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Low-lying pseudoscalar and vector mesons and their dynamics

How to describe radiative reactions with an odd number of pions

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Abstract. In chiral perturbation theory, the leading-order contribution to reactions with pions in the sector of odd intrinsic parity is defined by the Wess-Zumino-Witten structure. This structure is supplemented by a simple vector-meson Lagrangian where the vector mesons are described by antisymmetric tensor fields. With the rho-omega-pion coupling as the only parameter in the sector of odd intrinsic parity, i.e. without additional contact terms, one can achieve a proper description of the single- and double-virtual pion transition form factor and the three-pion production in electron-positron collisions.

1 Introduction

One of the open challenges in particle physics is the description of strong interactions at low energies. Due to the running coupling constant in quantum chromodynamics, perturbation theory is not applicable for this energy regime. Instead, chiral perturbation theory (ChPT) [1] is used for very low energies where the light pseudoscalar mesons are the only relevant degrees of freedom.

In the sector of odd intrinsic parity (odd number of pseudoscalars) in ChPT, the leading-order action is determined by the chiral anomaly yielding the Wess-Zumino-Witten action. For two flavors the relevant leading-order terms for radiative reactions read

$$\mathcal{L}^{\rm ChPT} = w_1 \varepsilon^{\mu\nu\alpha\beta}\, \partial_\mu \pi^0\, \partial_\nu A_\alpha\, A_\beta \;+\; i w_2 \varepsilon^{\mu\nu\alpha\beta}\, \partial_\mu \pi^+\, \partial_\nu \pi^-\, \partial_\alpha \pi^0\, A_\beta \tag{1}$$

including the photon field A and describing a direct $\pi^0 - 2\gamma$ and $3\pi - \gamma$ vertex. The values of the parameters w_1 and w_2 and their relative sign are determined by the axial anomaly,

$$w_1 = -\frac{e^2}{8\pi^2 f_\pi},\; w_2 = \frac{e}{4\pi^2 f_\pi^3}\,. \tag{2}$$

To allow for the description of reactions at somewhat higher energies ($\lesssim 1\,\text{GeV}$) and of form factors we supplement (1) by the vector-meson Lagrangian

$$\mathcal{L}^{\rm vec.} = x_1 \left(3\rho^0_{\mu\nu} + \omega_{\mu\nu}\right) \partial^\mu A^\nu + x_2\, \mathrm{tr}\left(V_{\mu\nu}\, \partial^\mu \Phi \partial^\nu \Phi\right) + x_3 \epsilon^{\mu\nu\alpha\beta}\, \mathrm{tr}\left(\left\{V_{\mu\nu}, \partial^\tau V_{\tau\alpha}\right\} \partial_\beta \Phi\right)\,. \tag{3}$$

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Here, $V_{\mu\nu}$ denotes the flavor matrix for the light vector mesons given in antisymmetric tensor representation and Φ the pion matrix,

$$V_{\mu\nu} = \begin{pmatrix} \rho^0_{\mu\nu} + \omega_{\mu\nu} & \sqrt{2}\rho^+_{\mu\nu} \\ \sqrt{2}\rho^-_{\mu\nu} & -\rho^0_{\mu\nu} + \omega_{\mu\nu} \end{pmatrix}, \ \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}. \tag{4}$$

The parameters x_1 and x_2 including their relative signs and the modulus of x_3 can be fixed from the reactions $e^+e^- \to \pi^+\pi^-$ and $\omega \to \pi^+\pi^-\pi^0$ (see [2] for details). This Lagrangian does not describe a direct $\pi^0 - 2\gamma$ or $3\pi - \gamma$ vertex but instead vertices with vector mesons, i.e. a $V - \gamma$, $V - 2\pi$ and $2V - \pi$ vertex for $V = \rho, \omega$. It has been shown in [1] that the low-energy constants of ChPT at next-to-leading order are essentially saturated by vector-meson exchange.

