Six-body calculations for hyperhydrogen ${}_{\Lambda}^{6}\mathrm{H}$

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Abstract . The binding energy and the structure of superheavy hydrogen-Λ are studied within the method of hyperspherical harmonics. The ${}_{\Lambda}^{6}\mathrm{H}$ hypernucleus is considered as a six body system consisting of Λ particle and five nucleons.

1. Introduction

A FINUDA experiment [1] reported a very important observation of superheavy hydrogen-Λ hypernucleus (with a bound state $B_{\Lambda} = 4.0 \pm 1.1$ MeV.), which goes far toward the nuclear drip line. Dalitz and Levi Setti [2] first pointed out the existence of ${}_{\Lambda}^{6}\mathrm{H}$ by using shell model; Akaishi et al. [3] have calculated the binding energy for ${}_{\Lambda}^{6}\mathrm{H}$ in a framework based on a coherent ΛN–ΣN coupling. The different properties of the hyperhydrogen were studied applying different methods of calculations using shell model [4] and three [5] and four body variational models [6]. Therefore, it is reasonable to study the structure of ${}_{\Lambda}^{6}\mathrm{H}$ within a microscopic approach. Below we present the computational approach based on the hyperspherical harmonics (HH) method developed in Ref. [7] and for the first time used to study superheavy hydrogen-Λ. The numbers of equations that have to be retained in any calculation using the (HH) method will, of course, depend on the nature of the potential used. In the present work, the Numerov method was used to obtain a converged set of coupled differential equations in a single variable. The eigenvalues and eigenfunctions of the hypernucleus ${}_{\Lambda}^{6}\mathrm{H}$ were determined. The Λ separation energy, B_{Λ}, depends on the spatial structure (size) of the core nucleus. Therefore, before calculating the binding energy of ${}_{\Lambda}^{6}\mathrm{H}$, it is necessary to analyze the property of the ${}^{5}\mathrm{H}$.

2. Method of HH

In the present section we give a brief description of the HH basis showing some properties of the basis that allow to use symmetrized basis elements to describe a system of few identical particles and one nonidentical particle. We start with the definition of the Jacobi coordinates for a six body system, with masses $m_1,..m_6$ and Cartesian coordinates $\mathbf{r_1},\dots\mathbf{r_6}$ [7,8]:

$$\mathbf{x} = \sqrt{\frac{m_i m_j}{m_i + m_j}}\left(\mathbf{r}_i - \mathbf{r}_j\right), \qquad \mathbf{y} = \sqrt{\frac{(m_i + m_j)m_k}{m_i + m_j + m_k}}\left(-\mathbf{r_k} + \frac{m_i\mathbf{r}_i + m_j\mathbf{r_j}}{m_i + m_j}\right),$$

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$$\mathbf{z} = \sqrt{\frac{(m_i + m_j + m_k)m_l}{m_i + m_j + m_l}}\left(-\mathbf{r}_l + \frac{m_i\mathbf{r_i} + m_j\mathbf{r_j} + m_k\mathbf{r_k}}{m_i + m_j + m_k}\right), \quad \mathbf{R} = \frac{m_1\mathbf{r}_1 + \ldots + m_6\mathbf{r_6}}{M}$$

$$\mathbf{w} = \sqrt{\frac{(m_i + m_j + m_k + m_l)m_q}{m_i + m_j + m_k + m_l + m_q}}\left(-\mathbf{r_q} + \frac{m_i\mathbf{r_i} + m_j\mathbf{r_j} + m_k\mathbf{r_k} + m_l\mathbf{r}_l}{m_i + m_j + m_k + m_l}\right), \quad M = \sum_{i=1}^{6}m_i, \tag{1}$$

$$\mathbf{u} = \sqrt{\frac{(m_i + m_j + m_k + m_l + m_q)m_p}{M}}\left(-\mathbf{r}_p + \frac{m_i\mathbf{r}_i + m_j\mathbf{r}_j + m_k\mathbf{r}_k + m_l\mathbf{r}_l + m_q\mathbf{r}_q}{m_i + m_j + m_k + m_l + m_q}\right)$$

We introduce hyperspherical variables

$$|\mathbf{x}| = \rho\cos\alpha\sin\beta\sin\gamma\sin\delta; \quad |\mathbf{y}| = \rho\sin\alpha\sin\beta\sin\gamma\sin\delta;$$

$$|\mathbf{z}| = \rho\cos\beta\sin\gamma\sin\delta; \quad |\mathbf{w}| = \rho\cos\gamma\sin\delta; \quad |\mathbf{u}| = \rho\cos\delta. \tag{2}$$

