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# Centrifugal stretching of $^{170}$Hf in the interacting boson model

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**Abstract.** We present the results of a recent experiment to deduce lifetimes of members of the ground state rotational band of $^{170}$Hf, which show the effect of centrifugal stretching in this deformed isotope. Results are compared to the geometrical confined beta-soft (CBS) rotor model, as well as to the interacting boson model (IBM). Two methods to correct for effects due to the finite valence space within the IBM are proposed.

## 1 Introduction

The nuclear shape as a function of nucleon numbers, as well as a function of spin or excitation mode, is important for our understanding of the nuclear many-body system, since it directly reflects details of the mean-field resulting from nucleon-nucleon interactions. The geometrical nuclear model [1] introduced shape parameters $\beta$ and $\gamma$ which are typically used to describe rigid ellipsoidal shapes [2], while effective values can be defined in soft cases [3]. Considering the atomic nucleus in a liquid drop model, one may expect that its long axis stretches with increasing spin due to centrifugal forces. This effect is in fact predicted within the confined beta-soft (CBS) rotor model [4], which uses an infinite square-well potential in $\beta$, based on the X(5) work of Iachello [5], but allows the inner wall of the potential to approach the outer wall. That way, one varies between a beta-soft wide potential well and a delta function at a given deformation value, hence, a beta-rigid potential. Within the CBS model, the centroid of the wave function density of states shifts as a function of increasing spin towards the outer wall of the square well potential, i.e. to larger deformation values. The result is a centrifugal stretching of the nucleus, which can be observed as an increase moments of inertia, and of transition quadrupole moments

$$Q_t(J) = \sqrt{\frac{16\pi}{5}} \frac{\sqrt{B(E2; J \to J-2)}}{\langle J, 0, 2, 0 | J-2, 0 \rangle} \tag{1}$$

with spin. Nuclei approaching mid-shell for protons and neutrons are typically assumed to be axially-symmetrically deformed. This is the case for a wide range of rare earth isotopes. However, it is not clear to what degree softness in $\beta$ is present, even in strongly deformed nuclei with $R_{4/2} = E(4_1^+)/E(2_1^+)$ ratios larger than 3. Only due to the finite width of the nuclear potential in $\beta$, an intrinsic excitation such as a $\beta$-vibration would be expected at finite energy, as pointed out in [4]. Available data on excited $0^+$ states agrees well with soft CBS potentials [6], and centrifugal stretching has been observed in a highly deformed isotope, $^{168}$Hf [7], with $R_{4/2} = 3.11$.

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The microscopic shell model typically works within a valence space, neglecting effects of the bulk of the nucleus, which are then taken into account through the use of effective parameters. Deformation of the nuclear body needs to be taken into account, e.g., by introducing an axially deformed basis and angular momentum projection (see, eg., Ref [8]). Another particularly successful model used in the description of spectra and electro-magnetic properties of excited states over a broad range of structures, including deformed nuclei, is a vast truncation of the shell model, namely, the interacting boson model (IBM) [9]. The simplest version of the IBM only considers valence nucleons, which are paired to angular momentum $L = 0$ or 2, corresponding to $s$ and $d$ bosons, respectively. Well-deformed nuclei are compared to the dynamical symmetry limit SU(3), which is considered the algebraic analog of a rigid, axially-deformed rotor. A specific parameter range of the model corresponds to nuclei in-betwen X(5)-like, $\beta$-soft rotors and SU(3) rigid rotors. No deformed core, in fact, no core at all is used, except for defining the valence space.

In this work, we present results from a recent measurement of $Q_t$ values in $^{170}$Hf, which are in good agreement with CBS predictions of centrifugal stretching. A reproduction of this data within the standard sd-IBM-1, however, fails due to the neglection of the rotating nuclear body. We show that this defect can be accounted for by using a large-N version of the model, or by effectively correcting finite-N effects.

## 2 $Q_t$ values in $^{170}$Hf

The nucleus $^{170}$Hf has an $R_{4/2}$ value of 3.20, hence, even larger deformation and likely higher rigidity than its neighbor isotope $^{168}$Hf. At the Wright Nuclear Structure Laboratory (WNSL) of Yale University, we obtained excited state lifetimes within the ground state band up to spin $J = 16$ in a recoil distance Doppler-shift (RDDS) experiment [10]. The $^{124}$Sn($^{50}$Ti,4n) fusion-evaporation reaction was used to populate excited states of $^{170}$Hf, with a beam of 195 MeV delivered by the 20 MV ESTU tandem accelerator of WNSL. Excited state lifetimes have been deduced using the Yale plunger device [11], surrounded by YRAST-Ball clover detectors [12] at forward and backward angles, in order to observe Doppler-shifts of $\gamma$-rays emitted in-flight.

Reduced $B(E2)$ transition probabilities have been extracted from measured lifetimes and $Q_t$ values were obtained from Eq. (1). The latter can then be compared with model predictions. For details on the data analysis and the corresponding calculations within the CBS model, see Ref. [10]. Relative $Q_t$ values

$$R_{Q_t}(J) = \frac{Q_t(J)}{Q_t(2_1^+)} , \tag{2}$$

obtained for $^{170}$Hf are shown in Fig. 1. The rigid rotor value would be unity for all spins. The high quality of the data reveals a rising trend of $R_{Q_t}$ as a function of spin in the ground state band, that is a rise in deformation, hence, centrifugal stretching. This result by itself is remarkable for such a highly deformed nucleus. We find good comparison to the CBS model, which incorporates only one parameter fitted to the $R_{4/2}$ energy ratio of 3.20.

