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Application of adjustment calculus in the nodeless Trefftz method for a problem of two-dimensional temperature field of the boiling liquid flowing in a minichannel

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Abstract. The paper presents application of the nodeless Trefftz method to calculate temperature of the heating foil and the insulating glass pane during continuous flow of a refrigerant along a vertical minichannel. Numerical computations refer to an experiment in which the refrigerant (FC-72) enters under controlled pressure and temperature a rectangular minichannel. Initially its temperature is below the boiling point. During the flow it is heated by a heating foil. The thermosensitive liquid crystals allow to obtain two-dimensional temperature field in the foil. Since the nodeless Trefftz method has very good performance for providing solutions to such problems, it was chosen as a numerical method to approximate two–dimensional temperature distribution in the protecting glass and the heating foil. Due to known temperature of the refrigerant it was also possible to evaluate the heat transfer coefficient at the foil-refrigerant interface. For expected improvement of the numerical results the nodeless Trefftz method was combined with adjustment calculus. Adjustment calculus allowed to smooth the measurements and to decrease the measurement errors. As in the case of the measurement errors, the error of the heat transfer coefficient decreased.

1 Introduction

Progress in technology leads to miniaturization of technical devices which can be downsized without loss of their functionality. This trend is followed by engineers' efforts to construct miniature heat exchangers which, due to phase change of a refrigerant, can be employed for efficient high heat flux removal at low temperature difference between the refrigerant and the surface to be cooled.

2 Experimental research

The experiment referring to the considered problem, presented and discussed in detail in [1-3], will be described shortly in this paper. The main element of experimental stand is a module with a vertical minichannel of rectangular cross section, designed for flow experiments with FC-72 fluid. One of the walls of the minichannel is a heating foil made of Haynes-230 superalloy with uniformly distributed micro-holes machined by laser technology. DC power supplied to the foil can be regulated. The liquid crystal film which is spread over the heating foil allows to obtain visualization of two-dimensional temperature distribution of the surface. The heating foil is separated from ambient space by a protecting glass barrier. Due to the other glass pane isolating the minichannel from the opposite side, one can observe flow structures and void fraction. The liquid (FC-72) whose temperature is below saturation temperature and whose pressure is known (and controlled), flows in the minichannel where it is heated by the foil at temperature exceeding the saturation temperature. Measurements of temperature and pressure of the two phase liquid are made at the minichannel outlet. The other measured parameter concerning the flow is mass flux and those concerning the power supply are output voltage and current. Collecting measurement data begins when the process is in a steady state.

3 Mathematical model

For simplicity, we assume two-dimensional flow conditions, neglecting temperature and velocity variation along the width of the minichannel. Our considerations focused on the central part of the measurement module (along its height) so that the physical phenomena on the side edges did not affect thermodynamic parameters within the investigated segment.

To use a clear notation in further mathematical equations, the subscripts G, F, f refer to protecting glass, heating foil and fluid, respectively. It is assumed that stationary temperature distributions in the protecting

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glass and heating foil are described, as in [1,4,5], by Laplace and Poisson equation, respectively:

a) for glass

$$\nabla^2 T_G = 0 \ \text{ for } \ (x,y) \in \Omega_G \tag{1}$$

where

$$\Omega_G = \{(x,y) \in R^2 : 0 < x < L, \quad 0 < y < \delta_G\} \tag{2}$$

b) for foil

$$\nabla^2 T_F = -\frac{q_V}{\lambda_F} \ \text{ for } \ (x,y) \in \Omega_F \tag{3}$$

where

$$\Omega_F = \{(x,y) \in R^2 : x_1 < x < x_P, \quad \delta_G < y < \delta_G + \delta_F\} \tag{4}$$

The symbols used in formulas (1) – (4) mean: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, L – minichannel height, q_V – volumetric heat flux, λ – thermal conductivity, x_1 and x_P – x– coordinate of the n-th measurement point where n=1 and n=P, respectively, P – the number of measurement points.

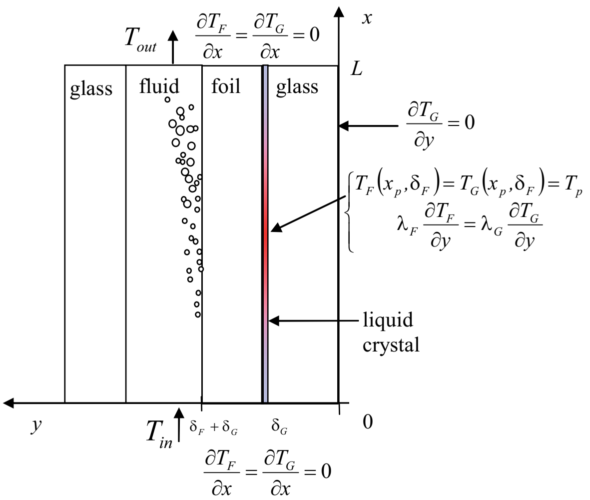

Figure 1. Measurement module and boundary conditions. (Picture not scaled proportionally).

