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Nucleon mean-free path in the medium

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Abstract. A microscopic determination of the mean-free path of a nucleon in symmetric nuclear matter is presented. Calculations are based on self-consistent Green's functions theory within the ladder approximation and use different realistic nucleon-nucleon potentials supplemented by semi-phenomenological three-body forces. Temperature and density dependence are discussed. At zero temperature and nuclear saturation density we find that, for energies above 50 MeV, a nucleon has a mean-free path of 4 to 5 fermi.

1 Introduction

Homogeneous nuclear matter gives access to a rich insight into properties of correlated fermionic systems and has always represented a major focus for nuclear theory. Extensive work has been devoted to study its equation of state, or more in general its equilibrium properties [1]. Starting solely from the knowledge of two- and three-nucleon interactions, state-of-the-art calculations nowadays are able to reproduce thermodynamic observables for different isospin, density and temperature conditions with quantified theoretical uncertainties [2, 3].

The determination of transport properties, in contrast, involves additional difficulties which have hindered in the past a fully microscopic description. The extension of the many-body treatment to near- or non-equilibrium conditions often involves an artificial coupling of different methods or the introduction of ad-hoc ingredients (e.g. non-locality corrections) that may lead to uncontrolled errors in the calculations. An accurate determination of transport quantities is nevertheless highly desirable. In astrophysics, viscosities and mean-free paths in the dense hadronic medium govern the dynamics of instability modes or the cooling properties of compact stars. In heavy-ion collisions, the nucleon mean-free path is a basic coefficient that enters Glauber model cross sections or transport simulations.

A method that incorporates many-body treatment and non-equilibrium dynamics in a consistent way is provided by Green's function theory [4]. Even without evaluating explicitly the time evolution of the system, correlators computed near equilibrium give access to non-equilibrium features. We discuss here how the extension of standard self-consistent Green's functions to the complex energy plane provides a first-principles description of quasiparticle properties. In particular, we address the calculation of the mean-free path of a nucleon moving in a homogenous, isospin-symmetric medium.

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2 From the complex-plane propagator to the mean-free path

Our approach is based on non-relativistic Green's function theory [5]. We employ the self-consistent *T-matrix* or *ladder* approximation, in which particle-particle and hole-hole diagrams are resummed (for details see Refs. [6, 7]). The input is a realistic nucleon-nucleon potential, whereas three-body forces (3BF) are included effectively via an average over a third, correlated nucleon [6]. The resulting (retarded) Green's function $\mathcal{G}_R$ describes the propagation of an excitation in nuclear matter. The associated spectral function $\mathcal{A} = -2\,\mathrm{Im}\,\mathcal{G}_R$, which reflects the in-medium modifications of the nucleon properties, is depicted in Fig. 1 (upper panels). Close to the Fermi momentum k_F, nuclear matter retains a clear Fermi liquid character, i.e. $\mathcal{A}$ presents a δ-function peak that is commonly interpreted as a quasiparticle (qp). Away from the Fermi surface quasiparticles are not well defined or, more precisely, they are short-lived, with a typical inverse lifetime proportional to the width of the peak.

The qp inverse lifetime, Γ_k, and energy, ε_k, can be computed exactly from the poles of $\mathcal{G}_R$ in the complex energy plane. We work under the assumption of a single, simple pole in the lower-half plane, based on the single-peak structure of $\mathcal{A}$. By taking the Fourier transform of $\mathcal{G}_R$ in the energy domain and extending the integral over the lower-half plane, we obtain the real-time asymptotic form

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t} \mathcal{G}_R(k,\omega) \sim \int_C \frac{dz}{2\pi} e^{-izt} \frac{\eta(z)}{z - (\varepsilon_k - i|\Gamma_k|)} = -i\eta_k e^{-i\varepsilon_k t} e^{-|\Gamma_k| t} = \mathcal{G}_R(k,t)\,, \tag{1}$$

with η_k representing the qp strength. The spectrum is closely related to the group velocity, $v_k = \partial\,\varepsilon_k/\partial k$. These quantities give access to the typical length scale over which a qp exists, i.e. the (particle) mean-free path, $\lambda_k = v_k/|\Gamma_k|$.

