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Modelling of natural and bypass transition in aerodynamics

Jiří Fürst[1a], Petr Straka[2] and Jaromír Příhoda[3]

[1] Faculty of Mechanical Engineering, Czech Technical University in Prague, Czech Republic
[2] Aerospace Research and Test Establishment, Plc, Prague, Czech Republic
[3] Institute of Thermomechanics AS CR, v.v.i., Prague, Czech Republic

Abstract. Most of transition models are proposed for modelling of the bypass transition common in the internal aerodynamics especially in turbomachinery where free stream turbulence is the dominant parameter affecting the transition onset. Free-stream turbulence level in the external aerodynamics is usually noticeably lower and so the natural transition often occurs in flows around airfoils. The transition model with the algebraic equation for the intermittency coefficient proposed originally by Straka and Příhoda [3] for the bypass transition was modified for modelling of the transition at low free-stream turbulence. The modification is carried out using experimental data of Schubauer and Skramstad [18]. Further, the three-equation k-k_L-ω model proposed by Walters and Cokljat [10] was used for the modelling of the transition at low free-stream turbulence. Both models were tested by means of the incompressible flow around airfoils at moderate and very low free-stream turbulence.

1 Introduction

The modelling of the laminar/turbulent transition is crucial for the adequate simulation of flows in internal and external aerodynamics, especially for flows around blades and airfoils where substantially influences the prediction of energy losses and heat transfer.

Most of transition models are based on the transport and/or algebraic equation for the intermittency coefficient (see e.g. Langtry and Menter [1], Lodefier and Dick [2], Straka and Příhoda [3]). Nevertheless, these models need empirical correlations for the onset and length of the transition region and their application is limited for low free-stream turbulence. The so-called e^n method based on the linear stability theory is often used for low free-stream turbulence flows in aeronautics (see Drela and Giles [4], Stock and Haase [5]), but this method cannot do without a relation between the critical amplification factor n and free-stream turbulence Tu. Besides, three-equation k-k_L-ω model by Walters and Leylek [6] with the equation for the energy of non-turbulent fluctuations can be used for modelling of the natural and bypass transition as well.

The algebraic transition model and the three-equation model were tested from the viewpoint of their application for modelling of transition at very low free-stream turbulence level. Both models were used for simulation of the incompressible flow around airfoils, namely the NACA0012 airfoil measured by Lee and Kang [7] at Tu = 0.3 % and the XIS40MOD airfoil investigated by Würz [8] at Tu = 0.012 %. In addition, various transition criteria were compared for the flat-plate flow at low free-stream turbulence level.

2 Mathematical model

The Reynolds-averaged Navier-Stokes equations are closed partly by the explicit algebraic Reynolds stress turbulence model (EARSM) according to Hellsten [9] connected with the algebraic transition model modified by Straka and Příhoda [3] and partly by the three-equation k-k_L-ω model of Walters and Cokljat [10].

The algebraic model was implemented into the in-house numerical code. The code is based on the finite volume method of the cell-centered type with the Osher's-Solomon's approximation of the Riemann solver and a two-dimensional linear reconstruction with the Van Albada's limiter. The governing equations are discretized using a multi-block quadrilateral structured grid with a block overlapping implementation.

The free accessible program OpenFOAM with the own implementation of the k-k_L-ω model was used for the

[a] Corresponding author: Jiri.Furst@fs.cvut.cz

simulation of the transitional flows. The system of governing equations was solved by the SIMPLE algorithm for the incompressible flow. The convective terms were discretized by the upwind method with the reconstruction of the second degree and the viscous terms by means of the central scheme.

The k-k_L-ω model of Walters and Cokljat [10] can be applied for the simulation of complex shear flows on unstructured grids without any restriction. On the other hand, the algebraic transition model was adapted for local variables only but structured grids are preferred near walls.

