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Systematic study of α half-lives of superheavy nuclei

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Abstract. Two different descriptions of the α-decay process, namely, the shell model rate theory and phenomenological description are emphasized to investigate the α-decay properties of SHN. These descriptions are shortly presented and illustrated by their results. Special attention is given to the shell structure and resonance scattering effects due to which they exist and decay. A first systematics of α-decay properties of SHN was performed by studying the half-life vs. energy correlations in terms of atomic number and mass number. Such a systematics shows that the transitions between even-even nuclei are favored, while all other transitions with odd nucleons are prohibited. The accuracy of experimental and calculated α-half-lives is illustrated by the systematics of these results.

1 Theoretical model

The approach used here for studying the essential features of α-decay of SHN is presented in Ref.[1]. The procedure is to match smoothly the four shell model wave functions of individual nucleons with a general solution of the system of differential equations [2]:

$$\left[\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)-V_{nn}(r)+Q_n\right]u_n^0(r)+\sum_{m\neq n}V_{nm}(r)u_m^0(r)=0,$$
$$\left[\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)-V_{nn}(r)+Q_n\right]u_n^k(r)+\sum_{m\neq n}V_{nm}(r)u_m^k(r)=I_n^{k[SM]}(r).$$

These equations define an α-particle of kinetic energy Q_α and angular momentum l moving in the potential $V(r)$. $I_n^{k[SM]}(r)$ denote the shell model (SM) formation amplitude (FA) of the outgoing α-particle in channel n from the resonance state k. The solutions of the above system describe the radial motion of the fragments at large and small separations, respectively, in terms of the reduced mass of the system m, the kinetic energy of the emitted particle $Q_\alpha = Q_n = E - E_D - E_\alpha$, the FA $I_n^k(r)$, and the matrix elements of the interaction potential $V_{nm}(r)$.

The effective decay energy used in the above relations is $Q_\alpha = \frac{A}{A-4}E_\alpha^{exp}+\left(6.53Z_d^{7/5}-8.0Z_d^{2/5}\right)10^{-5}$, where A is the mass number of the parent nucleus, E_α^{exp} denote the measured kinetic energy of α-particle and the second term is the screening correction.

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2 Methods

- Shell model α-formation amplitude (SM)
  The SM α-particle formation amplitude is defined as the antisymmetrized projection of the parent wave function on the channel wave function:

$$I_n^{k[SM]}(r) = r\langle\Psi_k^{SM}(r_i) \mid \mathcal{A}\left\{\left[\Phi_D^{SM}(\eta_1)\Phi_p(\eta_2)Y_{lm}(\hat{r})\right]_n\right\}\rangle, \quad (1)$$

  where $\Phi_D^{SM}(\eta_1)$ and $\Phi_\alpha(\eta_2)$ are the internal (space-spin) wave functions of the daughter nucleus and respectively of the α-particle, $Y_{lm}(\hat{r})$ is the wave function of the angular motion, $\mathcal{A}$ is the inter-fragment antisymmetrizer, r connects the centers of mass of the fragments, and the symbol $\langle\,|\,\rangle$ means integration over the internal coordinates and the angular coordinates of relative motion.

  The shell model overlap integral given by Eq.(1) is estimated using the harmonic oscillator single-particle wave functions for the definition of the total wavefunctions of the parent and daughter nuclei. However, in order to integrate the Eq.(1), the channel wavefunction Ψ_k^{SM} must be transformed from the individual coordinates $\{r_i\}_{i=1,A}$ to the center of mass radius r and internal coordinates η_1 and η_2 of the fragments [5]. For nuclei with $Z = 102 - 120$ we use the single proton states $\{1i_{13/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}\}$ [3] and for nuclei with $N = 150 - 178$ the single neutron states $\{2g_{9/2}, 2g_{7/2}, 3d_{5/2}, 3d_{3/2}, 4s_{1/2}\}$ were used.

  The α-decay width is given by:

$$\Gamma_n^{k[SM]} = 2\pi \left| \frac{\int_{r_{min}}^{r_{max}} I_n^{k[SM]}(r)u_n^0(r)dr}{\int_{r_{\min}}^{r_{\max}} I_n^k(r)u_n^{k[SM]}(r)dr} \right|^2, \quad (2)$$

  where the lower limit in the integrals is an arbitrary small radius $r_{min} > 0$, while the upper limit r_{max} is close to the first exterior node of $u_n^0(r)$.

