

# Strings and extra dimensions: Mass hierarchies and experimental signatures

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**Abstract.** Lowering the string scale in the TeV region provides a theoretical framework for solving the mass hierarchy problem and unifying all interactions. The apparent weakness of gravity can then be accounted by the existence of large internal dimensions, in the submillimeter region, and transverse to a braneworld where our universe must be confined. I review the main properties of this scenario and its experimental implications.

## 1 Introduction

During the last few decades, physics beyond the Standard Model (SM) was guided from the problem of mass hierarchy. This can be formulated as the question of why gravity appears to us so weak compared to the other three known fundamental interactions corresponding to the electromagnetic, weak and strong nuclear forces. Indeed, gravitational interactions are suppressed by a very high energy scale, the Planck mass  $M_P \sim 10^{19}$  GeV, associated to a length  $l_P \sim 10^{-35}$  m, where they are expected to become important. In a quantum theory, the hierarchy implies a severe fine tuning of the fundamental parameters in more than 30 decimal places in order to keep the masses of elementary particles at their observed values. The reason is that quantum radiative corrections to all masses generated by the Higgs vacuum expectation value are proportional to the ultraviolet cutoff which in the presence of gravity is fixed by the Planck mass. As a result, all masses are “attracted” to about  $10^{16}$  times heavier than their observed values.

Besides compositeness, there are two main theories that have been proposed and studied extensively during the last years, corresponding to different approaches of dealing with the mass hierarchy problem. (1) Low energy supersymmetry with all superparticle masses in the TeV region. Indeed, in the limit of exact supersymmetry, quadratically divergent corrections to the Higgs self-energy are exactly cancelled, while in the softly broken case, they are cutoff by the supersymmetry breaking mass splittings. (2) TeV scale strings, in which quadratic divergences are cutoff by the string scale and low energy supersymmetry is not needed. Both ideas are experimentally testable at high-energy particle colliders and in particular at LHC.

## 2 Strings and extra dimensions

The appropriate and most convenient framework for low energy supersymmetry and grand unification is the perturbative heterotic string. Indeed, in this theory, gravity and gauge interactions have the same

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origin, as massless modes of the closed heterotic string, and they are unified at the string scale  $M_s$ . As a result, the Planck mass is predicted to be proportional to  $M_s$ :

$$M_P = M_s/g, \quad (1)$$

where  $g$  is the gauge coupling. In the simplest constructions all gauge couplings are the same at the string scale, given by the four-dimensional (4d) string coupling, and thus no grand unified group is needed for unification. In our conventions  $\alpha_{\text{GUT}} = g^2 \simeq 0.04$ , leading to a discrepancy between the string and grand unification scale  $M_{\text{GUT}}$  by almost two orders of magnitude. Explaining this gap introduces in general new parameters or a new scale, and the predictive power is essentially lost. This is the main defect of this framework, which remains though an open and interesting possibility.

The other other perturbative framework that has been studied extensively in the more recent years is type I string theory with D-branes. Unlike in the heterotic string, gauge and gravitational interactions have now different origin. The latter are described again by closed strings, while the former emerge as excitations of open strings with endpoints confined on D-branes [1]. This leads to a braneworld description of our universe, which should be localized on a hypersurface, i.e. a membrane extended in  $p$  spatial dimensions, called  $p$ -brane (see Fig. 1). Closed strings propagate in all nine dimensions of string theory: in those extended along the  $p$ -brane, called parallel, as well as in the transverse ones. On the contrary, open strings are attached on the  $p$ -brane. Obviously, our  $p$ -brane world must have at least the three known dimensions of space. But it may contain more: the extra  $d_{\parallel} = p - 3$  parallel dimensions must have a finite size, in order to be unobservable at present energies, and can be as large as  $\text{TeV}^{-1} \sim 10^{-18} \text{ m}$  [2]. On the other hand, transverse dimensions interact with us only gravitationally and experimental bounds are much weaker: their size should be less than about 0.1 mm [3]. In the following, I review the main properties and experimental signatures of low string scale models [4].

