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Molecular resonances in ^{28}Si + ^{28}Si

Wobbling motions observed by angular correlation measurements

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Abstract. High-spin resonances observed in ^{28}Si + ^{28}Si collisions are studied with a dinuclear molecular model. At high spins, a stable dinuclear configuration of the oblate-oblate system (^{28}Si + ^{28}Si) is found to be an equator-equator (E-E) touching one. Normal modes have been investigated around the equilibrium, which are expected to be an origin of a large number of the resonances observed. Analyses of physical quantities are made and compared with the recent experimental data measured at Strasbourg. Since the E-E configuration is slightly triaxial, rotations of the total system induce mixing of K quantum numbers, called wobbling motion, which clearly explains the particle-γ angular correlations observed as well as the disalignments observed in the angular distributions, in a simple and natural way. Furthermore, predictions are given for the angular correlations of the wobbling excited states. The importance of the angular correlation measurements is stressed, which provide identification of the dinuclear configurations by spin orientations of the constituent nuclei ^{28}Si.

1 Introduction

The ^{28}Si+^{28}Si system exhibits narrow resonances well above the Coulomb barrier. They are correlated among the elastic and inelastic channels [1]. The maximum spin is over 40$\hbar$ [2], and the mass-symmetric decays are much larger than asymmetric ones. In addition, high level density more than 'one per MeV' is observed. Those observations suggest compound systems in an extreme condition, with much longer life time than those of the lighter systems such as ^{12}C + ^{12}C resonances. However, the structures of the resonance states in ^{28}Si + ^{28}Si as well as their reaction mechanism are not known yet.

In the experimental angle-averaged excitation functions of ^{28}Si + ^{28}Si, bumps of widths about 2 MeV are seen to correspond with the grazing angular momenta [2], and several sharp peaks of widths about 150 keV are found on each bump [1]. Recently new experimental data were obtained for one of those sharp peaks at $E_{\rm cm}$ = 55.8 MeV [3]. Disalignments between the orbital angular momentum and the spins of the emitted fragments ^{28}Si have been found as a novel aspect of heavy-ion resonances [3], which is never observed in any heavy-ion system so far and is considered mysterious. At the same time, the particle-γ angular correlation measurements were made, in which fragments ^{28}Si are detected at $\theta_{\rm cm} = 90^\circ$ and γ-rays of the $E2$-transition from the first excited state of the ^{28}Si nuclei are measured with 4π gamma detectors. The results show very characteristic features.

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By using a molecular model the authors found the equilibrium configuration of the interacting two oblate nuclei to be the *equator-equator* (E-E) one as displayed below in fig. 2 [4]. The axis of the largest moment of inertia for this configuration is normal to the plane defined by the two pancake-like ^{28}Si nuclei, which is assigned approximately to be the reaction plane of the molecular ground-state resonance of a high-spin state. Note that the E-E configuration is *triaxial* as a whole system. Therefore, at a given high angular momentum J, the triaxial configuration rotates preferentially about the axis corresponding to the largest moment of inertia, in the state with the lowest energy, which is called *wobbling* [5]. In such a case, the spins of the ^{28}Si fragments are in the reaction plane, not normal to the plane, since no rotation can occur about the symmetry axes of ^{28}Si. Thus, the spins of ^{28}Si are expected to be disaligned with the orbital angular momentum, which naturally explains the experimental observation.

In order to study distribution of the spin orientations of the constituent nuclei in the dinuclear molecule, we have investigated the rotational state of the total system, including mixing of K-quantum numbers [6], and we further have analyzed the particle-γ angular correlations [7]. The results have turned out to clarify that the angular correlation measurements for ^{28}Si + ^{28}Si provide crucially important information on resonances. Furthermore, we expect that some of the other sharp peaks observed on the same bump correspond to wobbling excited states, and thus we also have analyzed angular correlations of the calculated excited states. The results for the molecular ground state and a couple of the excited states are compared among them, which show quite different characteristics, respectively. The wobbling ground state has '$m = 0$' dominance with z-axis normal to the reaction plane (spin disalignments), while the first excited state exhibits '$m = 2$' dominance, which means strong *alignments* of the fragment spins to the orbital angular momentum. Thus it is clear that the angular correlation measurements provide a powerful tool to distinguish nuclear structures of dinuclear molecular states.