We assume a perfect thermal contact between the foil and the glass pane, which gives

$$T_F(x_p, \delta_G) = T_G(x_p, \delta_G) = T_p \ \text{ for } p = 1,2,...,P \tag{5}$$

$$T_F(x, \delta_G) = T_G(x, \delta_G) \ \text{ for } 0 \le x \le L \tag{6}$$

$$\lambda_F \frac{\partial T_F}{\partial y} = \lambda_G \frac{\partial T_G}{\partial y} \ \text{ for } y = \delta_G \text{ , } 0 \le x \le L \tag{7}$$

where T_p denotes temperature measurement at the p-th point.

The remaining boundaries of the foil and glass are insulated which is indicated in figure 1 by proper equations.

3 The nodeless Trefftz method

The two-dimensional temperature distribution in glass and in foil was computed by Trefftz method, both classic and nodeless. According to the concept of Trefftz method, the unknown solution to a governing differential equation is approximated with a linear combination of Trefftz functions satisfying the governing equation. Trefftz functions suitable for equations (1) and (3) are harmonic polynomials. Temperature measurements obtained by liquid crystal thermography and required for numerical computations were smoothed by Trefftz functions, [4]. To determine semi-analytical solution for two-dimensional temperature distribution in the glass, T_G, one can use Trefftz method following a computational procedure described in detail in [5]. Approximate temperature distribution T_F , in the part (4) of the heating foil can be obtained by discontinuous Trefftz method (the nodeless Trefftz method). In order to solve for T_F , the region Ω_F will be divided into J subregions

$$\Omega_F^j = \{(x,y) \in R^2 : x_j \le x \le x_{j+1}, \quad \delta_G \le y \le \delta_G + \delta_F\} \quad \text{for } j = 1, 2, ..., J \tag{8}$$

In each of the subregions Ω_F^j the temperature of the heating foil T_F^j, is approximated with a formula containing a linear combination of Trefftz functions

$$T_F^j(x,y) = u(x,y) + \sum_{i=1}^{M} a_{ij} v_i(x,y) \tag{9}$$

where $u(x,y)$ denotes a particular solution to (3) and $v_i(x,y)$ are Trefftz functions (harmonic polynomials) for Laplace equation, M - the number of Trefftz functions. The coefficients a_{ij} in formula (9) are calculated from the minimum of a functional H_F

$$\begin{aligned} H_F = & \int_{\delta_G}^{\delta_G+\delta_F} (T_F^1(x_1,y) - T_1)^2 \, dy + \\ & + \int_{\delta_G}^{\delta_G+\delta_F} (T_F^K(x_P,y) - T_P)^2 \, dy + \\ & + \sum_{p_j=1}^{P} (T_F^j(x_{p_j},\delta_G) - T_{p_j})^2 + \\ & + \sum_{j=1}^{K} \int_{x_j}^{x_{j+1}} (T_F^j(x,\delta_G) - T_G(x,\delta_G))^2 \, dx + \\ & + \sum_{j=1}^{K} \int_{x_j}^{x_{j+1}} (\lambda_F \frac{\partial T_F^j}{\partial y}(x,\delta_G) - \lambda_G \frac{\partial T_G}{\partial y}(x,\delta_G))^2 \, dx + \\ & + \sum_{j=1}^{K-1} \int_{\delta_G}^{\delta_G+\delta_F} (T_F^j(x_{j+1},y) - T_F^{j+1}(x_{j+1},y))^2 \, dy + \\ & + \sum_{j=1}^{K-1} \int_{\delta_G}^{\delta_G+\delta_F} (\frac{\partial T_F^j}{\partial x}(x_{j+1},y) - \frac{\partial T_F^{j+1}}{\partial x}(x_{j+1},y))^2 \, dy \end{aligned} \tag{10}$$

The first terms of the functional (10) represent average squared error between computed and prescribed values of the heating foil temperature and heat flux in every subregion Ω_F^j while the remaining terms are responsible for good agreement between temperature and its gradient along the common boundaries of the subregions Ω_F^j. The obtained approximate temperatures T_G and T_F satisfy the differential equations (1) and (3), respectively, but the boundary conditions are fulfilled by the approximants T_G and T_F only in variational sense.

