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Chiral Structure of Baryon and Scalar Tetraquark Currents

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Abstract. We investigate chiral properties of local fields of baryons consisting of three quarks with flavor $SU(3)$ symmetry. We construct explicitly independent local three-quark fields belonging to definite Lorentz and flavor representations. We discuss some implications of the allowed chiral symmetry representations on physical quantities such as axial coupling constants and chiral invariant Lagrangians. We also systematically investigate chiral properties of local scalar tetraquark currents, and study their chiral transformation properties.

1 Introduction

As the chiral symmetry of QCD is spontaneously broken, $SU(N_f)_L \otimes SU(N_f)_R \rightarrow SU(N_f)_V$ (N_f being the number of flavors), the observed hadrons are classified by the residual symmetry group representations of $SU(N_f)_V$. The full chiral symmetry may then conveniently be represented by its non-linear realization and this broken symmetry plays a dynamical role in the presence of the Nambu-Goldstone bosons to dictate their interactions.

Yet, as pointed out by Weinberg [1, 2], there are situations when it makes sense to consider algebraic aspects of chiral symmetry, i.e. the chiral multiplets of hadrons. Such hadrons may be classified in linear representations of the chiral symmetry group with some representation mixing. While the chiral representation can also be used as a theoretical probe for the internal structure of hadrons [3–7].

Motivated by these arguments, we perform a complete classification of baryon fields written as local products of three quarks according to chiral symmetry group $SU(3)_L \otimes SU(3)_R$ [8–11]. We also systematically classified the scalar tetraquark currents and study their chiral transformation properties [12–14].

2 Baryon Currents

Local fields for baryons consisting of three quarks can be generally written as

$$B(x) \sim \epsilon_{abc} \left(q_A^{aT}(x) C \Gamma_1 q_B^b(x) \right) \Gamma_2 q_C^c(x) \,, \tag{1}$$

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where a, b, c denote the color and A, B, C the flavor indices, $C = i\gamma_2\gamma_0$ is the charge-conjugation operator, $q_A(x) = (u(x),\ d(x),\ s(x))$ is the flavor triplet quark field at location x, and the superscript T represents the transpose of the Dirac indices only (the flavour and colour $SU(3)$ indices are *not* transposed). The antisymmetric tensor in color space ϵ_{abc}, ensures the baryons' being color singlets. For local fields, the space-time coordinate x does nothing with our studies, and we shall omit it.

The three-quark fields may belong to one of several different Lorentz group representations which fact imposes certain constraints on possible chiral symmetry representations. This is due to the Pauli principle and can be explicitly verified by the method of Fierz transformations. As shown in Ref. [8], for Dirac fields without Lorentz index, there are one singlet field Λ and two octet fields N_1^N and N_2^N:

$$\begin{aligned}
\Lambda_1 &= \epsilon_{abc}\epsilon^{ABC}(\bar{q}_A^a C q_B^b)\gamma_5 q_C^c\,, \\
N_1^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(\bar{q}_A^a C q_B^b)\gamma_5 q_C^c\,, \\
N_2^N &= \epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(\bar{q}_A^a C \gamma_5 q_B^b) q_C^c\,.
\end{aligned}$$

For the Rarita-Schwinger fields with one Lorentz index, there are two non-vanishing independent fields (also independent of the previous three Dirac fields):

$$\begin{aligned}
N_\mu^N = P_{\mu\nu}^{3/2} N_{3\nu}^N &= P_{\mu\nu}^{3/2}\epsilon_{abc}\epsilon^{ABD}\lambda_{DC}^N(\bar{q}_A^a C\gamma_\mu\gamma_5 q_B^b)\gamma_5 q_C^c \\
\Delta_\mu^P = P_{\mu\nu}^{3/2}\Delta_{5\nu}^P &= P_{\mu\nu}^{3/2}\epsilon_{abc} S_P^{ABC}(\bar{q}_A^a C\gamma_\mu q_B^b) q_C^c\,,
\end{aligned}$$

where $P_{\mu\nu}^{3/2}$ is the projection operator:

$$P_{\mu\nu}^{3/2} = (g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu)\,. \tag{2}$$

For tensor fields with two antisymmetric Lorentz indices, there is only one non-vanishing independent field (also independent of the previous three Dirac fields and two Rarita-Schwinger fields):

$$\Delta_{\mu\nu}^P = \Gamma^{\mu\nu\alpha\beta}\Delta_{7\mu\nu}^P = \Gamma^{\mu\nu\alpha\beta}\epsilon_{abc} S_P^{ABC}(\bar{q}_A^a C\sigma_{\mu\nu} q_B^b)\gamma_5 q_C^c\,,$$

where

$$\Gamma^{\mu\nu\alpha\beta} = (g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}g^{\nu\beta}\gamma^\mu\gamma^\alpha + \frac{1}{2}g^{\mu\beta}\gamma^\nu\gamma^\alpha + \frac{1}{6}\sigma^{\mu\nu}\sigma^{\alpha\beta})\,. \tag{3}$$

We also perform chiral transformations and verify that Λ and $N_1^N - N_2^N$ are together combined into one chiral multiplet $(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$; N_μ^N and Δ_μ^P are together combined into another chiral multiplet $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$; while $N_1^N + N_2^N$ and $\Delta_{\mu\nu}^P$ are transformed into themselves under chiral transformation, and they belong to chiral representations $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ and $(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$, respectively.