2 Dinuclear molecular model for the ^{28}Si + ^{28}Si system

2.1 Stable dinuclear configurations at high spins and molecular normal modes

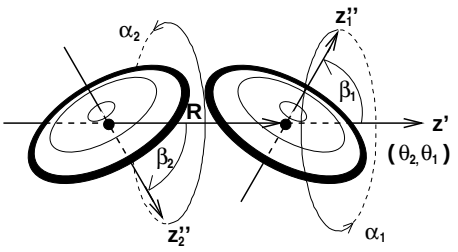

Figure 1. Dinuclear molecular coordinates; seven degrees of freedom of the ^{28}Si + ^{28}Si system are displayed.

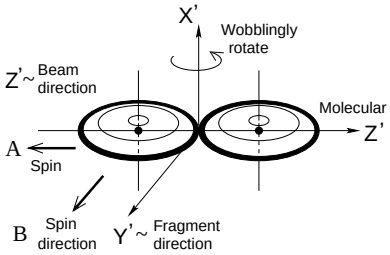

Figure 2. Spin directions of the molecular ground state with wobbling rotation, which give rise to disalignments.

For simplicity, assuming a constant deformation and an axial symmetry of the constituent nuclei, we have seven degrees of freedom, $(q_i) = (\theta_1, \theta_2, \theta_3, R, \alpha, \beta_1, \beta_2)$ illustrated in fig. 1, where (R, θ_2, θ_1) is the relative vector of the two ^{28}Si. As internal degrees of freedom, the orientations of the symmetry axes of the two constituent ^{28}Si nuclei are described with the Euler angles (α_i, β_i) which refer to the molecular axes. α_1 and α_2 are combined into $\theta_3 = (\alpha_1 + \alpha_2)/2$ and $\alpha = (\alpha_1 - \alpha_2)/2$. Consistently with the coordinate system, at first, we introduce a rotation-vibration type wave function as basis one,

$\Psi_\lambda \sim D^J_{MK}(\theta_i)\chi_K(R,\alpha,\beta_1,\beta_2)$, where χ_K describes the internal motions. A multidimensional energy surface including centrifugal energies has been investigated [4], and a local minimum is obtained around $\beta = 90^\circ$ and $R = 7.6$ fm with the barrier at ~ 9.5 fm, which is an *equator-equator* (E-E) configuration illustrated in fig. 2.

In order to solve the internal motions, we expand the effective potential at the energy minimum point and adopt the harmonic approximation to obtain the normal modes. Motions associated with β-degrees are well confined to be vibrational, which are classified into new two modes, *butterfly*: $\beta_+ = (\Delta\beta_1 + \Delta\beta_2)/\sqrt{2}$ and *antibutterfly*: $\beta_- = (\Delta\beta_1 - \Delta\beta_2)/\sqrt{2}$ around $\alpha = \pi/2$, respectively. In fig. 3(a), the spectrum of the molecular normal modes of ^{28}Si +^{28} Si with spin 38 is displayed, classified with K-quantum numbers. Butterfly and antibutterfly modes are indicated with (b) under the levels. As for the α-degree, $\alpha = (\alpha_1 - \alpha_2)/2$, the confinements in the present folding potential appear to be unexpectedly weak, and hence the motion is close to a free rotation. We call it, *twisting mode*, associated levels of which are indicated by (t). All those are due to internal degrees of freedom, i.e., intrinsic excitations. These excitations exist over the range of spin $J = 34 - 40$ with stable E-E configurations, and are expected to be an origin of the narrow resonances [4, 6].

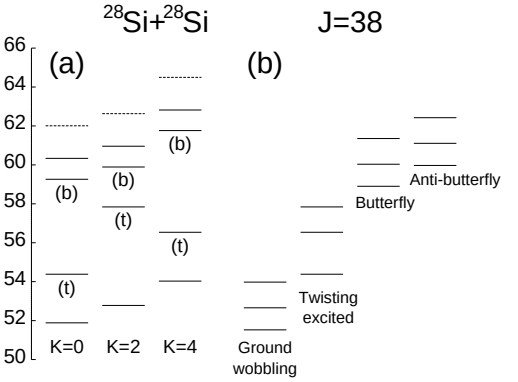

Figure 3. Energy spectra of the ^{28}Si + ^{28}Si system for $J = 38$. (a) Molecular normal modes without K-mixing. (b) After K-mixing, with indications of the modes under the levels.