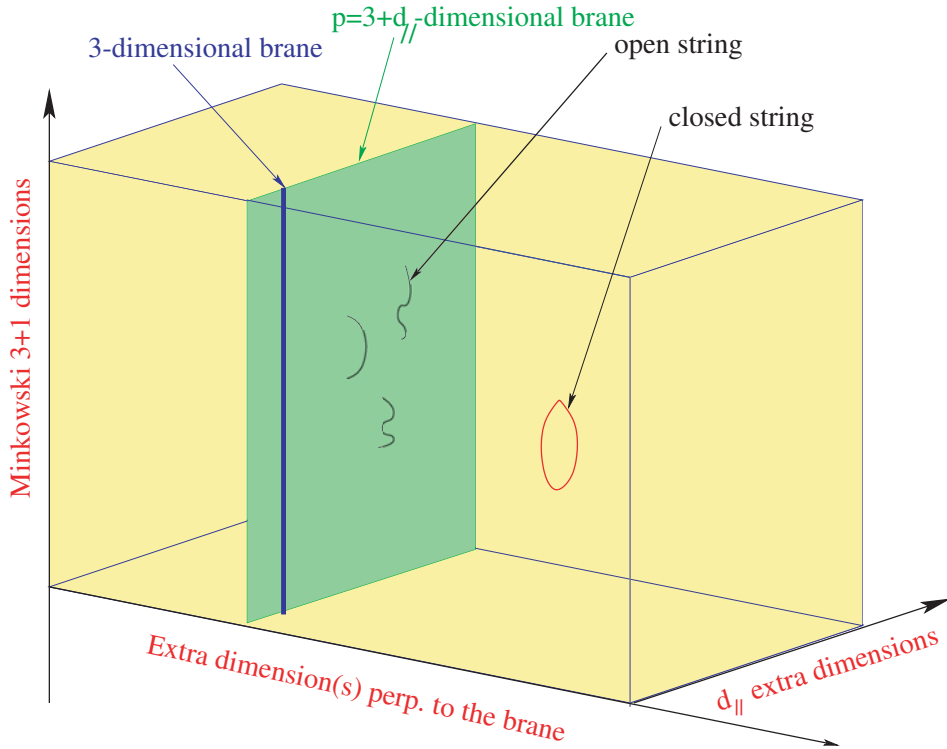
## 2.1 Framework of low scale strings

In type I theory, the different origin of gauge and gravitational interactions implies that the relation between the Planck and string scales is not linear as (1) of the heterotic string. The requirement that string theory should be weakly coupled, constrain the size of all parallel dimensions to be of order of the string length, while transverse dimensions remain unrestricted. Assuming an isotropic transverse space of  $n = 9 - p$  compact dimensions of common radius  $R_{\perp}$ , one finds:

$$M_P^2 = \frac{1}{g_s^2} M_s^{2+n} R_{\perp}^n, \quad g_s \simeq g^2. \quad (2)$$

where  $g_s$  is the string coupling. It follows that the type I string scale can be chosen hierarchically smaller than the Planck mass [4, 5] at the expense of introducing extra large transverse dimensions felt only by gravity, while keeping the string coupling small [4]. The weakness of 4d gravity compared to gauge interactions (ratio  $M_W/M_P$ ) is then attributed to the largeness of the transverse space  $R_{\perp}$  compared to the string length  $l_s = M_s^{-1}$ .

An important property of these models is that gravity becomes effectively  $(4 + n)$ -dimensional with a strength comparable to those of gauge interactions at the string scale. The first relation of Eq. (2) can be understood as a consequence of the  $(4 + n)$ -dimensional Gauss law for gravity, with  $M_*^{(4+n)} = M_s^{2+n}/g^4$  the effective scale of gravity in  $4 + n$  dimensions. Taking  $M_s \simeq 1 \text{ TeV}$ , one finds a size for the extra dimensions  $R_{\perp}$  varying from  $10^8 \text{ km}$ ,  $.1 \text{ mm}$ , down to a Fermi for  $n = 1, 2$ , or 6 large dimensions, respectively. This shows that while  $n = 1$  is excluded,  $n \geq 2$  is allowed by present experimental bounds on gravitational forces [3, 6]. Thus, in these models, gravity appears to



**Figure 1.** D-brane world universe in type I string framework.

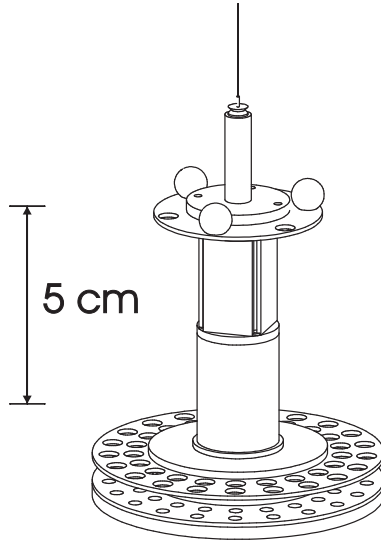
us very weak at macroscopic scales because its intensity is spread in the “hidden” extra dimensions. At distances shorter than  $R_{\perp}$ , it should deviate from Newton’s law, which may be possible to explore in laboratory experiments (see Fig. 2).