Having calculated temperature distribution in the heating foil in every subregion Ω_F^j, one can easily determine the heat transfer coefficient from the formula

$$\alpha^j(x)=\frac{-\lambda_F \dfrac{\partial T_F^j(x,\delta_G+\delta_F)}{\partial y}}{T_F^j(x,\delta_G+\delta_F)-T_f(x)} \tag{11}$$

where fluid temperature $T_f(x)$ is approximated linearly from the value T_{in} at the channel inlet to the value T_{out} at the channel outlet

$$T_f(x)=T_{in}+\frac{T_{out}-T_{in}}{L}x \tag{12}$$

3.1 Adjustment calculus

For the sake of accuracy in determining two-dimensional temperature distribution in the glass and foil, temperature measurements will be approximated with Trefftz functions as follows

$$T_p \approx T(x_p,\delta_G)=\sum_{i=1}^{I} b_i v_i(x_p,\delta_G) \tag{13}$$

Trefftz functions in eq. (13) are selected so that the columns of the matrix

$$V_{pi}=v_i(x_p,\delta_G) \tag{14}$$

be linearly independent. Estimation according to eq. (13) assumes that the temperature measurements are biased. It seems to be more realistic that the temperature measurements T_p suffer from random measurement errors. In fact, in our experiment we obtain P measurement values; each can be expressed by two parts – a „true" temperature and an error which is unsystematic (random) and different for different measurements. The following formula allows to express it in terms of mathematical equations

$$\sum_{i=1}^{I} b_i v_i(x_p,\delta_G)-T_p=\Delta_p \neq 0 \tag{15}$$

Temperature measurements T_p approximated by the Trefftz functions (13) can be corrected by adjustment calculus as shown in [4,6]. In such an approach we replace measurements T_p with "new" (corrected) measurements $\tilde{T}_p$, demanding that the corrections ε_p contained in the formula

$$\tilde{T}_p=T_p+\varepsilon_p \tag{16}$$

have a normal distribution with expected value equal to zero and a finite variance σ_p^2, [6]. The σ_p value reflects the error of heating foil temperature estimation based on the hue indicated by liquid crystals, [5]. Corrections ε_p are determined so as to minimize Lagrange'a function

$$\Phi=\sum_{p=1}^{P}\left(\frac{\varepsilon_p}{\sigma_p}\right)^2+2\sum_{p=1}^{P}\omega_k\left(\tilde{T}_p-T(x_p,\delta_G)\right)\rightarrow \min \tag{17}$$

where ω_p – Lagrange multipliers.

Having corrected temperature measurements, we recalculated measurement errors σ_p^{corr} according to the error propagation law, [4,6]. With known values of $\tilde{T}_p$ it was possible to recalculate approximate temperatures of the protecting glass, $\tilde{T}_G$, and the heating foil, $\tilde{T}_F$, as described in [4].

4 Results

Numerical calculations were performed for the data coming from the experiments described in [1-3] and related to the forced flow of FC-72 through an asymmetrically heated minichannel, see figure 2.

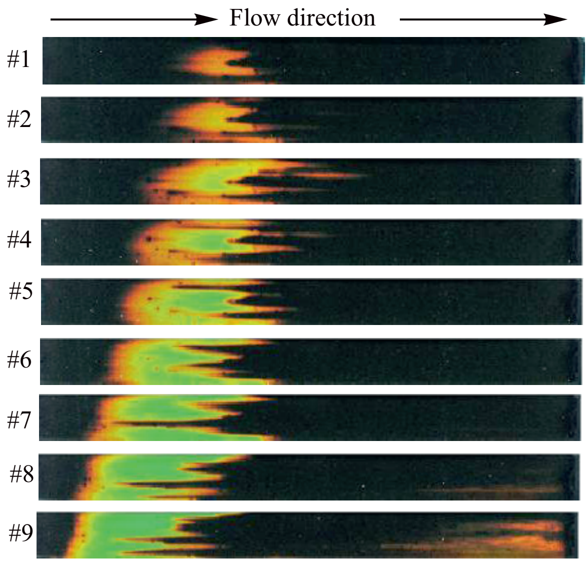

Figure 2. Hue distribution on the minichannel external surface while increasing the heat flux supplied to the heating foil. Experimental parameters of the runs: mass flux 285 kg/(m^{2}s); Re =884; inlet presser 124 kPa, volumetric heat flux 7.96·10^4÷ 1.24·10^5 kW/m^3; foil parameters: δ_F= 1.02·10^{-4} m, L= 0.3 m, λ_F= 8.3 W/(mK); glass parameters: δ_G= 0.005 m, λ_G= 0.71 W/(mK).