2 Which Lagrangian should one use?

In ChPT, the leading-order contribution (1) to radiative reactions with an odd number of pseudoscalars is of order p^4 for a typical momentum p. Furthermore, all vector mesons are treated as heavy. Therefore, the Lagrangian (3) including vector mesons would be of order p^6 for momenta much smaller than the vector meson mass m_V. For $p \approx m_V$ the counting scheme of [3] treats both the low-lying pseudoscalar and vector-meson nonet as light. According to this counting scheme, the Lagrangian (3) is of order p^2 for momenta $p \approx m_V$ whereas the ChPT contribution (1) is still of order p^4. Thus, the two Lagrangians are formally never of the same order and it is not clear which one should be used for reactions covering energies between threshold and the resonance region: While for low energies $p \ll m_V$, the ChPT contribution (1) is the leading contribution, for higher energies $p \approx m_V$ only the Lagrangian (3) with vector mesons should be used in leading order. Here, we study a more phenomenological approach where we use both Lagrangians (1) and (3).[1]

3 Decay $\pi^0 \to \gamma e^+ e^-$: Add or subtract the two Lagrangians?

For the Lagrangians (1) and (3), the parameters and the relative signs within a Lagrangian are known. However, the sign of x_3 (relative to (1)) is not known yet. In other words, the relative sign between the two Lagrangians is unknown, i.e. whether one has to add or subtract them, is not determined yet. However, it can be determined via the decay $\pi^0 \to \gamma e^+ e^-$.

This decay can happen directly, described by (1), (see first diagram in Fig. 1) or via a virtual ω- and ρ^0-meson as described by (3) (see second diagram in Fig. 1). Adding / subtracting the two Lagrangians yields the normalized π^0-γ transition form factor

$$F(q^2 = m^2_{e^+e^-}) = 1 \pm \frac{x_1^2\,|x_3|}{w_1\, m^2_{\rho/\omega}}\,\frac{q^2}{q^2 - m^2_{\rho/\omega}} \tag{5}$$

with the averaged ρ-ω mass $m_{\rho/\omega}$. In Fig. 2, the form factor is plotted for both added (left-hand side) and subtracted Lagrangians (right-hand side) in comparison to experimental data [4]. Note that the absolute values of the parameters have *not* been fitted to the data. Only the form factor calculated with added Lagrangians is able to describe the data. Furthermore, the comparison to the experimental data shows that one needs both Lagrangians (1) and (3): If only the ChPT Lagrangian (1) is used, i.e. $x_1 = x_3 = 0$, the form factor is equal to one, $F \equiv 1$ and, thus, cannot describe the data. On the other hand, the ChPT contribution is needed to be able to define the normalized form factor with $F(0) = 1$.

[1] Note that although $\mathcal{L}^{\rm ChPT}$ is of next-to-leading order for $p \approx m_V$ this is no full next-to-leading order calculation. For such a calculation, the next-to-leading order Lagrangian including vector mesons and one-loop contributions would be needed in addition.

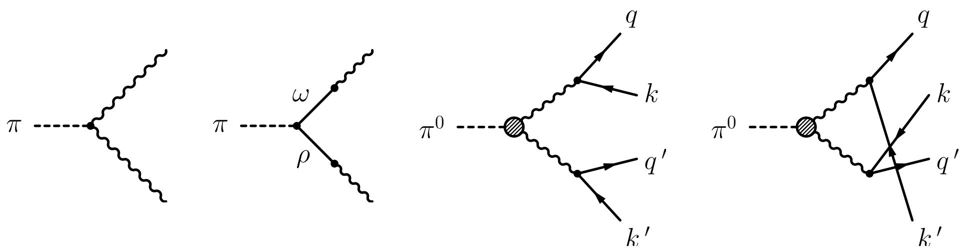

Figure 1. First and second diagram: Possibilities for the decay of a neutral pion into two (real or virtual) photons. **Third and forth diagram:** Possibilities for the decay into two electrons with momenta q and q' and two positrons with momenta k and k'. The shaded area represents the vertices from the first two diagrams.