  The α half-life is then expressed as $T_n^{k[SM]} = \ln 2 \cdot \frac{\hbar}{\Gamma_n^{k[SM]}}$. Finally, the shell model half-lives are corrected by even-odd terms h_{e-o} extracted from the available decay data:

$$\log T_n^{k[SM]}(s) \Longrightarrow \log T_n^{k[SM]}(s) + h_{e-o}. \quad (3)$$

  The α half-lives derived from these equations depend on the nuclear single-particle wavefunctions and finite sizes of nucleons and α particles [5].

- Empirical approximation
  Here we consider a phenomenological formula of Viola and Seaborg [6] which writes as

$$\textbf{(VS)} : \log T_\alpha(s) = (aZ_d + b)Q_\alpha^{-1/2} + (cZ_d + d) + h_{e-o}, \quad (4)$$

  where Q_α is the decay energy in MeV units, Z_d is the charge number of the daughter nucleus, a, b, c, d are parameters fitted from data and h_{e-o} is the even-odd hindrance term. The parameters used here are taken from Ref.[7]:

3 Results

The first systematics of α-decay lifetimes of natural emitters was obtained by plotting the experimental values of $\log T_\alpha{}^{exp}$ vs. $Q_\alpha^{-1/2}$. Further, as previously noted in Ref.[8], the plot $\log T_\alpha$ vs. $Z_d Q_\alpha^{-1/2}$ may be a better way to plot the data because in this case the data points are also ordered over the Z_d value

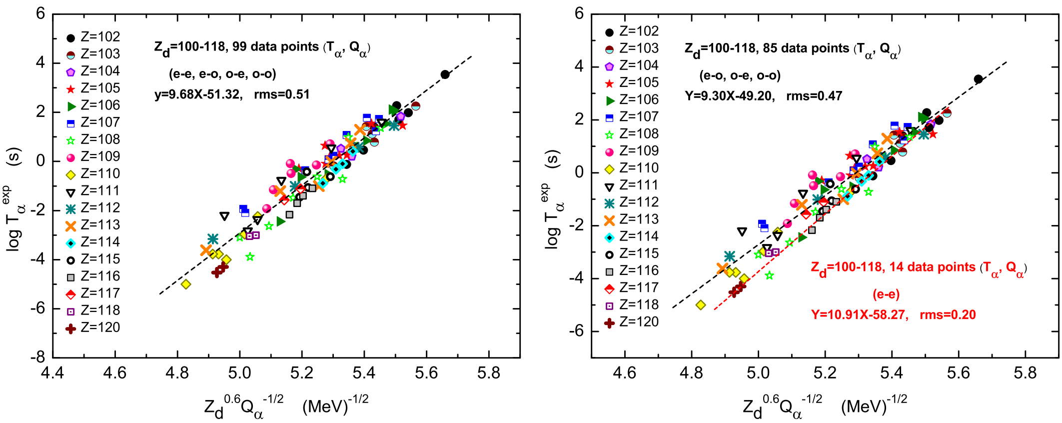

Figure 1. The experimental α half-lives plotted vs. $Z_d^{0.6}Q_\alpha^{-1/2}$ for 99 data points with Z=102-120. The line from the left figure represents the linear fit of the all data points while the straight lines from the right figure represent the results of the linear fits for the even-even (e-e) and even-odd (e-o), odd-even (o-e), odd-odd (o-o) nuclei.

and the scatter is less pronounced than in the plot vs. $Q_\alpha^{-1/2}$. However the best linear fit is obtained between these two pictures when $\log T_\alpha$ values are plotted vs. $Z_d^{0.6}Q_\alpha^{-1/2}$, as it is shown in Fig.1(left).

In fact, $\log T_\alpha$ vs. $Z_d^{0.6}Q_\alpha^{-1/2}$ appears as a simple interpolation between the plots $\log T_\alpha$ vs. $Q_\alpha^{-1/2}$ and $Z_dQ_\alpha^{-1/2}$ for which the *rms* values have a sharp minimum. Although, this dependence does not have a physical interpretation, it also came out numerically from the WKB calculations performed by Brown in Ref.[8]. It is obvious that such a dependence is not bounded to the used theoretical description, but rather to the general property of the α decay. This fact is also supported by the present results, where a similar linear dependence was obtained by means of a fully microscopic formalism. Although, the Brown systematics pointed evident regularities in the $\log T_\alpha$ vs. $Z_d^{0.6}Q_\alpha^{-1/2}$ representation of the nuclei known until 1992 it doesn't contain the odd-A and odd-odd nuclei, for which an additional even-odd correction term h_{e-o} is needed. Thus, we added this correction only in the microscopic approach employed in the present study, Viola-Seaborg formula having this correction already included by default.