As simple applications of the present mathematical formalism, we can extract the (diagonal) axial coupling constants g_A for these baryons, as shown in Table. 1; while we can also construct all $SU_L(3)\times SU_R(3)$ chirally invariant non-derivative one-meson-baryon interactions [10, 11]. These baryon fields can be used in the Lattice QCD and QCD sum rule calculations [15–17].

3 Scalar Tetraquark Currents

Multi-quark currents can be used to study exotic hadrons, such as hybrid states, glueballs, tetraquark states and molecular states, etc.. Particularly, the light scalar mesons $\sigma(600)$, $\kappa(800)$, $a_0(980)$ and $f_0(980)$ are good tetraquark candidates [18–24].

Table 1. Axial coupling constants g_A^0, g_A^3 and g_A^8. In the last column $\alpha = g_A^D/(g_A^F + g_A^D)$.

$SU(3)_L \otimes SU(3)_R$	$SU(3)_F$		g_A^0	g_A^3	g_A^8	α
$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	**1**	Λ	-1	–	–	–
	8	N_-	-1	1	-1	1
		Σ_-	-1	0	2	
		Ξ_-	-1	-1	-1	
		Λ_-	-1	–	-2	
$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	**8**	N_+	3	1	3	0
		Σ_+	3	1	0	
		Ξ_+	3	1	-3	
		Λ_+	3	–	0	
$(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})$	**8**	N_μ	1	5/3	1	3/5
		Σ_μ	1	2/3	2	
		Ξ_μ	1	-1/3	-3	
		Λ_μ	1	–	-2	
	10	Δ_μ	1	1/3	1	–
		Σ^*_μ	1	1/3	0	
		Ξ^*_μ	1	1/3	-1	
		Ω_μ	1	–	-2	
$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$	**10**	$\Delta_{\mu\nu}$	3	1	3	–
		$\Sigma^*_{\mu\nu}$	3	1	0	
		$\Xi^*_{\mu\nu}$	3	1	-3	
		$\Omega_{\mu\nu}$	3	–	-6	

In this section we shall study the scalar tetraquark currents. Let us consider currents for the tetraquark $ud\bar{s}\bar{s}$ having $J^P = 0^+$. Here we consider only local currents. To write a current, Lorentz and color indices are contracted with suitable coefficients ($L^{abcd}_{\mu\nu\rho\sigma}$) to provide necessary quantum numbers,

$$\eta = L^{abcd}_{\mu\nu\rho\sigma} \bar{s}^\mu_a \bar{s}^\nu_b u^\rho_c d^\sigma_d \, , \tag{4}$$

where the sum over repeated indices ($\mu, \nu, \cdots$ for Dirac spinor indices, and $a, b, \cdots$ for color indices) is taken. Again, due to the Pauli principle, there are five independent currents which can be explicitly verified by the method of Fierz transformations [12]:

$$\begin{aligned} S_6 &= (\bar{s}_a \gamma_5 C \bar{s}_b^T)(u_a^T C \gamma_5 d_b) \, , \\ V_6 &= (\bar{s}_a \gamma_\mu \gamma_5 C \bar{s}_b^T)(u_a^T C \gamma^\mu \gamma_5 d_b) \, , \\ T_3 &= (\bar{s}_a \sigma_{\mu\nu} C \bar{s}_b^T)(u_a^T C \sigma^{\mu\nu} d_b) \, , \\ A_3 &= (\bar{s}_a \gamma_\mu C \bar{s}_b^T)(u_a^T C \gamma^\mu d_b) \, , \\ P_6 &= (\bar{s}_a C \bar{s}_b^T)(u_a^T C d_b) \, . \end{aligned} \tag{5}$$

The similar results can be obtained for the general scalar tetraquark currents [13].

We also investigated the chiral structure of tetraquarks, and found all the chiral multiplets [14]. Since these calculations for tetraquark are much more complicated than those for baryon, at the first stage we only considered the chiral (flavor) structure and other degrees of freedom remain undetermined. We concentrated on the tetraquarks belonging to the "non-exotic" $[(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})]$ and

$[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ chiral multiplets as well as their mirror multiplets, which have the same representations as the lowest level $\bar{q}q$ chiral multiplets. We have studied their behaviors under the $U(1)_V$, $U(1)_A$, $SU(3)_V$ and $SU(3)_A$ chiral transformations. We found that most of them contain one pair of quark and antiquark which have the same chirality and can be combined to be a chiral singlet $\bar{q}q$, and so they can be constructed by adding one chiral singlet quark-antiquark pair to the lowest level $\bar{q}q$ chiral multiplets. Consequently, under chiral transformations they transform exactly like the lowest level $[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$ chiral multiplet (σ,π) and the $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ chiral multiplet (ρ,a_1). There is only one exception, $\mathcal{B}_3^{(\bar{\mathbf{3}},\mathbf{3})}$, whose quark-antiquark pairs all have the opposite chirality, and it transforms differently from other $[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$ chiral multiplets.

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