a quadratic IMME. A need for a cubic term may come from isospin-mixing, or be due to the role played by three-body forces. We obtained the $A = 35,\ 3/2^+$ quartet mass excesses from the diagonalization of the newly constructed INC Hamiltonian. The procedure is similar to the study of $A = 32,\ 0^+$ quintet [4, 10]. The isobaric quartet consists of $3/2^+$ ground state for ^{35}K and ^{35}S, the sixth $3/2^+$ for ^{35}Ar and ^{35}Cl. To identify the IAS's in $T_z = \pm 1/2$ nuclei, we calculated the corresponding Fermi matrix elements and compared it to the model independent value. Overall the b and c coefficients are close to experimental values, with only an offset of ~20 keV and ~4 keV, respectively. By comparing the theoretical and experimental IMME d coefficients, both sets of data clearly show that IMME d coefficients is not negligible in obtaining nuclear mass excess, c.f. Table 1. As was shown in Ref. [10], within the INC shell model, the breaking of the isospin of the quadratic IMME is due to the isospin mixing with neighboring states. Results of further analysis will be published elsewhere [11].

Isospin-symmetry breaking and superallowed Fermi β ***decay.*** The constancy of the absolute $\mathscr{F}t$ value of superallowed $0^+ \to 0^+$ β transitions could verify the conserved vector current (CVC) hypothesis. If CVC holds, the $\mathscr{F}t$ value can serve to extract the vector coupling constant for a nuclear beta decay, G_V. Combined with the data from the muon decay, the latter can be used for the unitarity test on the Cabibbo-Kobayashi-Maskawa (CKM) matrix, The three top-row elements of CKM matrix, i.e. V_{ud}, V_{us}, and V_{ub}, provide the most definitive test. The unitary of the first row is largely contributed from the V_{ud}, ~94.9% [2], which can be determined from the relation $G_V = G_F g_V |V_{ud}|$, G_F is the fundamental weak-interaction coupling constant, $g_V = 1$ according to CVC. and G_V is related to $\mathscr{F}t$ via

$$\mathscr{F}t \equiv ft(1 + \delta_R')(1 + \delta_{NS} - \delta_C) = \frac{K}{G_V^2 |M_{FO}|^2 \left(1 + \Delta_V^R\right)}\,, \tag{2}$$

(see Ref. [2, 4] for notations). Recently, the nuclear-structure dependent ISB corrections, δ_C, defined as $|M_F|^2 = |M_{FO}|^2(1 - \delta_C)$, for a $1^+ \to 1^+$ dominant superallowed Fermi branch of ^{32}Cl ground state beta decay has been determined and found to be quite large [12].

Within the shell model, the nuclear structure corrections can with a good level of precision be separated into two parts [2, 13], $\delta_C = \delta_{IM} + \delta_{RO}$, where the first refers to the isospin-symmetry breaking in the mixing of many-body harmonic-oscillator configurations, while the second describes the correction outside the model space (due to the introduction of the realistic tail of the spherically-symmetric basis wave functions). In the present work, we apply the INC Hamiltonian from Ref. [4] to calculate the isospin-mixing part of the nuclear structure correction, δ_{IM}, to Fermi β decay branch of ^{32}Cl ($1^+ \to 1^+$). Table 2 shows the comparison of the present results with those of Ref. [12]. The major difference in our approach and that of Ref. [12] is that their INC Hamiltonian strengths were adjusted case-by-case for each considered multiplet to reproduce the isobaric mass splitting, as well as, some truncations of *sd* shell-model space have been imposed [14], while in the present work a global parametrization of the Hamiltonian in a full *sd* shell-model space.

We obtain the unscaled value of δ_{IM} with the USD interaction 50% higher than that of Ref. [12], while it is 3 times smaller than that of Ref. [12] for USDA and USDB based INC Hamiltonians. As pointed out in Ref. [2, 12], δ_{IM} is proportional to $1/\Delta E^2$, where ΔE is the energy difference between two admixed states. We find those theoretical energy differences ΔE_{theo} equal to 76 keV (USD), 203 keV (USDA) and 297 keV (USDB), which is different from Ref. [12]. The scaled values of δ_{IM} obtained in the present work are quite close to each other for different interactions and represent the highest isospin-mixing part of the nuclear structure correction when compared to superallowed $0^+ \to 0^+$ transitions. However, they are much lower than those found in Ref. [12], probably, due to differences in our approaches. Detailed results and further analysis will be published elsewhere [11].