2.1 Algebraic transition model

The algebraic transition model was used together with the EARSM model proposed by Hellsten [9]. The Reynolds stress is given by the anisotropy tensor defined by the relation

$$a_{ij} = \frac{\overline{u_i u_j}}{k} - \frac{2}{3}\delta_{ij} \tag{1}$$

The anisotropy tensor is expressed by the polynomial

$$\begin{aligned} a_{ij} &= \beta_1 S_{ij} + \beta_3(\Omega_{ik}\Omega_{kj} - \frac{1}{3}\delta_{ij}II_\Omega) + \beta_4(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}) \\ &+\beta_6(S_{ik}\Omega_{kl}\Omega_{lj} + \Omega_{ik}\Omega_{kl}S_{lj} - \frac{2}{3}\delta_{ij}IV) \\ &+\beta_9(\Omega_{ik}S_{kl}\Omega_{lm}\Omega_{mj} - \Omega_{ik}\Omega_{kl}S_{lm}\Omega_{mj}) \end{aligned} \tag{2}$$

where non-dimensional strain-rate and vorticity tensors are defined by relations

$$S_{ij} = \tau\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \text{ and } \Omega_{ij} = \tau\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \tag{3}$$

The time scale τ is given by the relation

$$\tau = \max\left(\frac{1}{\beta^*\omega}; C_\tau\sqrt{\frac{\nu}{\beta^*\omega k}}\right) \tag{4}$$

where the Kolmogorov time scale is used near the wall with constants $\beta^* = 0.09$ and $C_\tau = 6$. Coefficients β in Eq. (2) are functions of invariants $II_S = S_{kl}S_{lk}$, $II_\Omega = \Omega_{kl}\Omega_{lk}$, $III_S = S_{kl}S_{lm}S_{mk}$, and $IV = S_{kl}\Omega_{lm}\Omega_{mk}$.

The transport equations for the turbulent energy k and the specific dissipation rate ω are given by equations

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho U_j k)}{\partial x_j} = \rho P_k - \rho\,\varepsilon + \frac{\partial}{\partial x_j}\left[(\mu + \sigma_k\mu_t)\frac{\partial k}{\partial x_j}\right] \tag{5}$$

$$\begin{aligned} \frac{\partial(\rho\omega)}{\partial t} + \frac{\partial(\rho U_j\omega)}{\partial x_j} &= \rho\frac{\omega}{k}(C_{\omega1}P_k - C_{\omega2}\varepsilon) \\ &+ \frac{\partial}{\partial x_j}\left[(\mu + \sigma_\omega\mu_t)\frac{\partial\omega}{\partial x_j}\right] + \sigma_d\frac{\rho}{\omega}\max\left(\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j};0\right) \end{aligned} \tag{6}$$

Both equations are used in the two-layer form with two sets of model coefficients and with the blending function similarly as the SST model proposed by Menter [11]. The detailed description of the EARSM model is given by Hellsten [9]. For the prediction of the transitional flows, the production and destruction terms in the k-equation are multiplied by the intermittency coefficient γ.

The algebraic transition model is based on the concept of different values of the intermittency coefficient in the boundary layer γ_i and in the free stream γ_e. The intermittency coefficient in the boundary layer is given by the relation

$$\gamma_i = 1 - \exp\left[-\hat{n}\sigma\left(Re_x - Re_{xt}\right)^2\right] \tag{7}$$

proposed by Narasimha [12].

The onset and the length of the transition region is given by empirical correlations for the momentum Reynolds number $Re_{\theta t}$ and parameters characterizing the generation and the growth of turbulent spots. Most of transition models are proposed for modelling of the bypass transition where free stream turbulence induces so called bypass transition by the diffusion of turbulent fluctuations across the shear layer. The lower limit of empirical relations for the bypass transition onset is about $Tu \approx 0.15$-0.2 %.

The often used approach to the transition modelling at low turbulence level is the so called e^n method based on the linear stability analysis. The solution of the Orr-Sommerfeld equation for a given velocity profile is applied for the estimation of the spatial growth rate of disturbances and the transition onset occurs when the amplification of the most unstable disturbance reaches a critical value. However results of stability analysis cannot be directly used for the estimation of the transition onset as the e^n method is based on the disturbance ratio rather than on the actual disturbance amplitude.

On the basis of the stability analysis, Mack [13] proposed the correlation $n = -8.43 - 2.4 \ln(Tu/100)$ giving the link between the stability analysis and experimental data. For the determination of the maximum amplification factor by means of boundary layer parameters, Gleyzes et al. [14] proposed a simple empirical relation giving for the similar solution of Falkner and Skan [15] the linear relation between the maximum amplification factor and the momentum Reynolds number. This approach was modified for non-similar flows as well, and so the maximum amplification factor can be determined without the solution of the Orr-Sommerfeld equation, see Drela and Giles [4].