Where,

$$\rho^2 = \mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 + \mathbf{w}^2 + \mathbf{u}^2$$

The set of the hyperangles $\alpha, \beta, \gamma, \delta$ together with the direction of the Jacobi coordinates $\mathbf{x, y, z, w, u}$ form the hyperangular coordinates Ω. The six-body hyperspherical function is [8]:

$$\Phi_{K_3 K_4 K_5 K_6}^{l_1 l_2 l_3 l_{123} l_4 l_{1234} l_5 L}(\Omega) = N_{K_6+9/2}^{K_5+9/2, l_5}(\sin\delta)^{K_5}(\cos\delta)^{l_5}P_s^{K_5+5, l_5+1/2}(\cos 2\delta)[\Phi_5 Y_{l_5}(\hat{\mathbf{u}})]^L,$$

$$\Phi_5 = N_{K_5+3}^{K_4+3, l_4}(\sin\gamma)^{K_4}(\cos\gamma)^{l_4}P_p^{K_4+7/2, l_4+1/2}(\cos 2\gamma)N_{K_4+3/2}^{l_3, K_3+3/2}(\cos\beta)^{l_3} \times \tag{3}$$

$$(\sin\beta)^{K_3}P_m^{K_3+2, l_3+1/2}(\cos 2\beta)N_{K_3}^{l_1, l_2}(\cos\alpha)^{l_1}(\sin\alpha)^{l_2}P_n^{l_2+1/2, l_1+1/2}(\cos 2\alpha) \times$$

$$\left[\left[\left[Y^{l_1}(\hat{\mathbf{x}})Y^{l_2}(\hat{\mathbf{y}})\right]^{l_{12}}Y^{l_3}(\hat{\mathbf{z}})\right]^{l_{123}}Y^{l_4}(\hat{\mathbf{w}})\right]^{l_{1234}}.$$

Where,

$$N_a^{bc} = \sqrt{\frac{2d!(a+2)\Gamma(d+c+b+2)}{\Gamma(d+b+3/2)\Gamma(d+c+3/2)}}; \quad 2d = a - b - c. \tag{4}$$

Choosing the coordinates (1) the function (3) is a product of the five –particle hyperspherical function with the function, coming from the addition of the sixth particle.

$^6_\Lambda$H hypernucleus consist of five identical particles and Λ hyperon, the symmetrization with respect to five particles may be done easily with the use of five body symmetrization coefficients [9], because coordinates from $\mathbf{x, y, z, w, u}$ (1) coincide with five particle Jacobi coordinates. For six-body a construction scheme of the symmetrized HH basis for (5+1) configuration is given [8]:

$$\Phi_{K_6 L}^{[f]\lambda_{[\bar{f}]}\nu_{[\bar{f}]}}(\Omega) = \sum_{\alpha} C^{[\bar{f}]\lambda_{[\bar{f}]}\nu_{[\bar{f}]}\alpha}\Phi_{K_6 L}^{[\bar{f}]\lambda_{[\bar{f}]}\nu_{[\bar{f}]}\alpha}(\Omega), \quad \alpha \equiv \{l_{1234}l_5 K_5\} \tag{5}$$

Where $C^{[\bar{f}]\lambda_{[\bar{f}]}\nu_{[\bar{f}]}}$ are five body symmetrzation coefficients, [f] is the Young diagram of the six particle system, $\lambda_{[\bar{f}]}$ denotes rows of the [f] representation, and $\nu_{[\bar{f}]}$ is the [f] representation number with given K_6 and L. The transformation coefficients of the function $\Phi_{K_3 K_4 K_5 K_6}^{l_1 l_2 l_3 l_{123} l_4 l_{1234} l_5 L}(\Omega)$ under permutations were obtained with the use of six-body Raynal-Revai coefficients [8].

3. Results and Discussion

The Schrödinger equation for the $^6_\Lambda$H hypernucleus system expressed in terms of the hyperspherical functions becomes an infinite set of second order coupled differential equations:

$$-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial\rho^2}+\frac{17}{\rho}\frac{\partial}{\partial\rho}-\frac{K(K+16)}{\rho^2}-E\right)U_{[\alpha]}(\rho)+V_{[\alpha][\alpha']}(\rho)U_{[\alpha']}(\rho)=0 \qquad (6)$$

