The nucleon mass and pion-nucleon sigma term from a chiral analysis of $N_f = 2$ lattice QCD world data

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Abstract. We investigate the pion-mass dependence of the nucleon mass within the covariant $SU(2)$ baryon chiral perturbation theory up to order p^4 with and without explicit $\Delta(1232)$ degrees of freedom. We fit lattice QCD data from several collaborations for 2 and 2+1 flavor ensembles. Here, we emphasize our $N_f = 2$ study where the inclusion the $\Delta(1232)$ contributions stabilizes the fits. We correct for finite volume and spacing effects, set independently the lattice QCD scale by a Sommer-scale of $r_0 = 0.493(23)$ fm and also include one $\sigma_{\pi N}$ lQCD data point at $M_\pi \approx 290$ MeV. We obtain low-energy constants of natural size which are compatible with the rather linear pion-mass dependence observed in lattice QCD. We report a value of $\sigma_{\pi N} = 41(5)(4)$ MeV for the 2 flavor case and $\sigma_{\pi N} = 52(3)(8)$ MeV for 2+1 flavors.

1 Introduction

The current quark mass dependence of observables from lattice QCD simulations (lQCD) for unphysical values provides an additional perspective on QCD itself. Many simulations with two dynamical degenerated light quarks ($N_f = 2$) or two degenerated light and one heavy quark ($N_f = 2 + 1$) are now available with light quark masses spanning from unphysical heavy values down to nearly physical ones. To investigate this extensive data set the (baryon) chiral perturbation theory (BχPT) represents a prominent tool. By matching BχPT to lQCD data for unphysical quark masses, low-energy constants (LECs) can be extracted, which are then used for predictions at the physical point.

We performed such a matching for the quark-mass ($m_u = m_d = \overline{m}$) dependence of the nucleon mass $M_N(\overline{m})$ by separately fitting to $N_f = 2$ and $2 + 1$ lQCD data. We concentrate here on our $N_f = 2$ results and refer to [1] for more details on the $2 + 1$ ones.

An important derivative of $M_N(\overline{m})$ is given by the Hellmann-Feynman theorem:

$$\overline{m}\frac{\partial}{\partial\overline{m}}M_N(\overline{m}) = \sigma_{\pi N} = \overline{m}\langle N|\overline{u}u + \overline{d}d|N\rangle \quad , \tag{1}$$

which relates $M_N(\overline{m})$ to the so-called $\sigma_{\pi N}$ term. The $\sigma_{\pi N}$ term can be seen as a measure of the contribution from the explicit chiral symmetry breaking to the nucleon mass. Since the $\sigma_{\pi N}$ can also be defined by the nucleon scalar form factor at zero four-momentum transfer squared, it can also

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be isolated from πN-scattering data. In this sense, the quark-mass dependence of the nucleon mass relates lQCD, BχPT and experiment.

2 Nucleon mass and covariant baryon chiral perturbation theory

The chiral structure of the nucleon mass is parametrized by the covariant BχPT up to order p^4 as:

$$M_N^{(4)}\left(M_\pi^2\right) = M_0 - c_1 4M_\pi^2 + \frac{1}{2}\overline{\alpha}M_\pi^4 + \frac{c_1}{8\pi^2 f_\pi^2}M_\pi^4 \ln\frac{M_\pi^2}{M_0^2} + \Sigma_{loops}^{(3)+(4)}\left(M_\pi^2\right) + O\left(p^5\right) . \tag{2}$$

where $M_\pi^2 \sim \overline{m}$ and f_π are the pion mass and pion decay constant. The loop-contributions $\Sigma_{loops}^{(3)+(4)}$ can also contain explicit $\Delta(1232)$ contributions. We refer to Ref. [1] for all the details. We fit the nucleon chiral-limit mass M_0 and the two LECs c_1 and $\overline{\alpha}$ to lQCD data and obtain through Eq. (1) a $\sigma_{\pi N}$ value. Explicitly, we use the $N_f = 2$ lQCD data in its dimensionless from $(r_0 M_\pi, r_0 M_N)$, with r_0 being the Sommer-scale, and minimize the function:

$$\chi^2 = \sum_i \left[\frac{\widetilde{M}_N^{(4)}\left(\widetilde{M}_\pi^2\right) + \widetilde{\Sigma}_N^{(4)}\left(\widetilde{M}_\pi^2, L\right) + \tilde{c}_a \tilde{a}^2 - d_i\left(\widetilde{M}_\pi^2, L\right)}{\sigma_i}\right]^2 , \tag{3}$$

with $\quad \widetilde{M}_N^{(4)} = r_0 M_N^{(4)} , \quad \widetilde{M}_\pi^2 = (r_0 M_\pi)^2 , \quad \widetilde{\Sigma}_N^{(4)} = r_0 \Sigma_N^{(4)} , \tag{4}$