### 3 Large number of species

Here, we open a parenthesis to describe that low scale gravity with large extra dimensions is actually a particular case of a more general framework, where the ultraviolet (UV) cutoff is lower than the Planck scale due to the existence of a large number of particle species coupled to gravity [7]. Indeed, it was shown that the effective UV cutoff  $M_*$  is given by

$$M_*^2 = M_P^2/N, \tag{3}$$

where the counting of independent species  $N$  takes into account all particles which are not broad resonances, having a width less than their mass. The derivation is based on black hole evaporation but here we present a shorter argument using quantum information storage [8]. Consider a pixel of size  $L$  containing  $N$  species storing information. The energy required to localize  $N$  wave functions is then given by  $N/L$ , associated to a Schwarzschild radius  $R_s = N/LM_P^2$ . The latter must be less than the



**Figure 2.** Torsion pendulum that tested the validity of Newton's law at  $55 \mu\text{m}$ .

pixel size in order to avoid the collapse of such a system to a black hole,  $R_s \leq L$ , implying a minimum size  $L \geq L_{min}$  with  $L_{min} = \sqrt{N}/M_P$  associated precisely to the effective UV cutoff  $M_* = L_{min}$  given in eq. (3). Imposing  $M_* \approx 1 \text{ TeV}$ , one should then have  $N \sim 10^{32}$  particle species below about the TeV scale!

In the string theory context, there are two ways of realizing such a large number a particle species by lowering the string scale at a TeV:

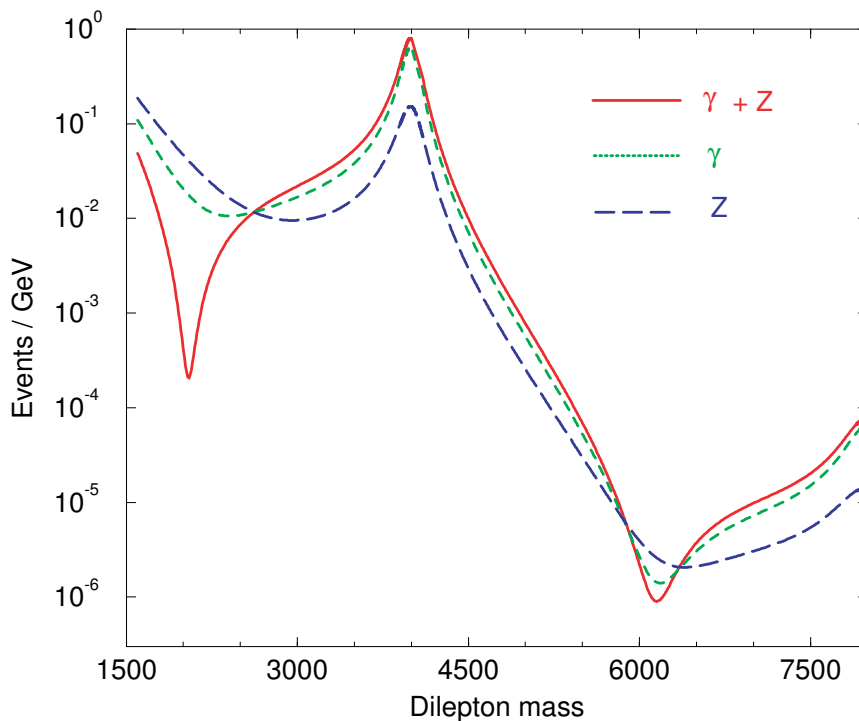
1. In large volume compactifications with the SM localized on D-brane stacks, as described in the previous section. The particle species are then the Kaluza-Klein (KK) excitations of the graviton (and other possible bulk modes) associated to the large extra dimensions, given by  $N = R_{\perp}^n l_s^m$ , up to energies of order  $M_* \approx M_s$ .
2. By introducing an infinitesimal string coupling  $g_s \approx 10^{-16}$  with the SM localized on Neveu-Schwarz NS5-branes in the framework of little strings [9]. In this case, the particle species are the effective number of string modes that contribute to the black hole bound [10]:  $N = 1/g_s^2$  and gravity does not become strong at  $M_s \sim \mathcal{O}(\text{TeV})$ .