$U_{[\alpha]}(\rho)$ -are the radial wave functions. In the case of a central two-body interaction $V=\sum_{i<j}V(i,j)$,

its matrix elements in terms of the HH basis are

$$V_{[\alpha][\alpha']}(\rho)=\sum_{[\alpha']}\left\langle\Phi_{K_6 L}^{[\bar{f}]\lambda_{[\bar{f}]}\nu_{[\bar{f}]}\alpha'}V(\rho)\Phi_{K_6 L}^{[\bar{f}]\lambda_{[\bar{f}]}\nu_{[\bar{f}]}\alpha}\right\rangle \qquad (7)$$

The renormalized Numerov method was then used to solve the set of coupled equations (6).
In order to study the convergence of the hypernucleus energy eigenvalue, using the Volkov nucleon-nucleon NN interaction

$$V(r)=V_1 e^{-\frac{r^2}{R_1^2}}+V_2 e^{-\frac{r^2}{R_2^2}} \qquad (8)$$

With V_1 = 144.86 MeV, R_1 = 0.82 fm, V_2 = −83.34 MeV, R_2 = 1.6 fm and for the ΛN interaction we employ an effective single-channel Nijmegen potential [10], where ΛN-ΣN coupling effect are renormalized into ΛN-ΛN transition. We have solved K coupled equations for K_6 = 0, 2, 4, 6, 8. In order to obtain for the $_\Lambda^6 H$ hypernucleus binding energy value of E_B=-5.12 MeV. Our calculation confirms the fast convergence for the ground state energy as is shown in the table 1. The first four partial waves are given in Figure 1.

Table1. Energy eigenvalues E_B, of $_\Lambda^6$H as a function of the number K_6 of coupled equations.

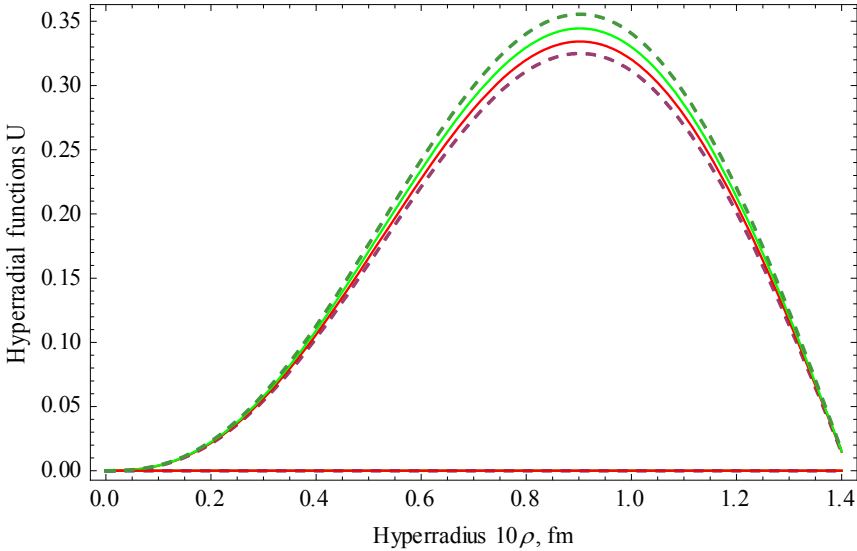| K_6	0	2	4	6	8
E_B, MeV	-3.48	-4.28	-4.83	-5.08	-5.12

Figure 1. First four hyperradial partial waves U for the $_\Lambda^6$H hypernucleus obtained in the approximation K_6= 0 (red dash curve), K_6= 0, 2(red solid curve), K_6 = 0, 2, 4 (green solid curve), and K_6 = 0, 2, 4, 6, 8 (green dash curve).

In order to demonstrate how B_Λ ($^6_\Lambda$H) depends on the energy of the core nucleus ^5H, we assume that the ^{5}H nucleus is represented by the *tnn* three-body system. There are several different ways of locating quantum resonances. In our paper we are using direct calculation of the Jost function using the method from Ref. [11]. For the V_{tn}, we employ a potential from Ref. [12]. Possible bound and resonant state of the (*tnn*) systems are required as zeros of the corresponding three body Jost functions calculated by using three body Hyperspherical approach.

We have the resonance pole at $E_r = 1.21$ MeV and $\Gamma = 1.97$ MeV close to the experimental value ($E_r = 1.7 \pm 0.3$ MeV and $\Gamma = 1.9 \pm 0.4$ MeV). In this case we have B_Λ=-3.78 MeV, which is in good agreement with the experimental value [1].

In this work we have shown results using the symmetrized HH expansion in the description of a 6-body $^6_\Lambda$H system. We have restricted the analysis to consider central potentials. This method makes possible to take care of interactions depending on spin and isospin degrees of freedom as the realistic two and three body baryon -baryon potentials. A preliminary analysis in this direction is in progress.

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