Isospin-Forbidden Proton Emission from ^{25}**Si.** This proton-rich nucleus decays by a β^+ emission to the excited states of ^{25}Al. The strongest branch is the Fermi decay to a state at 7.89 MeV excitation energy, an isobar-analogue state (IAS) of ^{25}Si ground state. Four proton branches from this IAS to the ground and three lowest states of ^{24}Mg at most have been observed in experiment [15]. We use the present INC Hamiltonian to calculate the spectroscopic factors for the proton decay width, $\Gamma_p(l) = 2\gamma^2\theta_l^2 P_l(Q_p)$, where γ^2 is the Wigner single-particle width, $\gamma^2 = 3\hbar^2c^2/(2\mu R_0^2)$, $P_l(Q_p)$ is the penetrability, Q_p is the Q-value for the proton emission (see Ref. [16] for details of a similar calculation). Branching ratios from shell model shows a good agreement with the experimental branching ratios, c.f. Table 3. A complete analysis of the decay scheme of ^{25}Si will be published elsewhere soon [11]. Further work on other precursors may provide more information of the INC Hamiltonian properties.

Table 2. Comparison of δ_{IM} for ^{32}Cl superallowed β decay.

	Present work			Ref. [12]		
δ_{IM} (%)	USD	USDA	USDB	USD	USDA	USDB
unscaled	5.2614	0.6537	0.3825	3.90	1.91	0.99
scaled	0.8581	0.7606	0.9525	3.73	3.32	4.19

[a] δ_{IM} is referred to as δ_{C1} in Refs. [2, 12].
[b] Scaled δ_{IM} values in the second row are δ_{IM} in the first row multiply with the respective factor $(\Delta E)^2_{theo}/(\Delta E)^2_{exp}$, where ΔE is the energy separation of the analogue and the nearby nonanalogue 1^+ states.

3 Conclusions

We constructed a new set of empirical INC Hamiltonians in *sd*-shell model space which provides an accurate theoretical description of isospin mixing in nuclear states. We will explore Coulomb potential based on the Woods-Saxon basis. Other applications of the INC Hamiltonians will be explored, and

Table 3. Branching ratios of β-delayed proton emission from ^{25}Si.

^{24}Mg J^{π}	E_{exc} (MeV) E_{exp}	E_{theo}	$E_{c.m.}$ (MeV)	$10^4\theta^2$ $l=0$	$l=2$	Γ_p (keV)	Branching ratios (%) Present work	Ref. [15]
0^+	0.000	0.000 (0)	5.624 (3)	0.0000 (0)	0.0595 (37)	11.28 (71)	10.05 (56)	18.60
2_1^+	1.369	1.495 (1)	4.252 (2)	0.0421 (100)	0.8220 (583)	94.19 (1178)	83.03 (153)	74.40
4_1^+	4.123	4.347 (4)	1.489 (7)	0.0000 (0)	0.0955 (85)	0.07 (1)	0.06 (0)	3.74
2_2^+	4.238	4.116 (5)	1.377 (6)	0.2333 (188)	0.0326 (43)	7.58 (61)	6.85 (100)	3.20
3_1^+	5.235	5.060 (5)	0.389 (5)	0.0850 (50)	0.2708 (282)	0.00 (0)	0.00 (0)	—

[a]Present calculations use V_{coul} and V_0 (USDB) combination.
[b]The error bars are provided from the standard deviation based on different SRC schemes.
[c]The E_{exp} of 3_1^+ is quoted from Ref. [17].

other fitting methods will be tested. The *psd*, *sdpf*, *pf*, and *pfg*-shell spaces' INC Hamiltonians are under construction.

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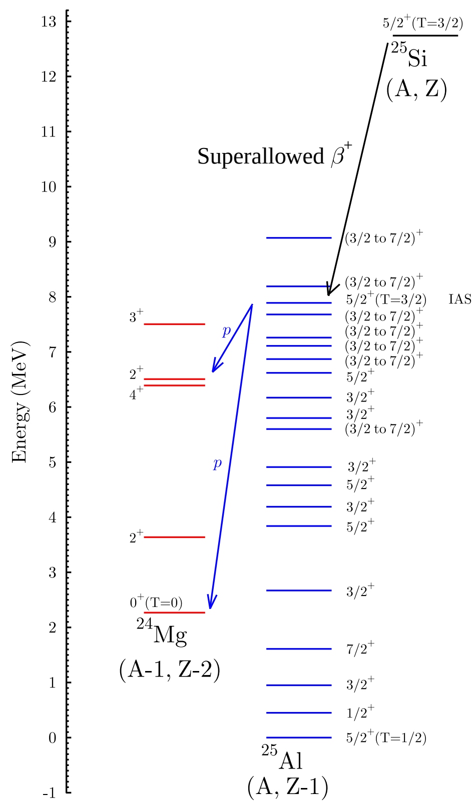

Fig. 1. Partial decay scheme of β-delayed proton emission from ^{25}Si, c.f. Ref. [15]