Note the both TeV string realizations above are compatible with the general expression (2), but in the second case there is no relation between the string and gauge couplings.

## 4 Experimental implications in accelerators

We now turn to the experimental predictions of TeV scale strings. Their main implications in particle accelerators are of three types, in correspondence with the three different sectors that are generally present:

1. New compactified parallel dimensions; In this case  $RM_s \gtrsim 1$ , and the associated compactification scale  $R_{\parallel}^{-1}$  would be the first scale of new physics that should be found increasing the beam energy [2, 11]. The main consequence is the existence of KK excitations for all SM particles that propagate along the extra parallel dimensions. These can be produced on-shell at LHC as new resonances [12] (see Fig. 3).

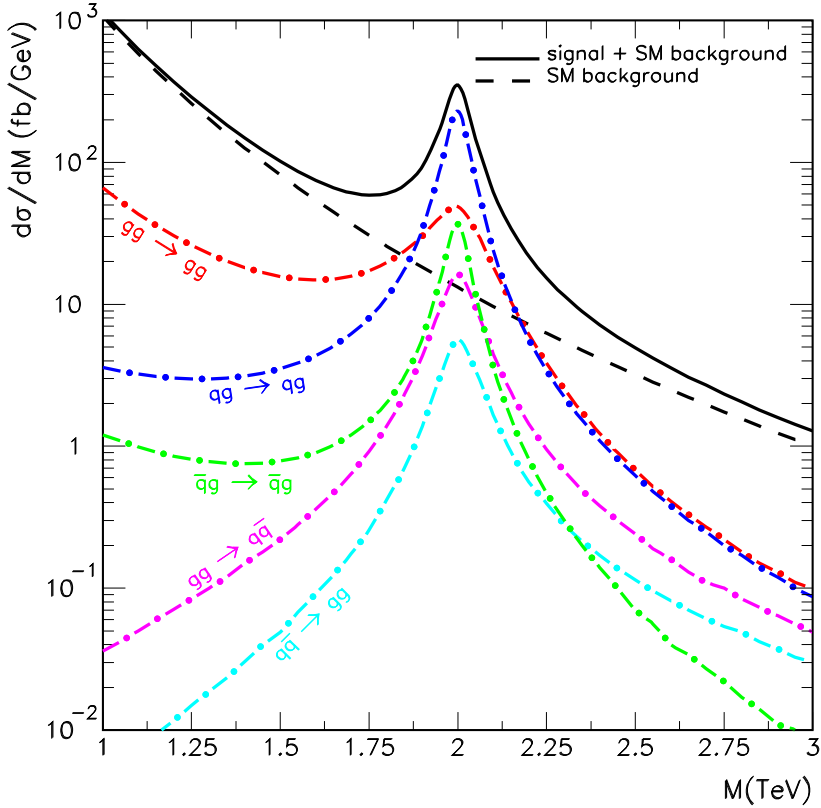


**Figure 3.** Production of the first KK modes of the photon and of the Z boson at LHC, decaying to electron-positron pairs. The number of expected events is plotted as a function of the energy of the pair in GeV.

2. New extra large transverse dimensions and low scale quantum gravity,. The main experimental signal is gravitational radiation in the bulk from any physical process on the world-brane [13]. The resulting bounds are given in Table 1.
3. Genuine string and quantum gravity effects. Direct production of string resonances in hadron colliders leads generically to a universal deviation from Standard Model in jet distribution [14]. In particular, the first Regge excitation of the gluon has spin 2 and a width an order of magnitude lower than the string scale, leading to a characteristic peak in dijet production; similarly, the first excitations of quarks have spin 3/2. The dijet cross-section is shown in Fig. 4 for LHC energies. Concerning possible micro-black hole production, note that a string size black hole