## 3 IBM calculations

### 3.1 Standard IBM

In the sd-IBM-1, we employed a simple schematic Hamiltonian,

$$H = (1-\zeta)\hat{n}_d - \frac{\zeta}{4N_B}\hat{Q}^\chi \cdot \hat{Q}^\chi , \tag{3}$$

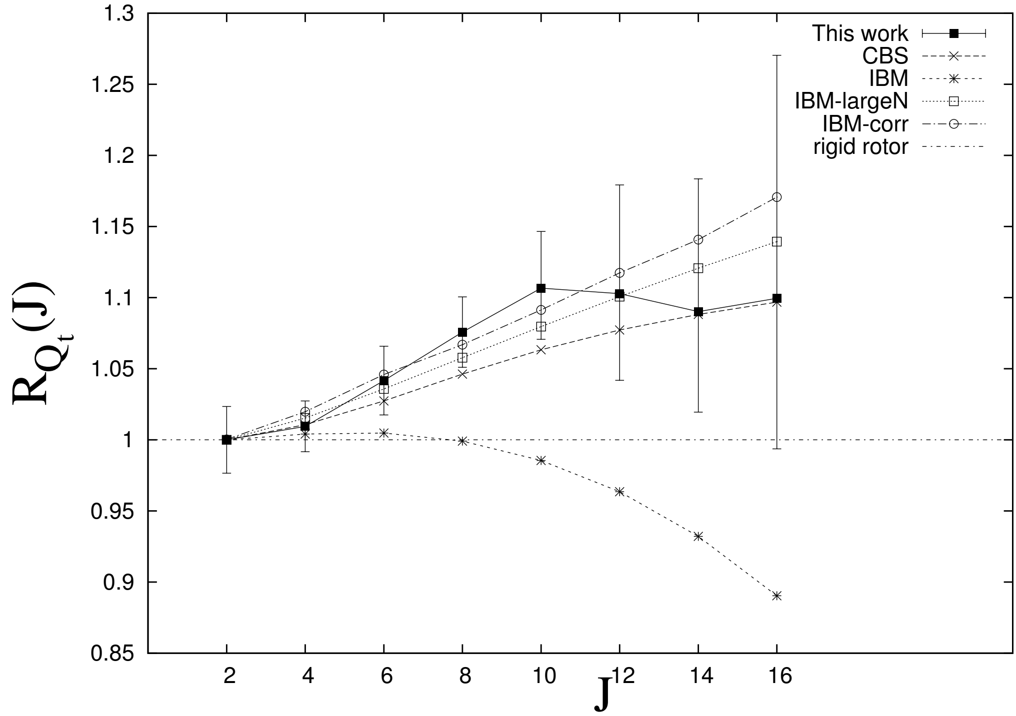


**Figure 1.** Relative transitional quadrupole moments as a function of spin. The dashed horizontal line represents the rigid rotor. Calculations within the CBS and the different IBM calculations as discussed in the text are included according to the legend.

within the extended consistent $Q$ formalism (ECQF) [13, 14], omitting an overall scale and using the standard method of counting valence bosons as pairs of nucleons from the nearest magic shell closure, resulting in $N_B = 13$. The structural parameters $\zeta$ and $\chi$ were adjusted to reproduce the $R_{4/2}$ ratio and the approximate positions of the $0_2^+$ and $2_2^+$ states, respectively. The transition quadrupole operator [9]

$$T(E2) = e_B\, Q^\chi \tag{4}$$

depends on the structural parameter $\chi$. It scales with the effective boson charge $e_B$, and was used to extract $Q_t$ values from the model. Even though with $\zeta = 0.58$ the derived parameters deviate significantly from exact SU(3) ($\zeta = 1$) toward $\beta$-softness, the model fails to describe the measured increase in deformation with spin, as evident from Fig. 1. This fact can be attributed to the boson cut-off, which leads to a serious under-prediction of the $Q_t$ value of the $6^+$ state already. At higher spin, the wave functions do not contain enough $d$ bosons in order to generate the observed $B(E2)$ strengths.

### 3.2 large-N IBM

In order to avoid artificial finite-N effects in the $Q_t$ values of $^{170}$Hf, a calculation with $N_B = 170/2 = 85$ bosons was performed using the code IBAR [15]. This extreme assumption considers the full nuclear body for collective rotations. The result, included in Fig. 1, is in excellent agreement with data. However, intrinsic excitations, upon which further rotational structures are built, such as the $2^+_\gamma$ or $0^+_\beta$ vibrations, are valence excitations. Hence, with the use of $N_B = 85$, their energies are significantly overpredicted.

### 3.3 Effective boson charge correction

In order to simultaneously describe the electro-magnetic properties within the ground state band, and the location of intrinsic excitations, we rescale the electro-magnetic matrix elements by their approximate $d$ boson content. To achieve this, a factor is introduced to rescale the effective charges in higher-lying excited states by that for the ground state through

$$\tilde{e}_B(J) = \sqrt{\frac{(2N+2+\delta)}{(2N+J+\delta)}}\sqrt{\frac{(2N-2)}{(2N-J)}} \cdot e_B \,, \tag{5}$$

where $\delta$ is a free parameter that depends on the location in the IBM symmetry space, and can be fitted to data. The dependence on $\delta$ is not strong, though, and for deformed nuclei a value about 2 works reasonably well. The result of a fit to the available $^{170}$Hf data, labeled "IBM-corr", is included in Fig. 1 and overall agrees as well with data as the large-N calculation, and the correct relative locations of intrinsic excitations are preserved.

A more detailed discussion on the method and the obtained results will be published in a forthcoming article.

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