The nucleon propagator in the above form, however, is not easily attainable. If the retarded Green's function is analytically continued to the whole complex energy plane, it results in a function that is analytic off the real axis [9], implying the absence of the qp pole postulated in Eq. (1). Instead, one imposes continuity on the self-energy. We adopt the prescription $\tilde{\Sigma}(k,z) \equiv \Sigma(k,z)$ for $\mathrm{Im}\,z > 0$ and $\tilde{\Sigma}(k,z) \equiv \Sigma^*(k,z)$ for $\mathrm{Im}\,z \leq 0$ and solve the corresponding Dyson equation for a complex-plane propagator, $\tilde{\mathcal{G}}$ [10]. This leads directly to an implicit equation for the complex pole,

$$z_k = \frac{k^2}{2m} + \mathrm{Re}\tilde{\Sigma}(k,z_k) + i\mathrm{Im}\,\tilde{\Sigma}(k,z_k)\,. \tag{2}$$

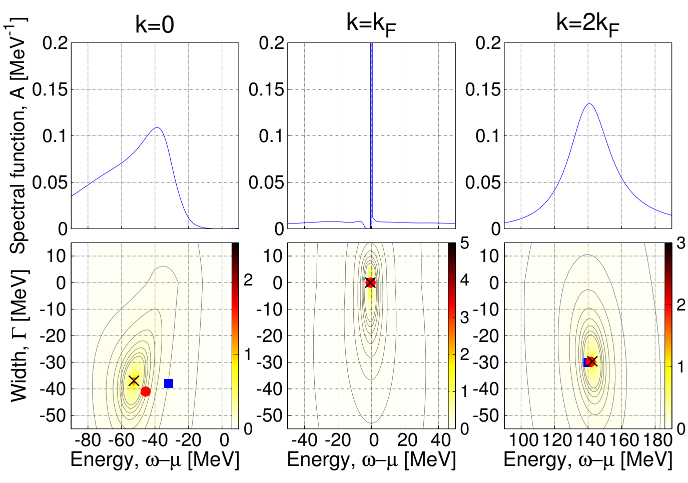

Figure 1. *Upper panels*: spectral function at density $\rho = 0.16$ fm^{-3} and temperature $T = 0$ MeV for the CDBonn interaction [8] for three characteristic nucleon momenta as a function of the nucleon energy (relative to the Fermi energy μ). *Lower panels*: absolute value of $\tilde{\mathcal{G}}$ in the same conditions. The fully dressed pole is indicated by a cross, while the circle (square) show the position of the first (second) renormalization quasi-particle.

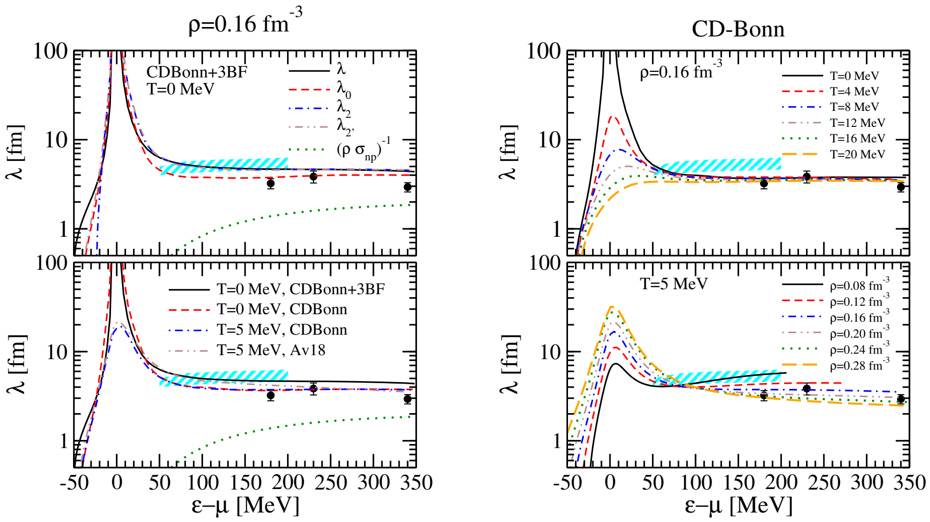

Figure 2. Mean-free path of a nucleon in nuclear matter as a function of energy. The shaded band and solid dots correspond to the experimental results of Refs. [12, 13], respectively. *Upper left*: results obtained with a CDBonn+3BF self-energy at $T = 0$ MeV. The different approximations are commented on in the text. *Lower left*: results obtained from the fully dressed pole for different NN forces and two different temperatures. *Upper right*: temperature dependence obtained with a CDBonn self-energy at a constant density of $\rho = 0.16$ fm^{-3}. *Lower right*: density dependence obtained with a CDBonn self-energy at a constant temperature of $T = 5$ MeV.