The scatter of experimental data is for low free-stream turbulence $Tu < 0.1$ % much greater because the transition in this region is influenced by other factors especially by acoustic disturbances. The effect of various disturbances especially acoustic noise on the transition at very low free-stream turbulence was firstly studied by Govindarajan and Narasimha [16]. In this region, the transition is caused by so-called residual disturbances specific to the used facility. Govindarajan and Narasimha [16] expressed these disturbances by the equivalent turbulence level Tu_o and modified the criterion for the transition to the relation $Re_{\theta to} = 110 + 340 / (Tu^2+Tu_o^2)^{1/2}$.

Similarly, the correlation proposed by Straka et al. [17] for the transition onset on the flat plate can be modified for $Tu \le 0.25$ % according to experiments of Schubauer and Skramstad [18] in the form

$$Re_{\theta to} = 900 + 215\left\{1 - \exp\left[-4\left(\frac{0.25 - Tu}{0.17}\right)^2\right]\right\} \tag{8}$$

where the parameter A = 215 depends on other disturbances than free-stream turbulence. This modified criterion follows up smoothly to the relation

$$Re_{\theta to} = 975.8 - 497.2Tu + \frac{11.4}{Tu} \quad \text{for } 0.25 \le \text{Tu} \le 1\%$$

$$Re_{\theta to} = 96.7 + \frac{340}{Tu} + \frac{53.3}{Tu^2} \quad \text{for } Tu > 1\% \tag{9}$$

The general criterion for the transition onset is still given by the relation

$$Re_{\theta t} = Re_{\theta to}\left[1 + \mathrm{F}(Tu)\frac{1-\exp(-40\lambda_t)}{1+0.4\exp(-40\lambda_t)}\right] \tag{10}$$

where is

$$\mathrm{F}(Tu) = 0.29\left[1 - 0.54\exp(-3.5Tu)\right]\exp(-Tu) \tag{11}$$

The present correlation is compared in figure 1 partly with experimental data of Schubauer and Skramstad [18] and Wells [19] where A = 575 and partly with empirical correlations of Govindarajan and Narasimha [16] and results of Gleyzes et al. [14] for the flat plate flow.

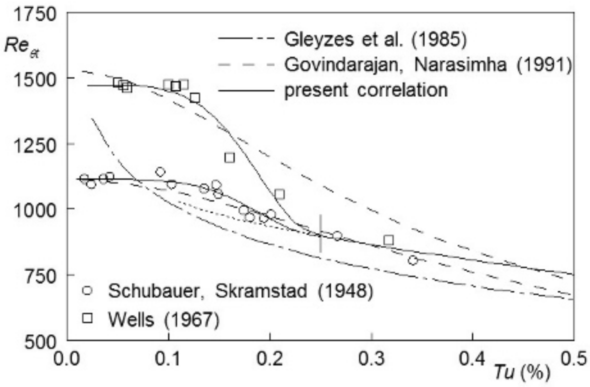

Fig.1. Correlations for the transition onset on the flat plate at low free-stream turbulence

The length of the transition region given by the spot generation rate $\hat{n}$ and the spot propagation rate σ is expressed by the parameter N introduced by Narasimha [20]. The effect of the free-stream turbulence and the pressure gradient on the parameter N is correlated by an empirical relation $N = \mathrm{f}(Tu, \lambda_t)$ proposed for the attached flow by Solomon et al. [21]. This correlation agrees with experiments Schubauer and Skramstad [18] at low free-stream turbulence as well.

The onset of transition in separated flow is given by the correlation proposed by Mayle [22] in the form

$$Re_{xt} = 300Re_{\theta s}^{0.7} + Re_{xs} \tag{12}$$

where $Re_{\theta s}$ is the momentum Reynolds number at the separation and Re_{xs} is the Reynolds number related to the distance of the separation position from the leading edge. The transition length is expressed in the form

$$\hat{n}\sigma = 2.28 \times 10^{-5}\, Re_{\theta s}^{-1.4} \tag{13}$$

and so the same approach can be applied as in the attached flow.