Measured values included temperature of FC-72 at the inlet and outlet of the minichannel, flow velocity, pressure at the minichannel inlet and outlet, voltage drop and electrical current supplied to the foil. Liquid crystal thermography helped calculate approximate temperature of the heating foil in contact with the protecting glass.

Approximate temperature of the glass was computed with a use of 16 Trefftz functions. The same number of Trefftz functions were used for approximation of temperature of the heating foil in the region Ω_F which, for accurate results, required division into 4 subregions according to (4). Computations were performed for the data smoothed by adjustment calculus in the manner described in paragraph **3.1**.

Figure 3 presents two-dimensional distribution of glass and heating foil temperature calculated with nodeless Trefftz method for the data smoothed by adjustment calculus.

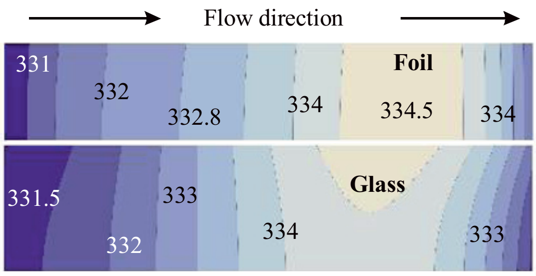

Figure 3. Temperature of the protecting glass by Trefftz method and temperature of the heating foil by nodeless Trefftz method (both after applying adjustment calculus). Setting #3.

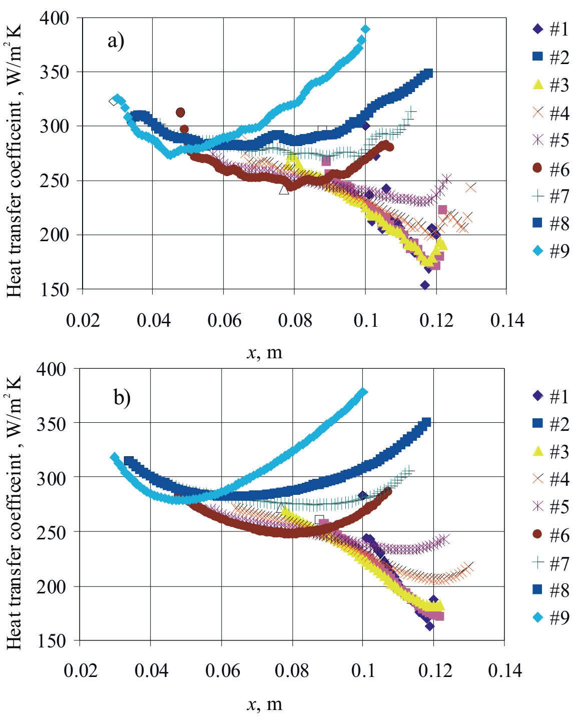

Figure 4. Heat transfer coefficient as a function of the minichannel height obtained by: a) nodeless Trefftz method, b) nodelees Trefftz method after applying adjustment calculus.

The figure 4 shows the heat transfer coefficient as a function of the minichannel height, calculated from (11) for the data presented in figure 2.

The mean error in estimation of the heat transfer coefficient (see [3]) for settings #1 to #9 turned to be equal to 5.4%. After application of adjustment calculus the error decreased to 3.5%.

In [7] were proposed mathematical models describing heat transfer in three areas (protecting glass, heating foil and fluid). Trefftz method was used to determine the two-dimensional temperature distribution also in the liquid in bubbly and bubbly-slug flow. The computations were based on temperature measurements of foil obtained by liquid crystal thermography and experimentally calculated void fraction.

5 Conclusions

1. Trefftz method and nodeless Trefftz method were applied for solving a direct problem of determining the temperature distribution in the protecting glass and also for solving an inverse problem of determining the temperature distribution in the heating foil.

2. Trefftz method (as well as nodeless Trefftz method) can be combined with adjustment calculus. In the considered problem both gave satisfying results.

3. Measurement data approximation with Trefftz functions helps to smooth the measurements and decreases measurement errors.

4. When calculated from measurements corrected by adjustment calculus, the heat transfer coefficient has significantly smaller error in comparison with that calculated without adjustment calculus.

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