

$\alpha+^{12}\text{C}$ rotational bands in ^{16}O

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Abstract. The total quantum number N of the $\alpha+^{12}\text{C}$ rotational bands in ^{16}O is determined by a study of $\alpha+^{12}\text{C}$ elastic scattering. The 8^+ and 9^- states are found around the excitation energy $E_x = 30$ MeV and they are the member of the known rotational bands. At the same time, the 0_2^+ state ($E_x = 6.05$ MeV) is found to be dominated by $N = 8$.

1 Introduction

The 0_2^+ state ($E_x = 6.05$ MeV) of ^{16}O is thought to have the $\alpha+^{12}\text{C}$ cluster structure, which generates subsequent rotational excitation modes (e.g. [1] and references therein). When considered with the model, the 0_2^+ state is generated by the wavefunction as $\phi_{16\text{O}} = \chi_N(r) \phi_\alpha \phi_{12\text{C}}$ with $N = 6$, because the ground state is dominated by the lowest quantum number ($N = 4$) from the Pauli principle. $\chi_N(r)$ is the relative wavefunction and N corresponds to the total quantum number of the harmonic oscillator for relative motion between α -particle and ^{12}C , $N = 2n + L$. (n is the number of radial nodes; L is the angular momentum.) ϕ_α and $\phi_{12\text{C}}$ are the internal wavefunctions of α -particle and ^{12}C nuclei. If the 0_2^+ state is described by $N = 6$, the resultant rotational band does not have the 8^+ ($L = 8$) state. On the other hand, the 0_2^+ state is also thought to be described by $N = 8$ of the $4p4h$ configuration. The quest for the 8^+ state at high excitation energies is the last piece of the study of the $\alpha+^{12}\text{C}$ rotational band.

Although the 8^+ state was thought to be found at $E_x \approx 20$ MeV from the naive extrapolation of the rotational band [2], it may be at $E_x \approx 30$ MeV with a large α -particle width. The experimental studies of $^{12}\text{C}(^{12}\text{C}, ^{16}\text{O}^*)^8\text{Be}$ show that the excited $^{16}\text{O}^*$ decays into the $^8\text{Be}+^8\text{Be}$ channel at $E_x \approx 30$ MeV with $J^\pi = 8^+$ [3]. The $L = 8$ broad peak has been observed in the excitation function of $^{12}\text{C}(\alpha, ^8\text{Be})^8\text{Be}$ [4]. The elastic cross section has been reported to be enhanced from the pure Coulomb scattering [5], and the possibility of the high-spin states has been discussed from elastic and inelastic scattering [6]. The $J^\pi = 8^+$ and 9^- states can be predicted in the rotational bands by the potential models [7, 8]. The tetrahedral router model predicts $J^\pi = 8^+$ at $E_x \approx 30$ MeV [9].

In the present paper, the elastic cross sections in the energy region of $E_{c.m.} = 21.15 - 26.625$ MeV [5] are evaluated with the optical model to investigate the 8^+ and 9^- states. In addition, the excitation functions below $E_{c.m.} = 5$ MeV [10] are also evaluated. The aim of the present work is to determine the total quantum number N of the $\alpha+^{12}\text{C}$ rotational bands and to show that the 0_2^+ state has $N = 8$.

2 Interaction potentials between α -particle and ^{12}C

Before illustrating the rotational bands, let us describe the internuclear potential between the α -particle and ^{12}C nuclei. We determined the potential so as to minimize the difference between the calculated

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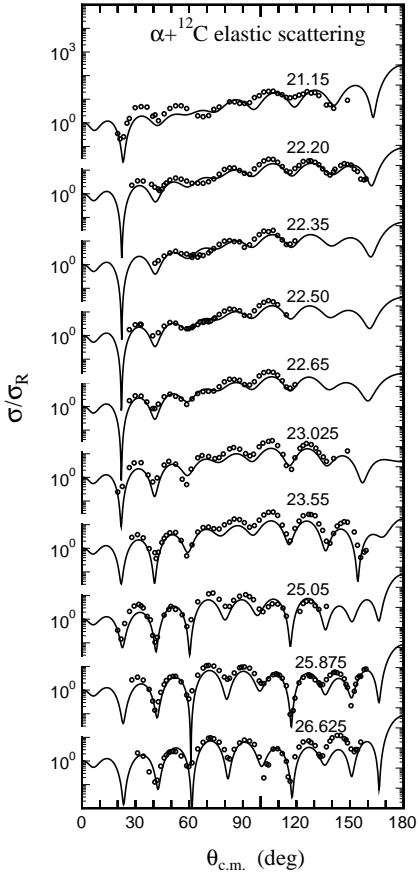


Figure 1. The differential cross sections of $\alpha+^{12}\text{C}$ elastic scattering for the center-of-mass energy $E_{c.m.} = 21.15 - 26.625$ MeV in Rutherford ratio. The solid curves are the results obtained from the simultaneous optimization in this energy region. The experimental data are taken from [5, 12].