To avoid the application of local variables, the maximum of the vorticity Reynolds number Re_{vmax} is used instead of the momentum Reynolds number Re_θ according to Langtry and Menter [1]. The vorticity Reynolds number is given for complex flows by the relation

$$Re_v = y^2\left|\Omega\right|/\nu \tag{14}$$

where y is the distance from the wall and Ω is the absolute value of the vorticity tensor. This link is expressed by the relation $Re_\theta = Re_{vmax}/C$ where the parameter C depends on the pressure gradient. Free-stream turbulence was taken at the boundary layer thickness δ estimated by means of the position of the maximum vorticity Reynolds number using the similar solutions of Falkner and Skan [15]. The algebraic transition model is described in detail by Straka at al. [17].

2.2 *k-k$_L$-ω* model

The three-equation model proposed by Walters and Leylek [6] and later modified by Walters and Cokljat [10] is based on the assumption that velocity fluctuations before the transition can be divided into small vortices contributing to the production of turbulence and large mainly longitudinal vortices near the wall contributing to the production of non-turbulent fluctuations.

The generation of the laminar energy is caused by the interaction of non-turbulent fluctuations with the shear flow before the transition where turbulent fluctuations are damped. After the transition onset, this damping is restricted to the viscous sublayer. The transition process is expressed by the transfer from the energy of non-turbulent velocity fluctuations k_L to the turbulent energy k_T of three-dimensional turbulent velocity fluctuations.

The transport equations for the turbulent energy k_T, laminar energy k_L and the specific dissipation rate ω are given by equations

$$\frac{Dk_T}{Dt} = P_{kT} + R_{BP} + R_{NAT} - D_T + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\alpha_T}{\sigma_k}\right)\frac{\partial k_T}{\partial x_j}\right] - \omega k_T \tag{15}$$

$$\frac{Dk_L}{Dt} = P_{kL} - R_{BP} - R_{NAT} - D_L + \frac{\partial}{\partial x_j}\left[\nu\frac{\partial k_L}{\partial x_j}\right] \tag{16}$$

$$\frac{D\omega}{Dt} = C_{\omega 1}\frac{\omega}{k_T}P_{kT} + \left(\frac{C_{\omega R}}{f_w} - 1\right)\frac{\omega}{k_T}\left(R_{BP} + R_{NAT}\right) - C_{\omega 2}f_w^2\omega^2 + C_{\omega 3}f_\omega\alpha_T f_w^2\frac{\sqrt{k_T}}{y^3} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\alpha_T}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] \tag{17}$$

The transfer of energy between non-turbulent and turbulent vortices is modelled in these equations by terms R_{BP} and R_{NAT} expressing the effect of the decay of laminar fluctuations during the natural and bypass transition. The used version of the three-equation model is in detail described by Fürst et al. [23].

3 Results

The transition models were tested by means of the incompressible flow around airfoils at relatively low free-stream turbulence. Two cases ware used for the comparison of the both transition models. Firstly, the numerical simulation of the flow around the NACA0012 airfoil investigated by Lee and Kang [7] at the moderate free-stream turbulence Tu = 0.3 % was carried out for the single airfoil at the angle of attack α = 0 deg and the Reynolds number $Re_c = 6 \times 10^5$. Further, the flow around the XIS40MOD airfoil measured by Würz [8] at very low free-stream turbulence Tu = 0.012 % was simulated for the Reynolds number $Re_{smax} = 1.2 \times 10^6$ and angles of attack α = 1 deg with the transition in attached flow and α = -3 deg with the transition in separated flow.

The multi-block structured grid with the block overlapping was used together with the combined C-H grid for the simulation by the EARSM model with the algebraic transition model. The structured quadrilateral grid was applied for the k-k_L-ω model. The grid was refined near the leading edge and near walls. A very refined grid had to be used in the case with the transition in the separation bubble. The distance of nearest node from the wall was $y^+ < 1$ in the all cases. The detail of the grid near the trailing edge of the XIS40MOD airfoil used for the k-k_L-ω model is shown in figure 2.

