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Comparison of multi-$\hbar\omega$ shell-model results with MCAS

J. P. Svenne[1,a], S. Karataglidis[2,3,b], K. Amos[3,2], L. Canton[4], P. R. Fraser[4,3], and D. van der Knijff[3]

[1] *Department of Physics and Astronomy, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2*
[2] *Department of Physics, University of Johannesburg, South Africa*
[3] *School of Physics, University of Melbourne, Victoria 3010, Australia*
[4] *Istituto Nazionale di Fisica Nucleare, Sezione di Padova, I-35131, Italy*

Abstract. A multi-channel algebraic scattering (MCAS) method has been used to obtain spectra of a number of light-mass nuclei, which are treated as a two-cluster system, here specifically a nucleon plus nucleus. To date, collective models have been used to specify the interactions between the nucleon and low-lying states of the nucleus that form the compound. For the case of the carbon isotopes, these studies have been complemented by sufficiently complex and complete shell-model calculations. Comparisons with the multi-$\hbar\omega$ shell-model results provide new insights into the validity of those from MCAS.

1 Introduction

MCAS is a multichannel algebraic scattering method [1] which has been used to obtain spectra of a number of light-mass nuclei, treating them as two-cluster systems, specifically a nucleon plus nucleus in the cases discussed herein. The MCAS method gives both sub-threshold and resonance states of the nuclei in question. To date, collective models have been used to specify the interactions between the nucleon and low-lying states of the nucleus that form the compound. In 2006, we published [2] results of MCAS calculations on the mirror system ^{15}C (as n + ^{14}C) and ^{15}F (as p + ^{14}O). For the ^{15}C calculations, parameters for the neutron−nucleus potentials were chosen to give a good description of the low-excitation energy levels of ^{15}C. A Coulomb potential was then added to that nuclear potential to describe properties of the $p+{}^{14}$O cluster. As well as fitting the two known resonances in ^{15}F, narrow resonances above those were predicted by the MCAS calculations. In 2009, experimental results were published by Mukha, *et al.* [3] , which showed the existence of such higher-energy narrow resonances in the energy region of our results. Thus, the method is shown to have predictive power.

For the case of the carbon isotopes, these studies have been complemented by sufficiently complex and complete shell-model (SM) calculations. Our MCAS mass-17 results [4], presented in sect. 2, prompted a paper [5] comparing our results with those of an adapted $0\hbar\omega$ shell model with three neutrons occupying the *sd* shell outside of an inert closed core of ^{14}C. The conjecture that the results of such a limited model would imply deficiencies in the MCAS method has been thoroughly refuted in ref. [6]. As further support, we consider here results obtained from shell-model calculations that are based on a complete $(0 + 2)\hbar\omega$ shell model. We conclude that a plain comparison and interpretation

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[a] e-mail: svenne@physics.umanitoba.ca
[b] e-mail: stevenka@uj.ac.za

of our mass-17 results with a $0\hbar\omega$ shell-model picture leads to serious misinterpretations. A more extensive publication on comparisons between MCAS and SM calculations is in preparation.

2 MCAS results for Mass-17 systems

The ^{17}C (n+^{16}C) system is studied with a rotational model prescription of the matrix of interaction potentials connecting three target states [the $0^+_{g.s.}(0)$, the 2^+ (1.77 MeV), and the 4^+ (4.14 MeV) states]. Higher-energy target states are expected to have little influence on the low-energy results displayed in fig. 1. The energy scale is set with the n + ^{16}C threshold as zero. The first column (labelled (0+2)) shows results published in an earlier paper [7] , where only two target states, the 0^+ and 2^+ ones, were included. Clearly, the 4^+ target state is essential to get the three lowest, bound, states of ^{17}C in the correct sequence, and nearly the correct spacing, compared to data. Also, some of the higher excited states are better described when three target states are included. Two different experimental data sets are included in the last column by the solid and dot-dashed horizontal lines. The numbers designate twice the spin of the states, and all states shown in this energy range have positive parity.

^{17}Na (p+^{16}Ne) is the mirror system to ^{17}C (n+^{16}C). We carry out MCAS calculations on this by using the same nuclear-force model and parameter set as those for ^{17}C and add a Coulomb potential derived from a deformed Woods-Saxon charge distribution having the same parameters (geometry and deformation) as the nuclear potential. The resulting spectrum for ^{17}Na is that labelled MCAS in fig. 2. ^{17}Na has not been observed, but is expected to decay by single proton emission or breakup into ^{14}O +3p. Shown in the second column, labelled 'T+D', is another theoretical spectrum, from [8], obtained using a microscopic cluster model. In both calculations, all states found are resonances, and their widths are also obtained. There are significant differences in the widths obtained from the two calculations. New experimental information on exotic nuclei, such as ^{17}C and ^{17}Na, would test the models.

