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Constituent Quarks and Gluons, Polyakov loop and the Hadron Resonance Gas Model *,**

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Abstract. Based on first principle QCD arguments, it has been argued in [1] that the vacuum expectation value of the Polyakov loop can be represented in the hadron resonance gas model. We study this within the Polyakov-constituent quark model by implementing the quantum and local nature of the Polyakov loop [2, 3]. The existence of exotic states in the spectrum is discussed.

1 Introduction

The Hadron Resonance Gas Model (HRGM) describes the confined phase of the QCD equation of state as a multicomponent gas of non-interacting massive stable and point-like particles [4], which are usually taken as the conventional hadrons listed in the review by the Particle Data Group. The Polyakov loop, a commonly used order parameter for the hadron–quark-gluon crossover [5, 6], admits a hadronic representation, as shown recently in [1]

$$\langle \mathrm{tr}_c \Omega_3 \rangle = \langle \mathrm{tr}_c \mathsf{P} e^{i\int_0^{1/T} A_0\, dx_0} \rangle \approx \frac{1}{2}\sum_\alpha g_{h\alpha}\, e^{-\Delta_{h\alpha}/T}\,, \qquad \Delta_{h\alpha} = M_{h\alpha} - m_h\,, \tag{1}$$

where $M_{h\alpha}$ are the masses of hadrons with exactly one heavy quark of mass $m_h \to \infty$, and $g_{h\alpha}$ are the degeneracies of the states. This model has been used to extract the spectrum of hadrons from a fit of lattice data for the renormalized Polyakov loop [1, 6]. Recent lattice data [5, 6] seem to indicate that conventional hadrons are not enough to reproduce the data, and this could signal the possible existence of exotic multipartonic states. In this communication we will study the realization of the HRGM within a particular constituent quark model coupled to the Polyakov loop, and address it as a diagnostic tool for the possible existence of non conventional hadrons.

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2 The Polyakov-constituent quark-gluon model

An effective approach to the physics of QCD at finite temperature is provided by chiral quark models coupled to gluon fields in the form of a Polyakov loop [7–9]. While most works remain within a mean field approximation, we stressed in [2, 3, 10] the need of quantum and local features. The partition function motivated in these works is given by

$$Z = \int \prod_{\boldsymbol{x}} d\Omega(\boldsymbol{x})\, e^{-S_{\rm PCM}(\Omega,T)}\,, \qquad S_{\rm PCM}(\Omega,T) = S_q(\Omega,T) + S_g(\Omega,T)\,, \tag{2}$$

where the Polyakov loop matrix $\Omega(\boldsymbol{x})$ is kept as a quantum and local degree of freedom. $d\Omega(\boldsymbol{x})$ is the invariant SU(N_c) group integration measure ($N_c = 3$) at each point $\boldsymbol{x}$. The action of the model depending on the quarks is obtained from the corresponding fermion determinant and reads

$$S_q(\Omega,T) = -2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left[{\rm tr}_c \log\left(1+\lambda\xi\Omega_3(\boldsymbol{x})\, e^{-E_q/T}\right) + {\rm tr}_c \log\left(1+\lambda\xi^{-1}\Omega_{\bar{3}}(\boldsymbol{x})\, e^{-E_q/T}\right) \right], \tag{3}$$

where $E_q = \sqrt{\boldsymbol{p}^2 + M_q^2}$ is the quark energy, M_q is the constituent quark mass, and $\Omega_{3(\bar{3})}$ is the Polyakov loop in the (anti)fundamental representation. We have introduced the parameter λ that counts the number of constituents (quarks plus antiquarks), and ξ for the number of quarks minus antiquarks. One can always replace $\lambda,\xi \to 1$ at the end. After a series expansion in Eq. (3), the quark Lagrangian reads

$$\mathscr{L}_q(\boldsymbol{x}) = 2N_f T \sum_{n=1}^{\infty} \frac{(-\lambda)^n}{n} J_n(M_q,T) \left[{\rm tr}_c(\Omega_3^n(\boldsymbol{x}))\xi^n + {\rm tr}_c(\Omega_{\bar{3}}^n(\boldsymbol{x}))\xi^{-n} \right], \tag{4}$$

where we have defined the functions

$$J_n(M,T) := \int \frac{d^3p}{(2\pi)^3}\, e^{-nE_p/T} = \frac{M^2T}{2\pi^2 n} K_2(nM/T) \overset{T \ll M}{\simeq} \left(\frac{MT}{2\pi n}\right)^{3/2} e^{-nM/T}\,, \tag{5}$$

$K_2(x)$ being the Bessel function. One can identify in the asymptotics of J_n the statistical Boltzmann factors appearing in the low T expansion of observables. The quark propagator behaves as $\sim e^{-M/T}$, so each Boltzmann factor is characteristic of a single quark state [2]. Mesonic contributions are identified by terms $\sim \lambda^2\xi^0$ and they behave as $\sim e^{-2M/T}$, while baryonic contributions are identified by terms $\sim \lambda^{N_c}\xi^{nN_c}$, and they behave as $\sim e^{-N_cM/T}$.