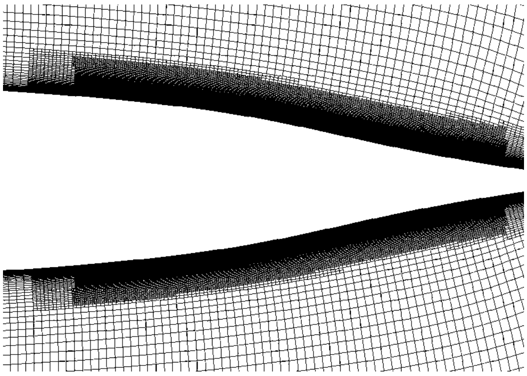

Fig.2. Detail of the grid for the XIS40MOD airfoil (k-k_L-ω model)

The computational domains correspond in both cases to the experimental arrangement. The angle of attack was adjusted by the corresponding turning of the outer boundary of the domain. The rectangular computational domain starts in the distance 1.5 c upstream the leading edge for the NACA0012 airfoil and 2 c for the XIS40 MOD airfoil respectively.

At the inlet boundary the mean velocity and turbulent scales were prescribed. The turbulent energy k and the specific dissipation rate ω are given by means of the free-stream turbulence level Tu and the ratio of viscosities μ_t/μ. The turbulence level Tu = 0.3 % and the viscosity ratio μ_t/μ = 8 were chosen as the inlet conditions for the flow around the NACA0012 airfoil. The flow around the XIS40MOD airfoil is characterized by the very low free-stream turbulence about Tu = 0.012 % and therefore the low ratio μ_t/μ = 0.1 was chosen. Besides, the laminar kinetic energy k_L = 0 was used in the k-k_L-ω model. The pressure was calculated using the homogeneous Neumann condition $\partial p/\partial n = 0$. At the outlet plane, the static pressure was prescribed and the homogeneous Neumann condition was applied for the other parameters. The slip boundary condition was used at the upper and lower boundary of the domain in the k-k_L-ω model while the no-slip condition was applied in the EARSM model as the transversal dimension of the domain was rather smaller than the height of the test section of the wind tunnel.

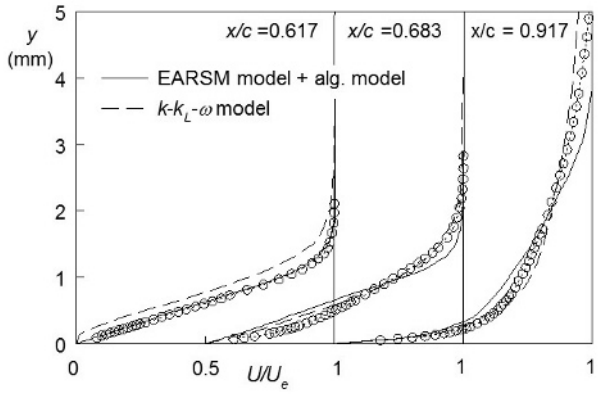

Fig.3. Mean velocity profiles on the NACA0012 airfoil

Mean velocity profiles typical for the laminar, transitional and turbulent regime are compared with the numerical simulation for the NACA0012 airfoil in figure 3. The agreement of predicted mean velocity profiles by various transition models is as good as good is the prediction of the transition onset. The transition onset predicted by the algebraic transition model with the EARSM model is shift rather downstream and so the thickness of the shear layer is smaller than in experiment. The k-k_L-ω model predicts a small separation bubble upstream the transition onset and therefore the velocity profile at x/c = 0.617 is just after the reattachment.