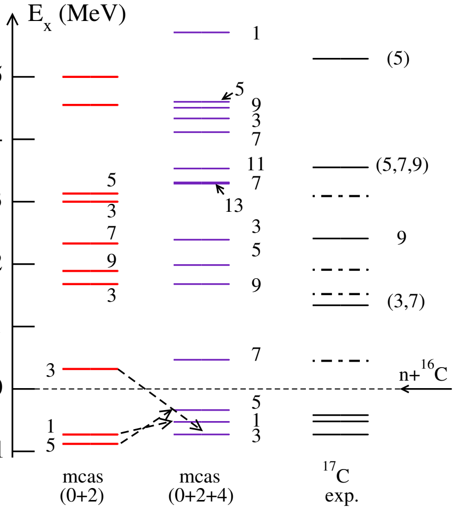

Figure 1. Spectra of ^{17}C, experiment and results from two MCAS calculations [4].

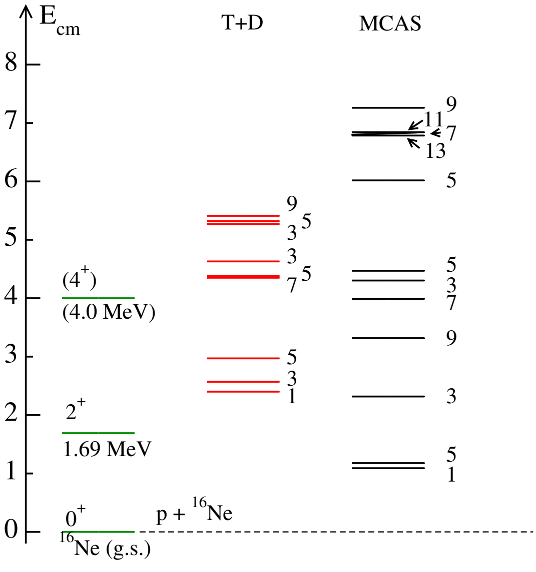

Figure 2. Predicted spectrum of ^{17}Na from MCAS [4] and an earlier publication [8].

3 Shell-model results for Carbon Isotopes

Comparisons with the shell-model results provide new insights into the validity of those from MCAS. We seek to describe $^{12-18}$C self-consistently using the same basis and, where possible, the same space and shell potential. The shell model we consider is that specified in the $spsdpf$ model space and $(0+2)\hbar\omega$ which is complete for ^{12,14}C, while for $^{16-18}$C the space is truncated of necessity. That truncation, for the $(0+2)\hbar\omega$ calculations, is the exclusion of the $0g1d2s$ shell; a shell required for a complete evaluation of $1p-1h$ excitations by $2\hbar\omega$ from the sd shell. For ^{15}C, the presence of the single neutron outside the $0p$-shell requires that the ground state and positive-parity excited states be calculated in a $(1+3)\hbar\omega$ model space, for which the truncation of the basis is more severe with regards to the $3\hbar\omega$ components. All calculations have been made using the OXBASH shell-model program [9] with two interactions: the WBP [10] and the MK3W [11] one as a comparison.

A first observation is that these models give ground state wave functions for the even mass isotopes, ^{12}C and ^{14}C, that have appreciable $2\hbar\omega$ components, namely, using the WBP interaction,

$$\left|^{12}\mathrm{C}\right\rangle = 87.0\%\,|0\hbar\omega\rangle + 13.0\%\,|2\hbar\omega\rangle \;\;;\;\; \left|^{14}\mathrm{C}\right\rangle = 84.9\%\,|0\hbar\omega\rangle + 15.1\%\,|2\hbar\omega\rangle\;. \tag{1}$$

The ground state of ^{15}C is given by

$$\left|\tfrac{1}{2}^{+};\tfrac{3}{2}\right\rangle_1 = 78.1\%\,|1\hbar\omega\rangle + 21.9\%\,|3\hbar\omega\rangle\ \text{(MK3W)} \;\;;\;\; = 84.5\%\,|1\hbar\omega\rangle + 15.5\%\,|3\hbar\omega\rangle\,\text{(WBP)}, \tag{2}$$

coming from the addition of the single sd-shell neutron to the ground state of ^{14}C. The negative-parity states in ^{15}C come from the $(2+4)\hbar\omega$ model space, with ($\sim$ 85%) being $2\hbar\omega$ in character, consistent with the heavier isotopes. The ground states of $^{16-18}$C in this model are purely $2\hbar\omega$ in character.

The MCAS collaboration is gradually moving to higher-mass systems, and systems of particular current interest to astrophysics; see also the article by Fraser, *et al* elsewhere in these proceedings. To aid in interpreting the spectra obtained from MCAS beyond arguments based on too simplistic models, we turn to complete $(0+2)\hbar\omega$ and $(1+3)\hbar\omega$ shell model (SM) results for $^{12-18}$C isotopes. Full results will be presented in an upcoming publication; here we show only some sample results. The first is the spectrum of ^{15}C. In fig. 3 the known low-energy spectrum of ^{15}C is compared to ones found using two SM calculations. In SM calculations for ^{17}C, the level diagram, fig. 4, is arranged according to spin values, up to spin $\frac{13}{2}^{+}$.