A convenient model for the gluonic action $S_g(\Omega,T)$ is [11]

$$S_g(\Omega,T) = 2\int \frac{d^3x d^3p}{(2\pi)^3} {\rm tr}_c \log\left(1-\lambda\Omega_8\, e^{-E_g/T}\right), \tag{6}$$

where $E_g = \sqrt{\boldsymbol{p}^2 + M_g^2}$, M_g represents a constituent gluon mass, Ω_8 is the Polyakov loop in the adjoint rep, and now λ counts the number of gluons. After performing the trace in color space, one gets [11]

$$\mathscr{L}_g(\boldsymbol{x}) = 2T \int \frac{d^3p}{(2\pi)^3} \log\left(\sum_{n=0}^{8} \lambda^n \gamma_n e^{-nE_g/T}\right), \tag{7}$$

where $\gamma_0 = \gamma_8 = 1$, $\gamma_1 = \gamma_7 = 1 - l_3 l_{\bar{3}}$, $\gamma_2 = \gamma_6 = 1 - 3l_3l_{\bar{3}} + l_3^3 + l_{\bar{3}}^3$, $\gamma_3 = \gamma_5 = -2 + 3l_3l_{\bar{3}} - (l_3l_{\bar{3}})^2$, $\gamma_4 = 2[-1 + l_3l_{\bar{3}} - l_3^3 - l_{\bar{3}}^3 + (l_3l_{\bar{3}})^2]$, and $l_\mu = {\rm tr}_c\Omega_\mu$ is the trace of the Polyakov loop in the representation μ.

3 Quantum and local features of the Polyakov loop

At low T, the gluonic action is small and Ω at any given point $\boldsymbol{x}$ is randomly distributed over the gauge group. A convenient model to account for correlations of two Polyakov loops at different points is $\langle \mathrm{tr}_c\Omega(\boldsymbol{x})\ \mathrm{tr}_c\Omega^{-1}(\boldsymbol{y})\rangle_{S_g} = e^{-\sigma|\boldsymbol{x}-\boldsymbol{y}|/T}$, with σ the string tension.[1] Different values of the spatial coordinate are suppressed due to screening of the color charge, and this defines independent confinement domains with volume $V_\sigma = 8\pi T^3/\sigma^3$. In view of this, instead of trying to model the higher-point correlation functions appearing in the thermal expansion, we adopt the following approach: we assume that the space is decomposed into domains of size V_σ, in such a way that two Polyakov loops are fully correlated if they lie within the same domain and are fully uncorrelated otherwise. This implies i) that each domain can be treated separately and ii) that the Lagrangian density is x-independent inside each domain. Therefore, the contribution to the partition function of any such domain is

$$Z = \int d\Omega\, e^{-\frac{V_\sigma}{T}(\mathscr{L}_q+\mathscr{L}_g)}\,. \tag{8}$$

This expression tells us that $e^{-\frac{V_\sigma}{T}(\mathscr{L}_q+\mathscr{L}_g)}$ represents the (unnormalized) probability density of the variable Ω, within the quark-gluon constituent model.

4 Partition function

By plugging Eqs. (4) and (7) into Eq. (8) and performing the integration in $\mathrm{SU}(N_c)$, one finds

$$\begin{aligned} Z &= 1+\lambda^2\frac{1}{2}\left(2Q_1\bar{Q}_1+G_1^2+G_2\right) \\ &\quad +\lambda^3\frac{1}{6}\left(Q_1^3+3Q_1Q_2+2Q_3+\bar{Q}_1^3+3\bar{Q}_1\bar{Q}_2+2\bar{Q}_3+6Q_1\bar{Q}_1G_1+2G_1^3+4G_3\right)+\mathscr{O}(\lambda^4)\,, \end{aligned} \tag{9}$$

where we have defined $Q_n(T) = 2N_f V_\sigma \xi^n J_n(M_q,T)$ and $G_n(T) = 2V_\sigma J_n(M_g,T)$, for quarks and gluons respectively. $\bar{Q}_n$ is numerically identical to Q_n but it accounts for n antiquarks, i.e. $\bar{Q}_n \sim \xi^{-n}$. Each factor Q_n, $\bar{Q}_n$ or G_n counts as n quarks, antiquarks or gluons, respectively. For instance, the term $Q_1\bar{Q}_1G_1$ has a content $[q\bar{q}g]$. We can present the result in a schematic way (including terms $\mathscr{O}(\lambda^4)$)