where $d_i\left(\widetilde{M}_\pi^2, L\right)$ are the data points with uncertainties σ_i for a lattice of size L and spacing a. The terms $\tilde{c}_a a^2$ parametrize finite spacing effects for each lQCD action separately. The self-energy $\Sigma_N^{(4)}$ contains also finite volume corrections. Furthermore, we use the physical nucleon mass to determine the Sommer-scale r_0 recursively and self-consistently inside the fit. In our $SU(2)$ fits to $N_f = 2 + 1$ data we assume that the strange quark contributions are integrated out and absorbed into the LECs.

To ensure controlled finite volume effects and an acceptable chiral convergence of our BχPT results, we restrict our fits to M_N-data points fulfilling $M_\pi L > 3.8$ and $r_0 M_\pi < 1.11$. Additionally, the QCDSF collaboration obtained one direct $\sigma_{\pi N}$ data point at $M_\pi \approx 290$ MeV [2] with which we also perform simultaneous fits to nucleon mass data and that $\sigma_{\pi N}(290)$ point.

3 Results and conclusions

We summarize some of our results of Ref. [1]. Figure 1 shows our fits to the lQCD data from the $N_f = 2$ ensembles of the BGR, ETMC, Mainz and QCDSF collaborations [2–5]. We fitted the data without and with explicit $\Delta(1232)$ contributions, left and right figures respectively. The dashed and solid lines correspond to the ex-/inclusion of the one $\sigma_{\pi N}(290)$ point, respectively. Our fits yield $2 < \chi^2/d.o.f. < 3$ reflecting that some of the data are marginally consistent.

By including the one $\sigma_{\pi N}(290)$ point in the $\Delta(1232)$-less fit we reduce the uncertainties, although, the shape of the pion-mass dependence changes noticeably. This is also seen in the obtained $\sigma_{\pi N}$ value at the physical point, which changes from $\sigma_{\pi N} = 62(13)$ MeV to 41(3) MeV. The situation is different when we include the explicit $\Delta(1232)$ contributions in our fit formula. For this case, the inclusion of the one $\sigma_{\pi N}(290)$ point does not change the pion-mass dependence much. Differences are mainly visible at higher pion masses and the $\sigma_{\pi N}$ term at the physical point turns out to be $\sigma_{\pi N} = 41(3)$ MeV for both cases. We conclude that for the present data situation the inclusion of $\Delta(1232)$ contributions stabilizes the fits and that the reported (only one) $\sigma_{\pi N}(290)$ point is more compatible with the BχPT with $\Delta(1232)$ rather than with the $\Delta(1232)$-less one. It will be interesting so see if further direct lQCD calculations of the $\sigma_{\pi N}$ at unphysical pion-masses will confirm this conclusion.
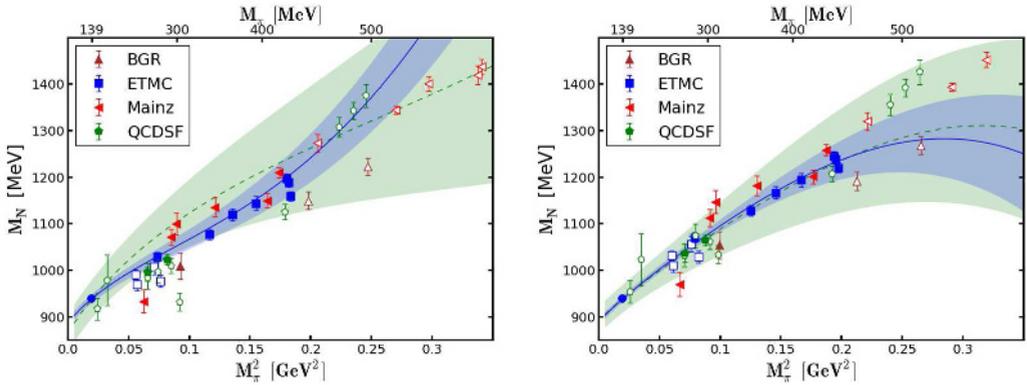
Figure 1. Pion-mass dependence of the nucleon mass from $N_f = 2$ fits. We show BχPT p^4 fits without (left) and with (right) explicit $\Delta(1223)$ degrees of freedom. The green-dotted lines correspond to fits to nucleon mass data alone and the blue-solid lines to simultaneous fits including also the $\sigma_{\pi N}(290)$ QCDSF data point. The lQCD data points are scaled by our $r_0 \approx 0.490$ fm obtained in the fits with $\Delta(1232)$.