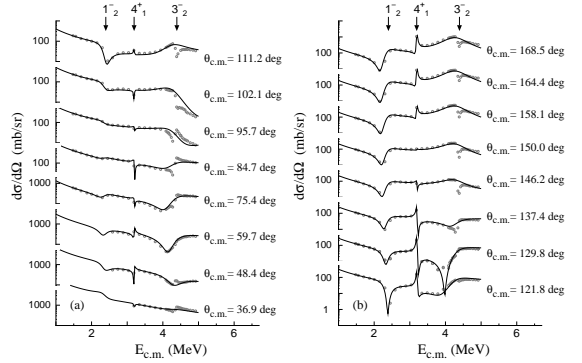


Figure 2. The excitation functions of $\alpha+^{12}\text{C}$ elastic scattering below $E_{c.m.} = 5$ MeV. The solid curves are the calculated results with the P4 potential. The experimental data are taken from [10, 12]. The arrows indicate the resonance energies of the states in the rotational bands.

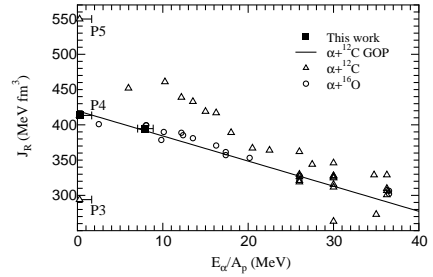


Figure 3. The volume integrals J_R of the real part of the potential. The solid squares are the results of the present work. The solid curve is obtained from the global optical potential [16]. The open triangles and open circles are the results in the studies of $\alpha+^{12}\text{C}$ and $\alpha+^{16}\text{O}$ [15, 17].

values and the experimental data of the cross sections and phase shifts for $\alpha+^{12}\text{C}$ elastic scattering [7, 11]. The resultant potential can reproduce the characteristic feature of the experimental data [5, 12], i.e. enhancement from the pure Coulomb scattering (figure 1). A good description of the excitation functions below $E_{c.m.} = 5$ MeV [10, 12] is also obtained (figure 2). The potential in the present study seems to be comparable to the folding potential with the M3Y interaction [7, 11, 13, 14].

Figure 3 indicates the volume integrals J_R of the real part of the potential as a function of incident energies. The solid squares indicate the present results. At low energies, we find the different strength of the potentials, labeled by P3, P4 and P5, which originates from the difference of n in $\chi_N(r)$ [7]. However, this ambiguity is known to be eliminated by evaluating the experimental data at high energies that lead to a semi-classical description of the quantum system (e.g. [13, 15]). Using the energy

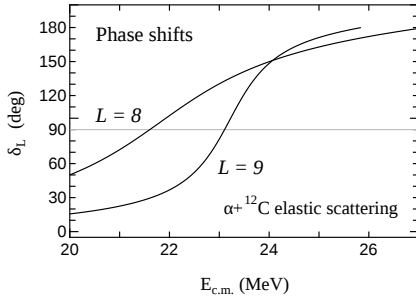


Figure 4. The theoretical phase shifts of $L = 8$ and $L = 9$ for $\alpha+^{12}\text{C}$ elastic scattering. The solid curves are calculated from the real part of the P4 potential. The resonance appears around the energy of $\delta_L = 90^\circ$.