2.2 Wobbling motion of triaxial molecule

Since the E-E configuration is triaxial, rotations of the total system induce mixing of quantum numbers K, which is known as *wobbling* [5]. At a given high angular momentum J, this configuration rotates in a triaxial way preferentially about the axis corresponding to the largest moment of inertia, in the state with the lowest energy. Therefore the whole system rotates about the axis normal to the plane defined by the two pancake-like nuclei. With such a characteristic rotation, the spins of the ^{28}Si fragments are preferentially in the reaction plane, because they are perpendicular to the symmetry axes of ^{28}Si. Thus wobbling motion is consistent with the disalignments in the inelastic scattering.

We describe the wobbling motion of the E-E configuration, i.e., K-mixing, with the hamiltonian of the asymmetric rotator,

$$\hat{T}_{\rm rot} = \frac{\hbar^2}{2}\left(\frac{\hat{J}_x^2}{I_x} + \frac{\hat{J}_y^2}{I_y} + \frac{\hat{J}_z^2}{I_z}\right) = \frac{\hbar^2}{2}\left\{\frac{\hat{J}^2}{I_{\rm av}} + \frac{\hat{J}_z^2}{I_K} - \frac{1}{2\Delta}\left(\hat{J}_+^2 + \hat{J}_-^2\right)\right\}, \quad (1)$$

where $(\hat{J}_x, \hat{J}_y, \hat{J}_z)$ denotes the angular momentum operator in the body-fixed frame, and (I_x, I_y, I_z) denote the moments of inertia about the intrinsic axes, respectively. $\hat{J}_+$ and $\hat{J}_-$ are the lowering and raising operators, $\hat{J}_\pm \equiv \hat{J}_x \pm i\hat{J}_y$, respectively, and $I_{\rm av}$, Δ and I_K in Eq. (1) are related to (I_x, I_y, I_z) by $1/I_{\rm av} = \left(1/I_x + 1/I_y\right)/2$, $\quad 1/\Delta = \left(1/I_y - 1/I_x\right)/2$, $\quad 1/I_K = 1/I_z - 1/I_{\rm av}$. After K-mixings by the diagonalization of $\hat{T}_{\rm rot}$, we obtain a *new energy spectrum* [6], displayed in fig. 3(b).

To obtain accurate K-mixing ratio, magnitudes of the moments of inertia of the molecular configuration displayed in fig. 2 are important. Those elements are calculated with relations,

$$I_x = \mu R^2 + I_3 \times 2, \quad I_y = \mu R^2 + I_1 \times 2, \quad I_z = I_1 \times 2, \quad (2)$$

where the moments of inertia of the constituent nuclei, individually in their principal axes, are written as (I_1, I_2, I_3), and $I_1 = I_2$ is assumed. Their moments of inertia can be estimated by integrating over nuclear volume, such as

$$I_i = \int_V \rho(\boldsymbol{r})(r^2 - x_i^2)dV, \tag{3}$$

where x_i denote the coordinates in the principal axes, and $\rho(\boldsymbol{r})$ is a nuclear density distribution. On the other hand, it is known that moments of inertia of rigid bodies are too large compared with the experimental values [5]. Thus the value of the moment of inertia I_1 for the states of the ^{28}Si ground band is determined from the excitation energy $E_{\rm ex} = 1.78$ MeV of the 2_1^+ state of the ^{28}Si nucleus. For I_3, by using eq. (3) with the nuclear density of ^{28}Si, we calculated the ratio I_3/I_1 and obtained a value by a factor $4/3$ larger than I_1.

In the high-spin limit ($|K|/J \sim 0$), we obtain analytical wobbling (K-mixing) solutions as

$$F_n(K) = H_n(K/b)\exp(-K^2/2b^2), \tag{4}$$

with the Hermite polynomial H_n for the n-th excited state, where the width b is given by the asymmetry in the moments of inertia as $b = (2J^2 I_K/\Delta)^{1/4}$ [6, 7]. Those analytical solutions provide a good perspective, namely, oscillatory behaviors of the coefficients of the K-substates give rise to wobbling excitations. For numerical calculations, however, we have exactly diagonalized the hamiltonian of the triaxial rotator, and have used the solutions.