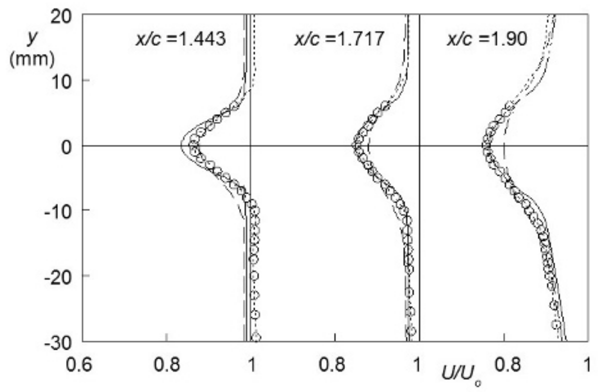

Fig.4. Mean velocity profiles in the wake of the NACA0012 airfoil

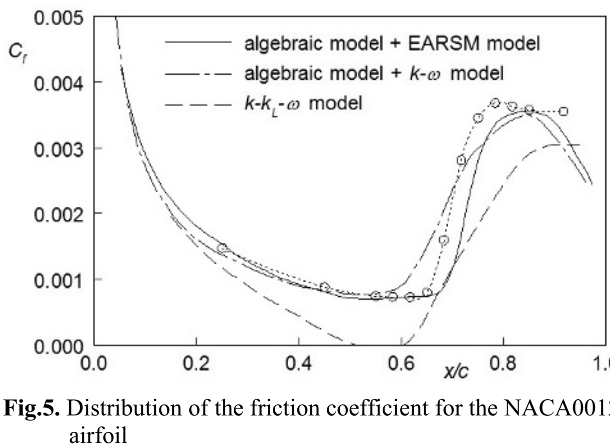

Fig.5. Distribution of the friction coefficient for the NACA0012 airfoil

Predicted velocity profiles in the wake are shown in figure 4. The agreement with experimental data is quite good although even though the k-k_L-ω model predicts a higher minimal velocity.

The distribution of the friction coefficient C_f predicted for the NACA0012 airfoil is compared with experimental data in figure 5. The agreement of the prediction by the algebraic transition model with the EARSM model or with the k-ω model (see Straka and Příhoda [3]) is quite good, while the k-k_L-ω model predicts the flow separation before the transition and so a short laminar separation bubble was predicted by the k-k_L-ω model.

Further, transition models were tested by means of the flow around the symmetrical XIS40MOD airfoil according to measurements of Würz [8]. The experimental and predicted data are presented by means of the coordinate s measured along the airfoil surface from the leading edge. The total length is about s_{max} = 1.025 c where c is the chord of the aerofoil. All experiments were realized on the suction side of the airfoil only.

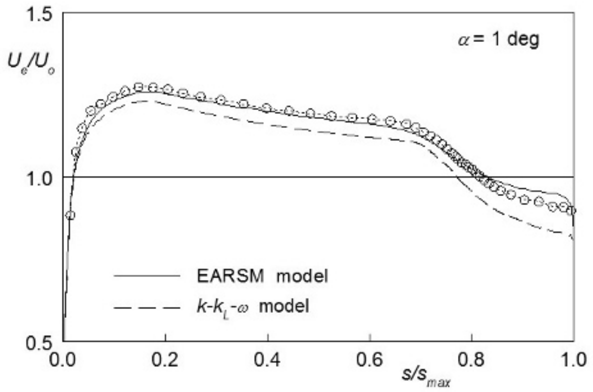

Fig.6. Velocity distribution on the XIS40MOD airfoil for α = 1 deg

The predicted velocity distribution is compared with experimental data in figure 6 for α = 1 deg where the transition in attached flow occurs. The k-k_L-ω model gives slightly lower free-stream velocity though the both predictions were carried out for the Reynolds number $Re_{smax} = 1.2 \times 10^6$. Similarly, the velocity distribution for α = -3 deg with the transition in a separation bubble is shown in figure 7.

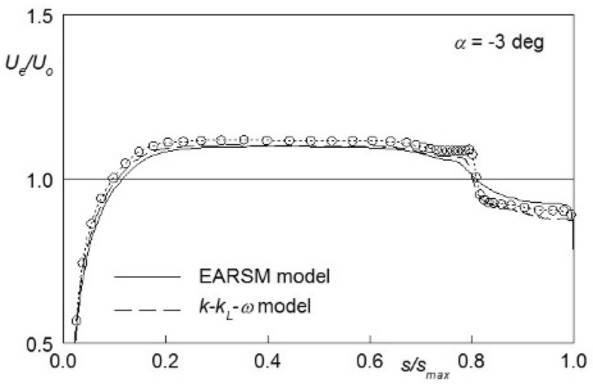

Fig.7. Velocity distribution on the XIS40MOD airfoil for α = -3 deg

The onset and the length of the transition region for the both regimes is well apparent from the distribution of the form parameter H and the friction coefficient C_f. Unfortunately, experimental data relating to the friction coefficient for α = 1 deg are not at disposal. The distribution of the form parameter H for α = 1 deg where the transition occurs in attached flow at s/s_{max} = 0.585 is shown in figure 8. The prediction by means of the algebraic transition model with the transition onset at s/s_{max} = 0.6 corresponds to the experiment while the k-k_L-ω model gives the transition onset at s/s_{max} = 0.69.