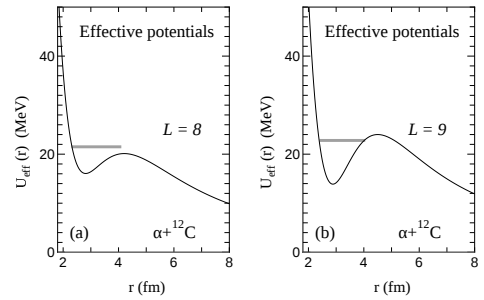


Figure 5. The effective potentials for (a) $L = 8$ and (b) $L = 9$. The solid curves are the sum of the nuclear, Coulomb and centrifugal potentials. The resonance energy is indicated by the gray bar.

dependence of the potential, we find that the potential of P4 is suitable for the evaluation of the $\alpha+^{12}\text{C}$ rotational bands. We use the label P4, because the P4 potential generates $\chi_{N=8}(r)$ for 0_2^+ and $\chi_{N=9}(r)$ for 1_2^- ($E_x = 9.59$ MeV). The cross sections in $E_{c.m.} = 21.15 - 26.625$ MeV cannot be reproduced by the potentials of P3, which do not generate the 8^+ and 9^- states. The P3 generates $\chi_{N=6}(r)$ for 0_2^+ .

3 $\alpha+^{12}\text{C}$ molecular resonances

We examine the possibility of the 8^+ and 9^- states. The theoretical phase shifts of $L = 8$ and $L = 9$ for $\alpha+^{12}\text{C}$ elastic scattering are shown by the solid curves in figure 4, and they go through 90° at $E_{c.m.} = 21.6$ MeV for $L = 8$ and $E_{c.m.} = 23.1$ MeV for $L = 9$. This means that the $\alpha+^{12}\text{C}$ molecular resonances of $J^\pi = 8^+$ and 9^- can be found around $E_x = 30$ MeV. In addition, the 8^+ and 9^- states belong to the known rotational bands. Figure 5 shows the effective potentials for $L = 8$ and $L = 9$. The solid curves are the sum of the nuclear, Coulomb and centrifugal potentials. The effective potentials have the pocket making the barrier-top resonances. From figure 5, we find that the nuclear potential is deep enough to bind the 8^+ and 9^- resonant states in short time. In other words, the nuclear force is strong, and it balances with the strong centrifugal force and Coulomb repulsive force. The P3 potential in the previous section (figure 3) is too weak to generate the resonances.

We finally illustrate the new rotational bands of ^{16}O in figure 6. The excited states from the present study (solid circles) are consistent with the evaluated values [2] (open squares). The 8^+ state appears to be located above the energy of the naive extrapolation of the even-parity rotational band.

4 Summary

The rotational bands in ^{16}O have been investigated by the study of $\alpha+^{12}\text{C}$ elastic scattering for $E_{c.m.} = 21.15 - 26.625$ MeV and $E_{c.m.} < 5$ MeV. The characteristic feature of the elastic cross sections seems to be reproduced with the optical model. The volume integral of the potential is consistent with that from other studies at high energies, and it shows the smooth energy variation. We find that the P4 potential is suitable at low energies, and that the nuclear force is strong enough to bind the 8^+ and 9^- states in short time. The 8^+ resonance is located at $E_x \approx 29$ MeV ($E_{c.m.} \approx 22$ MeV). The 9^- state is found at $E_x \approx 30$ MeV ($E_{c.m.} \approx 23$ MeV). From the present results, the total quantum number of the even-parity rotational band is confirmed to have $N = 8$. Likewise, the odd-parity band is $N = 9$. At the same time, we have found that the 0_2^+ state is generated by $N = 8$ of the $\alpha+^{12}\text{C}$ configuration.

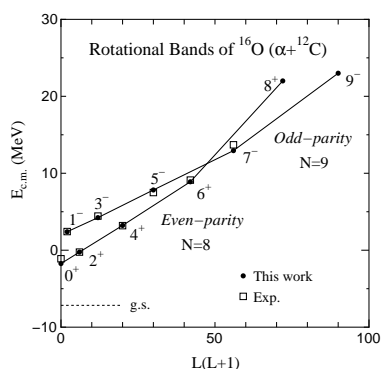


Figure 6. New $\alpha+^{12}\text{C}$ rotational bands of ^{16}O . The solid circles indicate the results of the present study. The even-parity band ($0^+ \dots 6^+$) and odd-parity band ($1^- \dots 7^-$) are calculated from the potential for $E_{c.m.} < 5$ MeV. The 8^+ and 9^- states are obtained from the potential for $E_{c.m.} = 21.15 - 26.625$ MeV. Both potentials are in the same class of the volume integrals, as shown in figure 3. The open squares are the evaluated values [2]. The even- and odd-parity rotational bands are confirmed to have $N = 8$ and $N = 9$.

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