3 Disalignments phenomenon observed in angular distributions

Figure 4 displays the angular distributions for the elastic and inelastic scattering of ^{28}Si + ^{28}Si at $E_{\rm cm} = 55.8$ MeV. From the bottom, the experimental data for the elastic, single 2^+ and mutual 2^+ channels are given. It is found that not only the elastic one, oscillations of those 2^+ channels accept good fits with the $L = J = 38$ Legendre polynomial, and thus disalignments between the orbital angular momentum and the fragment spins are expected [3].

Our molecular model naturally explains the mechanism of those disalignments. The triaxial configuration rotates preferentially around X'-axis, as called *wobbling*, and then the molecular X'-axis is normal to the reaction plane. The spins of the ^{28}Si fragments are approximately in the reaction plane, as is illustrated in fig. 2, because they are in $Y'Z'$-plane, as they are perpendicular to the symmetry axes of the constituent ^{28}Si nuclei. Thus the disalignments observed in the angular distributions, i.e., $L = J = 38$ oscillation patterns both in the single and mutual 2^+ channels are well reproduced by the analyses with R-matrix theory of the Wobbling ground state [7].

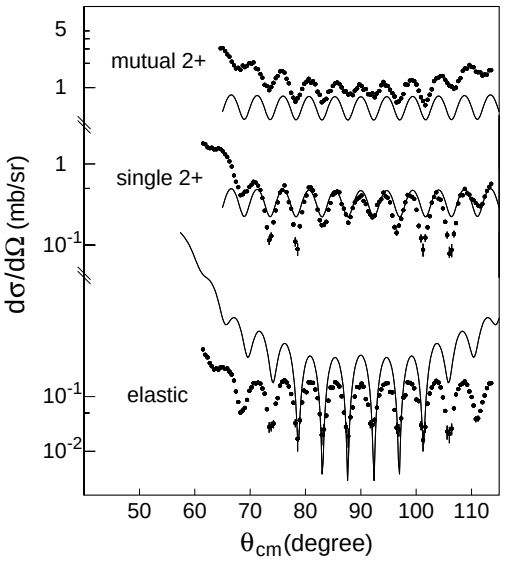

Figure 4. Fragment angular distributions for the elastic and inelastic scattering of ^{28}Si + ^{28}Si at $E_{\rm cm} = 55.8$ MeV. From the bottom those for the elastic, single 2^+ and mutual 2^+ are shown. Solid curves (theory) are obtained with the molecular ground state with wobbling motion, which are compared with the experimental data [3].

4 Fragment-fragment-γ angular correlations

4.1 evidence for spin orientations

In order to study motions of the constituent nuclei in the resonance state, i.e., distribution of their spin orientations in dinuclear molecule, we further investigate the rotational state of the total system with axial asymmetry [6]. In the high spin limit ($|K|/J \sim 0$), the analytical wobbling (K-mixing) weights are given by a Gaussian distribution, and we have the wave function for the wobbling ground state as

$$\Psi_{\lambda}^{JM} \sim \sum_{K} \exp(-K^2/2b^2) D^{J}_{MK}(\theta_i) \chi_K(R, \alpha, \beta_1, \beta_2), \quad (5)$$

where χ_K describes the motions of the internal degrees of freedom, which are in the zero-point vibrations at the equilibrium configuration, illustrated in fig. 2. As a matter of course, the K-mixing given above provides a quantitative description of the fragment spin distribution [7]. In fig. 5, dots display the experimental data of the angular correlations, i.e., γ-ray intensity distributions of the $E2$-transition observed in the mutual 2^+ channel [3]. Data on 4π solid angle are classified with three different quantization z axes: (a) beam direction, (b) normal to the reaction plane and (c) fragment direction taken perpendicular to the axes (a) and (b), respectively. Theoretical results of the molecular ground state are displayed by solid lines. The results show good agreements in all the three panels, and in panel (b), a typical '$m = 0$' pattern is obtained. Note that $m = 0$ means the structure of the state to be symmetric around the z-axis. Thereby it is expected that the spins are not only in the reaction plane, but also almost the same in their strengths of the Y'- and Z'-directions of fig. 2.