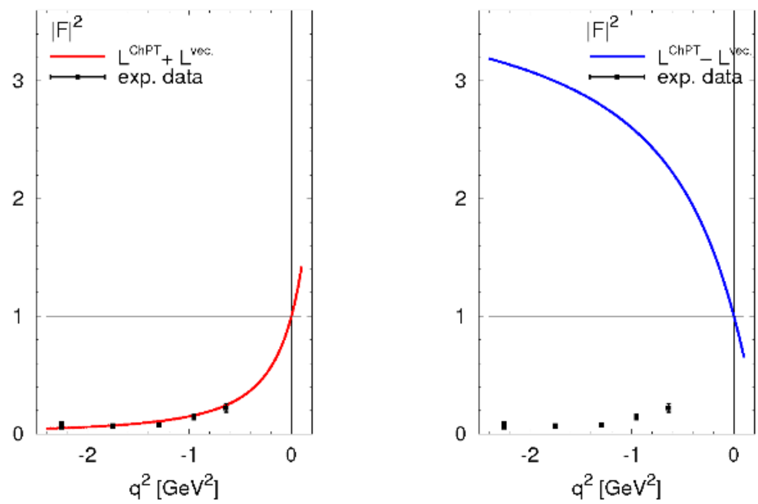

Figure 2. Normalized π^0-γ transition form factor (5) for added (**left**) and subtracted Lagrangians (**right**) compared to experimental data [4].

4 Applications

4.1 Decay $\pi^0 \to 2e^+2e^-$

Using the information from the previous section, one can calculate the decay $\pi^0 \to 2e^+2e^-$. There are two possible ways to produce the momenta of the electrons, (q, q'), and positrons, (k, k'), (see third and fourth diagram in Fig. 1) yielding two contributions to the partial decay width: An interference contribution depending on all possible combinations of two momenta and a direct contribution depending only on two such combinations. Thereby, the interference contribution is less than 1% of the direct contribution. The decay width is in very good agreement with experimental data,

$$\Gamma_{\pi^0\to 2e^+2e^-} = 2.68 \cdot 10^{-13}\,\text{GeV}, \tag{6}$$

$$\Gamma^{\text{exp.}}_{\pi^0\to 2e^+2e^-} = (2.58 \pm 0.13) \cdot 10^{-13}\,\text{GeV}\,. \tag{7}$$

4.2 Scattering $e^+e^- \to \pi^-\pi^+\pi^0$

This scattering reaction can happen directly, described by (1), or via a virtual ω- and ρ^0-meson, described by (3), (see left-hand side in Fig. 3). The cross section for momenta $\sqrt{s}$ between 0.6 and 1 GeV is plotted in Fig. 3 in comparison to experimental data [5, 6]. For higher momenta, the agreement is already good if only the vector Lagrangian (3) is used and the direct ChPT contribution has only a small influence whereas for small momenta both Lagrangians are needed in order to be able to describe the data.

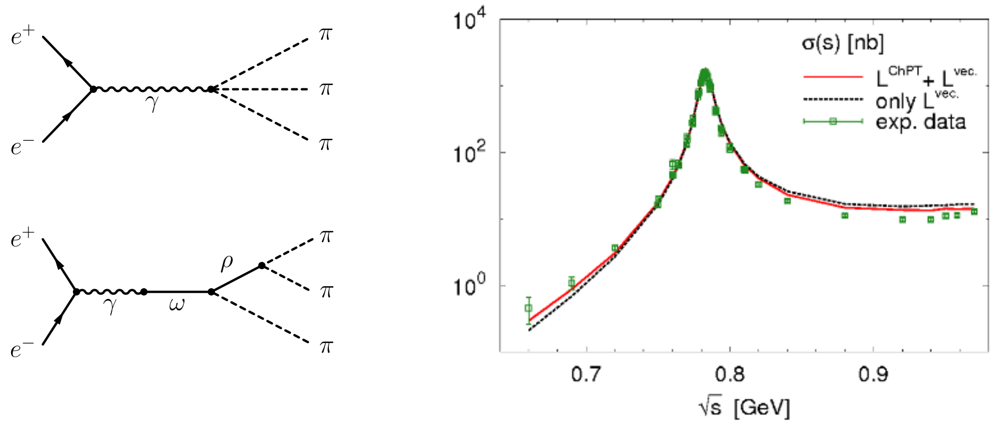

Figure 3. Left: Direct scattering reaction described by (1) (upper diagram) and reaction via intermediate vector mesons described by (3) (lower diagram). **Right:** Scattering cross section calculated with both Lagrangians (solid red) and with (3) only (dashed black) compared to experimental data [5, 6].

5 Summary

We calculated the π^0-γ transition form factor, the partial decay width for the decay $\pi^0 \to 2e^+2e^-$ and the scattering reaction $e^+e^- \to \pi^+\pi^-\pi^0$. Therefor, we used a simple vector-meson Lagrangian and the leading-order ChPT contribution to calculate radiative reactions with an odd number of pions. All results were in good agreement with the available experimental data.

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