The left panel of Fig. 2 shows the pion-mass dependence of the $\sigma_{\pi N}$ term obtained from our BχPT fits with $\Delta(1232)$ contributions, solid line, and without, dashed-line. As discussed above, the changes of the slope of the dashed line could indicate that the $\Delta(1232)$-less BχPT has problems to account for the $\sigma_{\pi N}(290)$ point. As for our final $\sigma_{\pi N}$ value at the physical point for the $N_f = 2$ fits we quote:

$$\sigma_{\pi N}^{N_f=2} = 41(5)(4) \text{ MeV} , \tag{5}$$

which corresponds to our result for the BχPT with $\Delta(1232)$ contributions. For the determination of the uncertainties we refer to Ref. [1].

The right panel of Fig. 2 compares our BχPT results from fits to $N_f = 2$ and $N_f = 2 + 1$ data. Differences are within the errorbars of the input data but translate into a ~ 11 MeV difference in the $\sigma_{\pi N}$ value. We obtain for our $N_f = 2 + 1$ fits the higher value of

$$\sigma_{\pi N}^{N_f=2+1} = 52(3)(8) \text{ MeV} . \tag{6}$$

Both values are compatible within the uncertainties, however, with the present data we cannot unambiguously determine the origin of the 11 MeV difference of the central values. As a first point, we see that the $N_f = 2$ data does not constrain the small pion-mass region much. This results in an up to ~ 13 % smaller c_1 value which determines solely the $\sigma_{\pi N}$ at leading order. As a second point, the $\sigma_{\pi N}(290)$ data point brought in the $N_f = 2$ case the $\Delta(1232)$ and $\Delta(1232)$-less results together. Such low-M_π $\sigma_{\pi N}$ data points are not available for the $N_f = 2 + 1$ case and our fits yield by $9 \sim 11$ MeV different $\sigma_{\pi N}$ results for the two cases. The above $N_f = 2 + 1$ value is the average of our $\Delta(1232)$ and $\Delta(1232)$-less results. Furthermore, since the strange quark is treated differently in the two lQCD data sets, one could expect that part of the difference comes also from this fact. Note also that the latest value extracted from pure π-N scattering data yields $\sigma_{\pi N} = 59(7)$ MeV [6].

In summary, we fitted lQCD data for $N_f = 2$ and $N_f = 2 + 1$ ensembles by a $SU(2)$ BχPT formula up to p^4 with and without $\Delta(1232)$ degrees of freedom. Even though the present data set is extensive, we observed systematic uncertainties stemming mostly from the distribution of the data points. New data for the following cases would further reduce these systematic effects: a) more $N_f = 2$ data points
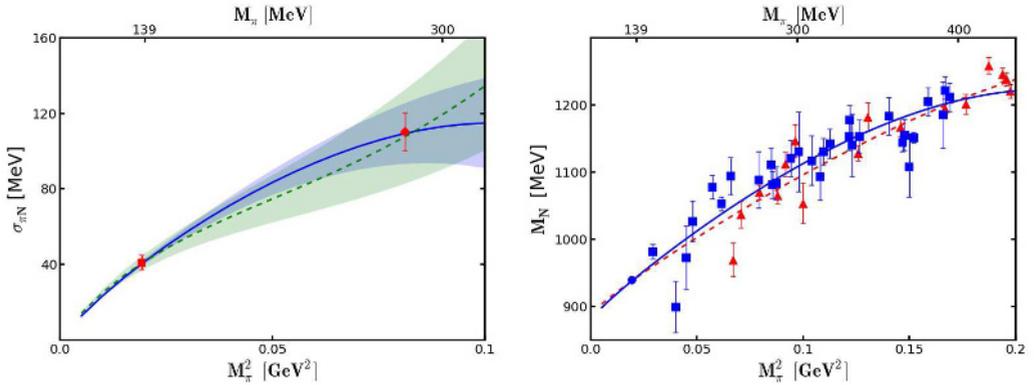
Figure 2. Left: Pion-mass dependence of the $\sigma_{\pi N}$ term from $N_f = 2$ fits. The blue-solid (green-dotted) line corresponds to the BχPT p^4 fit with(out) the $\Delta(1232)$ with including the $\sigma_{\pi N}(290)$ data point (red-diamond). The red-square is our predicted $\sigma_{\pi N}$ value at the physical point. Right: Comparison of BχPT p^4 fits with $\Delta(1232)$ to $N_f = 2$ (red-triangles) and $N_f = 2 + 1$ (blue-squares) lQCD data. The blue-circle is the physical nucleon mass.

in the low-M_π region, b) even one direct calculation of the $\sigma_{\pi N}$ at $M_\pi < 300$ MeV for the $N_f = 2 + 1$ case.

Acknowledgements

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