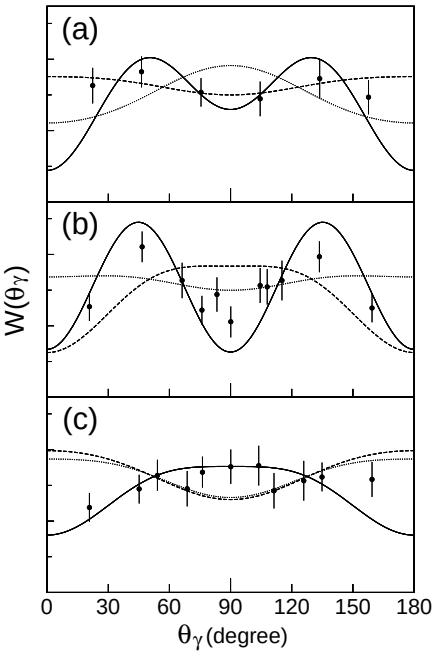

Figure 5. γ-ray intensities of particle-particle-γ angular correlations in the mutual 2^+ channel. 4π γ-ray data are integrated for three quantization z axes (see text for the axes). Dots display the experimental data [3]. Solid lines are theoretical results of the molecular ground state, dashed lines for the first excited state and dotted lines for the second excited state.

4.2 Predictions on angular correlations of wobbling excited states

Furthermore, we have predicted the first and second excited states of the wobbling motion (the states denoted 'Ground wobbling' in fig. 3(b)). The angular correlations of those states are calculated and are also shown in fig. 5. Apparently the excited states exhibit different characteristics from that of the molecular ground state. For the first excited state (dashed lines), with a dominant $m = 2$ pattern in (b), strong alignments are indicated. The second excited state (dotted lines) gives a dominant $m = 2$ pattern in (a). Those states are possible origin of the series of resonance peaks observed on the same bump of the 55.8 MeV resonance [1, 8].

4.3 Normal mode excitation

It is found that the angular correlations of the twisting excitation and the butterfly modes have clear

differences from those of the molecular ground state. The essential feature of the twisting excitation is '$m = 2$' dominance in (a) panel, where the z-axis of the spin quantization is parallel to the beam direction (not shown here [7]). This means that the spin directions are not in B, but in A of fig. 2, due to the twisting motion around the molecular Z'-axis. Note that in the observation condition of the decay particles [3], the direction of the molecular Z'-axis is close to the beam direction, since fragments are emitted to the tangential direction to the rotational circle of the dinuclear molecule in high spin rotation of $J = 38$, i.e., to the direction of the Y'-axis displayed in fig. 2. As for the butterfly excitation, '$m = 2$' dominance appears in (c). Thus each excited state of the normal modes has its own characteristics in angular correlations, and thereby one can identify the normal-mode excitations, i.e., intrinsic nuclear structures of the resonances. In other words, the angular correlation measurements for ^{28}Si + ^{28}Si made with 4π gamma detectors provide crucially important information on resonances [3].

5 Conclusions

By using a molecular model for ^{28}Si – ^{28}Si, the interaction between two deformed nuclei is described with the internal collective variables. The stable dinuclear configuration is found to be an equator-equator configuration, which means the structure of the resonance states to be a hyper-deformed molecule with axial asymmetry [4, 6]. Molecular normal modes around the equilibrium have been obtained, such as butterfly and antibutterfly ones in addition to the radial excitation. As a whole, they give rise to a variety of the molecular states. Those modes are expected to be an origin of a large number of the resonance states observed.

With the R-matrix theory, analyses of the decay properties have been made [7]. Due to the axial asymmetry of the stable configuration, the system rotates wobblingly around the axis normal to the reaction plane, which gives rise to the '$m = 0$' dominance of the fragment spins in the molecular ground state and thus to good agreement with the disalignments seen in the angular distributions of the inelastic scattering on the $E_{\rm cm} = 55.8$ MeV resonance. Thus, the dinuclear molecular model gives variety of molecular states [6], among which the molecular ground state with $J = 38$ is a candidate responsible for the resonance at $E_{\rm cm} = 55.8$ MeV [7]. Predictions of the angular correlations for the resonance states on the same bump are also made. It is found that contrary to the molecular ground state with the property of disalignments, the first excited state of wobbling motion shows strong alignments.

In conclusion, the ground state and the excited states of the normal modes exhibit their own different characteristics in the angular correlations, respectively. In other words, the particle-γ angular correlation measurements provide crucially important information to identify molecular intrinsic states of the resonances. For systematic study of excitations of the molecular modes, the same kind of information on the nearby resonances of ^{28}Si + ^{28}Si is strongly called for.

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