The solution, $z_k = \varepsilon_k + i\Gamma_k$, yields the *fully dressed* qp spectrum and inverse lifetime. In previous approaches, different approximations have been employed in the solution of Eq. (2), either completely neglecting the dependence on the imaginary part of z ("first renormalization", λ_0) or expanding the self-energy to first order in the imaginary part of z ("second renormalization", λ_2) [11]. In contrast, in our calculations we take into account the full momentum and energy dependence of the propagator and solve Eq. (2) fully self-consistently without further approximations.

Lower panels of Fig. 1 display the absolute value of $\tilde{\mathcal{G}}$ in the complex energy plane, illustrating the existence of an isolated pole. The location of this fully dressed pole is consistent with the numerical solution of Eq. (2), shown with a cross. Differences with first and second renormalization predictions are visible at $k = 0$. Comparing to the corresponding spectral functions (upper panels), one notices that the real energy position of the pole dies not necessarily coincide with the position of the spectral peak. Nevertheless, in the vicinity of the Fermi surface, $k = k_F$, the two agree and the quasiparticle width tends to zero, which provides a microscopic verification of Fermi liquid theory.

As discussed above, the mean-free path is derived from the real and imaginary part of the qp pole. A summary of our results is presented in Fig. 2. In the upper left panel we analyze different approximation schemes. As expected, we find that the largest differences occur for hole energies, below -20 MeV. In contrast, all approximations give similar results above 50 MeV. The classical kinetic theory prediction, $\lambda \sim (\rho\sigma_{np})^{-1}$ (dotted line), is well below all quantum in-medium mean-free paths. The latter flatten at high energies, and remain constant, at a value of around $4 - 5$ fm.

In the lower left panel, we study the dependence on the starting nuclear interaction. The $T = 0$ mean-free path with 3BF is slightly larger than that obtained without 3BF. We also vary the two-body NN interaction at a temperature of $T = 5$ MeV, with the largest differences observed again at negative energies. Taking the spread between different lines as an estimate of theoretical uncertainties, one estimates errors to be less than 1 fm beyond 50 MeV. This uncertainty is of the same order of that obtained from experimental estimates [12, 13].

The right panels of Fig. 2 display the temperature and density dependence of the mean-free path in nuclear matter. The effect of temperature is relevant in an area of about 20 MeV around the Fermi surface. By switching on temperature, the mean-free path at the Fermi surface becomes finite due to thermal damping. Thermal effects are otherwise negligible for the very high and low energy behaviour of the mean-free path. The density dependence is more pronounced at all energies. Near the Fermi surface, an increase in density leads to an increase of λ. This can be understood as a system at larger density is more degenerate and thus closer to the zero-temperature case, where λ diverges at the Fermi surface. At an energy of around 70 MeV the tendency is reversed and λ tends to decrease as density increases. This behaviour is more intuitive, but it remains to be seen whether the $1/\rho$ dependence predicted by classical transport theory is valid in this case. The low density results cover the band of experimental data of Ref. [12], which has been extracted phenomenologically from finite nuclei.

3 Conclusions

We have presented a new way of determining the mean-free path of a nucleon in homogeneous nuclear matter. Our calculation scheme involves the extension of Green's functions techniques to the complex energy plane and the extraction of the quasiparticle pole that appears in the one-body propagator. In the nuclear medium, the calculation of the pole within this method provides similar mean-free paths as those obtained with earlier approaches. The renormalization induced by this procedure is specially relevant for hole properties. With all many-body corrections properly implemented, we obtain a mean-free path of around $4 - 5$ fm at saturation density and energies above 50 MeV.

Acknowledgments

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