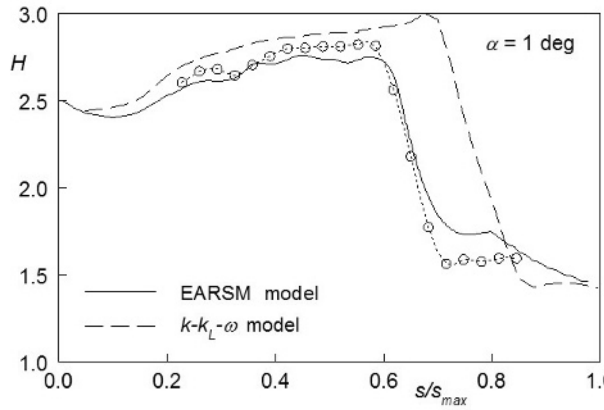

Fig.8. Distribution of the form parameter for α = 1 deg

Similarly, the predicted distribution of the form parameter H for α = -3 deg is compared with experimental data in figure 9. According to experiments, the separation arises at s/s_{max} = 0.717 and the reattachment at s/s_{max} = 0.82. The transition onset can be estimated at s/s_{max} = 0.795. The both transition models give quite acceptable results for this case with transition in the separation bubble.

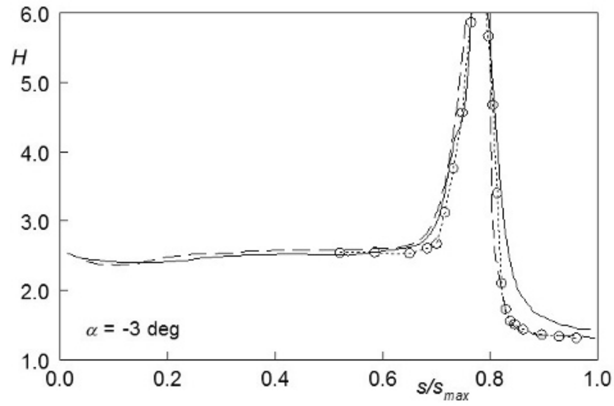

Fig.9. Distribution of the form parameter for α = -3 deg

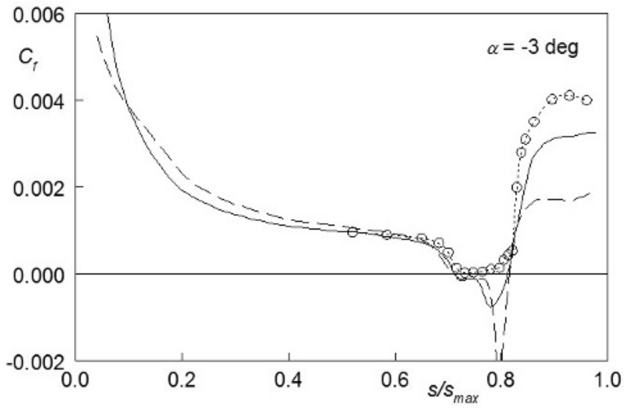

Fig.10. Distribution of the friction coefficient for α = -3 deg

The comparison of the predicted friction coefficient C_f with experimental data for α = -3 deg is shown in figure 10. The numerical results fully correspond to experimental data. The both transition models predict the separation at s/s_{max} = 0.71 and the reattachment at s/s_{max} = 0.815. Similar results were published by Windte et al. [24] obtained by means of the e^n method and various turbulence models.

Conclusions

Transition models based on the algebraic equation for the intermittency coefficient and on the transport equation for the energy of non-turbulent fluctuations were tested for transitional flows at relatively low free-stream turbulence. The empirical correlation for the transition onset was extended up to low free-stream turbulence levels.

The modified algebraic transition model of Straka and Příhoda [3] connected with EARSM turbulence model of Hellsten [9] and the k-k_L-ω model of Walter and Cokljat [10] were tested by means of the incompressible flow around airfoils at relatively low free-stream turbulence, partly the flow around the NACA0012 airfoil by Lee and Kang [7] at the moderate free-stream turbulence Tu = 0.3 % and partly the flow around the XIS40MOD airfoil by Würz [8] at very low free-stream turbulence Tu = 0.012 % in regimes with the transition in attached and separated flows. The algebraic model proposed initially for modelling of the bypass transition gives acceptable results as for the flow at low free-stream turbulence.

The k-k_L-ω model predicts in some cases the transition onset shift downstream which leads to the laminar separation in the decelerated flow and to the transition in the separation bubble.

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