$$\begin{aligned} Z &\simeq 1+g_f^2[q\bar{q}]+3[g^2]+\frac{1}{6}g_f(g_f+1)(g_f+2)\left([q^3]+[\bar{q}^3]\right)+2g_f^2[q\bar{q}g]+4[g^3] \\ &\quad +\frac{1}{2}g_f^2(g_f^2+1)[q^2\bar{q}^2]+\frac{2}{3}g_f(g_f^2-1)\left([q^3g]+[\bar{q}^3g]\right)+7g_f^2[q\bar{q}g^2]+7[g^4]+\mathscr{O}(\lambda^5)\,, \end{aligned} \tag{10}$$

where $g_f = 2N_f$. The factor in front of each term corresponds to the degeneracy. At order λ^2 there are contributions from mesons $[q\bar{q}]$ and bound states of two gluons $[g^2]$, while at order λ^3 there are contributions from baryons $[q^3]$, antibaryons $[\bar{q}^3]$, and other bound states involving quarks and gluons.[2] At any order, the model accommodates multiparton states with the correct counting. Some of them, like $[q\bar{q}g]$, are irreducible color clusters which correspond to hybrids, while other, like $[q^2\bar{q}^2]$, can be suitably arranged as a multihadron configuration.

[1]This formula is consistent with the group identity $\int d\Omega\, \mathrm{tr}_c\Omega\, \mathrm{tr}_c\Omega^{-1} = 1$.

[2]The identification of these quark and gluon states with hadrons and glueballs follow after quantization, as explained in [12].

5 Polyakov loop

The vacuum expectation value of the Polyakov loop in the representation μ can be computed as

$$\langle l_\mu \rangle = \frac{1}{Z} \int d\Omega \, l_\mu \, e^{-\frac{V_\sigma}{T}(\mathscr{L}_q + \mathscr{L}_g)} , \qquad l_\mu = \mathrm{tr}_c \Omega_\mu \, . \tag{11}$$

Following the same procedure as for the partition function, we get for the fundamental representation

$$\begin{aligned} \langle l_3 \rangle &= \lambda \bar{Q}_1 + \lambda^2 \frac{1}{2} \left(Q_1^2 + Q_2 + 2\bar{Q}_1 G_1 \right) + \lambda^3 \left(Q_1^2 + \bar{Q}_1 G_1 \right) G_1 + \mathscr{O}(\lambda^4) , \\ &\simeq g_f [h\bar{q}] + \frac{1}{2} g_f (g_f + 1) [hq^2] + 2 g_f [h\bar{q}g] + 2 g_f^2 [hq^2 g] + 4 g_f [h\bar{q}g^2] + \mathscr{O}(\lambda^4) \, . \end{aligned} \tag{12}$$

We have identified the Polyakov loop itself with a heavy quark source "h", which is screened by dynamical (anti)quarks and gluons from the medium. The first two terms $[h\bar{q}]$ and $[hq^2]$ correspond to mesons and baryons, respectively, with a heavy quark and one or several light (anti)quarks. This completes the connection with the HRGM for the Polyakov loop [1]. It is noteworthy that tetraquark states $[h\bar{q}^2 q]$ that do appear in numerator and denominator of (11), cancel in $\langle l_3 \rangle$. This implies that all the states of the type $[h\bar{q}^2 q]$ in the numerator (namely, the partition function in presence of the fundamental source h) can be accounted for by clusters of the type $[h\bar{q}][\bar{q}q]$, that is, light-heavy hybrid mesons $[h\bar{q}]$ from straight screening of the source, plus an ordinary meson $[q\bar{q}]$ occurring in the domain of the source. On the other hand, the other contributions can not be arranged into two or more singlet subclusters and should be regarded as genuine states screening the source. The situation becomes more involved when more partons are included [13].

6 Conclusions

We have studied the low temperature regime of QCD by using a chiral quark model with Polyakov loop. When the local and quantum nature of the Polyakov loop is taken into account, we find a clear connection with the hadron resonance gas description. In addition, the model definitely contains some quark-gluon hybrid states that could be exposed by saturation of the Polyakov loop expectation value sum rule in Eq. (1). The status of other exotic states of the type tetraquark or pentaquark is much less clear. A high precision computation of observables at low temperature in lattice can serve as a powerful tool to disentangle this rich content of the QCD spectrum.

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