The ratio between the results obtained with these two methods are gathered around the value of 1.3 which represents a very good agreement. The difference is that the Viola-Seaborg formula has four parameters and our microscopic estimations have none.

We repeat the plots from Fig.1 for the calculated values with Eq.(3). As we can see in Fig.2 this behavior also theoretically results from the numerical calculation of half-lives based on the shell model formation and resonance scattering amplitudes.

Our estimations are in a good agreement with the available experimental data [9–11] only for the proper α transitions unperturbed by other competing decay channels. The calculated α half-lives in the decay chains of $Z = 108, 110$ (Hs, Ds) and $Z = 114, 116$ (Fl, Lv) isotopes have well-defined energies, a fact which is pointing to the absence of hindrance in the observed decay. This is characteristic for the α decay of spherical nuclei. The large differences between theory and experiment for the α decay energies and half-lives are at the crossing of the proton $Z = 108, 114$ shells and $N = 152, 162, 172$ neutron shells. So, in a few cases (^{263}Hs, ^{266}Mt and ^{281}Ds) there are large discrepancies of about three

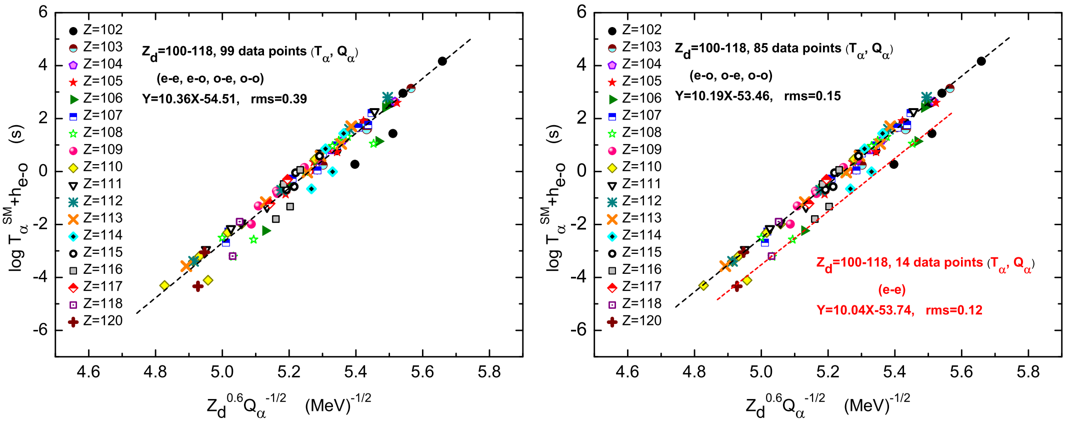

Figure 2. Calculated values for $\log T_\alpha^{SM}$, with even-odd corrections h_{e-o} added are plotted vs. $Z_d^{0.6} Q_\alpha^{-1/2}$ for 99 data points with Z=102-120. The line from the left figure represents the linear fit of the all data points while the straight lines from the right figure represent the results of the linear fits for the even-even (e-e) and even-odd (e-o), odd-even (o-e), odd-odd (o-o) nuclei.

orders of magnitude between the calculated and experimental half-lives which may be due to measurement errors. Detailed α-decay studies provide the access to the basic properties of SHN: masses, energy levels, lifetimes, spins, moments, reaction energies and emission rates. Moreover, α-decay has become a powerful tool to explore the nuclear structure (fine structure, shell effects, α clustering and deformation) and also of the most important aspects of reaction mechanisms (resonance tunneling, phase transitions and channel coupling). Studies of production and decay of SHN are revealing new competing decay modes and complex nuclear structures involving weakly bound states coupled to an environment of scattering states. For these nuclei it is of importance to predict the radioactive properties of unknown species. Such predictions can be made with a fair degree of confidence and this may help in the preparation and identification of new nuclear species.

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