Any model of structure that specifies states being an inert core plus a set of neutrons inherently has zero $B(E2)$ values. To match experiment, those models must invoke an effective charge to the active neutrons. That is an admission that the model of structure does not specify the states of the nucleus adequately, and that the protons in the nucleus are not inactive. $B(E2)$ values, as calculated without polarization charge from both the MK3W and WBP shell model interactions, for excitation of the 2^+_1 states in the ^{12,14,16,18}C isotopes, have been obtained. As an example, for ^{16}C, the carbon isotope relevant to the mass-17 study of sec. 2, the $B(E2)$ values are: 3.1(6) (exp.), and 1.76 (SM: WBP). Thus, only a small polarization charge (0.05) is required to match theory to this measured $B(E2)$ value for ^{16}C.

4 Conclusions

Our shell-model results show that the proton core in the Carbon isotopes is not passive. All shell-model states have components with 1 or 2 protons in the $0p_{\frac{1}{2}}$ sub-shell so that those proton aspects can contribute significantly to the 2^+ excitation of the core of ^{14}C. Adding the latter to the angular momenta allowable with, for example, three $0d_{\frac{5}{2}}$ neutrons, results in combinations with angular momenta

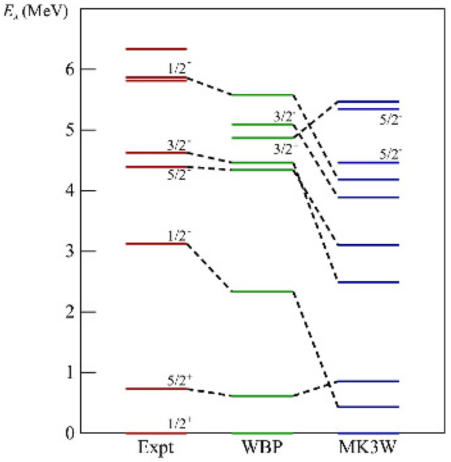

Figure 3. Shell-model results for ^{15}C, using two different interactions, compared to experiment.

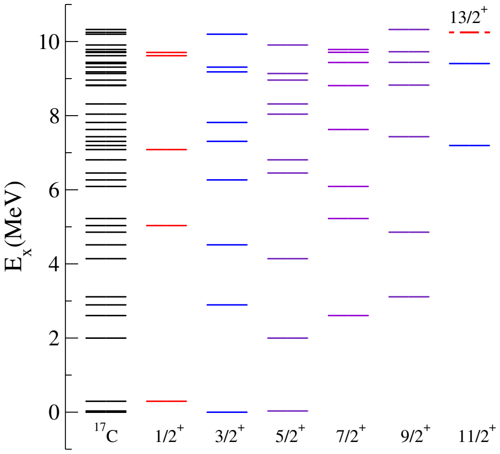

Figure 4. Level diagram to 10 MeV from shell-model calculations for ^{17}C.

ranging from $\frac{1}{2}^+$ to $\frac{13}{2}^+$ inclusive. That range of states is found with the coupled cluster approach and, in general, in order and at similar excitation energies. If some more basic SM calculations cannot include the 2^+ excited state of the of ^{14}C core, that would imply the absence of $\frac{11}{2}^+$ and $\frac{13}{2}^+$ states in the corresponding low-energy spectrum of ^{17}C. Thus the possible presence of those states also in the MCAS results, cannot be ascribed to a “violation of the Pauli principle” in the theory, as claimed erroneously in ref. [5]. In fact, it points to a too severe truncation implied by that $0\hbar\omega$ shell-model study of ref. [5].

References

[1] K. Amos *et al.*, Nucl. Phys. **A728**, 65 (2003).
[2] L. Canton *et al.*, Phys. Rev. Lett. **96**, 072502 (2006).
[3] I. Mukha *et al.*, Phys. Rev. C **79**, 061301 (R) (2009).
[4] K. Amos *et al.*, Nucl. Phys. **A879**, 132 (2012).
[5] H.T. Fortune, Nucl. Phys. **A890**, 25 (2012).
[6] K. Amos *et al.* Nucl. Phys. **A912** 7 (2013).
[7] S. Karataglidis *et al.*, Nucl. Phys. **A813**, 235 (2008).
[8] N.K. Timofeyuk and P. Descouvemont, Phys. Rev. C **81**, 051301 (2010).
[9] A. Etchegoyen *et al.*, MSUCL Report 524 (1986).
[10] E.K. Warburton & B.A. Brown, Phys. Rev. C **46**, 923 (1992).
[11] E.K. Warburton & D.J. Millener, Phys. Rev. C **39**, 1120 (1989).