**Table 1.** Limits on  $R_{\perp}$  in mm.

Experiment	$n = 2$	$n = 4$	$n = 6$
LEP 2	$5 \times 10^{-1}$	$2 \times 10^{-8}$	$7 \times 10^{-11}$
Tevatron	$5 \times 10^{-1}$	$10^{-8}$	$4 \times 10^{-11}$
LHC	$4 \times 10^{-3}$	$6 \times 10^{-10}$	$3 \times 10^{-12}$



**Figure 4.** Production of the first Regge excitations at LHC in the dijet channel, for  $M_s = 2$  TeV. The cross-section is plotted as a function of the dijet invariant mass  $M$ .

has a horizon radius  $r_H \sim 1$  in string units, while the Newton's constant behaves as  $G_N \sim g_s^2$ . It follows that the mass of a  $d$ -dimensional black hole is [15]:  $M_{\text{BH}} \sim r_H^{d/2-1}/G_N \simeq 1/g_s^2$ . Using the value of the SM gauge couplings  $g_s \simeq g^2 \sim 0.1$ , one finds that the energy threshold  $M_{\text{BH}}$  of micro-black hole production is about four orders of magnitude higher than the string scale, implying that one would produce  $10^4$  string states before reaching  $M_{\text{BH}}$ .

## 5 Electroweak symmetry breaking

Non-supersymmetric TeV strings offer also a framework to realize gauge symmetry breaking radiatively. Indeed, from the effective field theory point of view, one expects quadratically divergent one-loop contributions to the masses of scalar fields. The divergences are cut off by  $M_s$  and if the corrections are negative, they can induce electroweak symmetry breaking and explain the mild hierarchy between the weak and a string scale at a few TeV, in terms of a loop factor [16]. More precisely, in the minimal case of one Higgs doublet  $H$ , the scalar potential is:

$$V = \lambda(H^\dagger H)^2 + \mu^2(H^\dagger H), \quad (4)$$

where  $\lambda$  arises at tree-level. Moreover, in any model where the Higgs field comes from an open string with both ends fixed on the same brane stack, it is given by an appropriate truncation of a supersymmetric theory. Within the minimal spectrum of the SM,  $\lambda = (g_2^2 + g'^2)/8$ , with  $g_2$  and  $g'$  the  $SU(2)$  and  $U(1)_Y$  gauge couplings. On the other hand,  $\mu^2$  is generated at one loop:

$$\mu^2 = -\varepsilon^2 g^2 M_s^2, \quad (5)$$

where  $\varepsilon$  is a loop factor that can be estimated from a toy model computation and varies in the region  $\varepsilon \sim 10^{-1} - 10^{-3}$ .

Indeed, consider for illustration a simple case where the whole one-loop effective potential of a scalar field can be computed. We assume for instance one extra dimension compactified on a circle of radius  $R > 1$  (in string units). An interesting situation is provided by a class of models where a non-vanishing VEV for a scalar (Higgs) field  $\phi$  results in shifting the mass of each KK excitation by a constant  $a(\phi)$ :

$$M_m^2 = \left( \frac{m + a(\phi)}{R} \right)^2, \quad (6)$$

with  $m$  the KK integer momentum number. Such mass shifts arise for instance in the presence of a Wilson line,  $a = q \oint \frac{dy}{2\pi} gA$ , where  $A$  is the internal component of a gauge field with gauge coupling  $g$ , and  $q$  is the charge of a given state under the corresponding generator. A straightforward computation shows that the  $\phi$ -dependent part of the one-loop effective potential is given by [17]:

$$V_{eff} = -Tr(-)^F \frac{R}{32\pi^{3/2}} \sum_n e^{2\pi i n a} \int_0^\infty dl l^{3/2} f_s(l) e^{-\pi^2 n^2 R^2 l} \quad (7)$$

where  $F = 0, 1$  for bosons and fermions, respectively. We have included a regulating function  $f_s(l)$  which contains for example the effects of string oscillators. To understand its role we will consider the two limits  $R \gg 1$  and  $R \ll 1$ . In the first case only the  $l \rightarrow 0$  region contributes to the integral. This means that the effective potential receives sizable contributions only from the infrared (field theory) degrees of freedom. In this limit we would have  $f_s(l) \rightarrow 1$ . For example, in the string model considered in [16]:

$$f_s(l) = \left[ \frac{1}{4l} \frac{\theta_2}{\eta^3} \left( il + \frac{1}{2} \right) \right]^4 \rightarrow 1 \quad \text{for} \quad l \rightarrow 0, \quad (8)$$

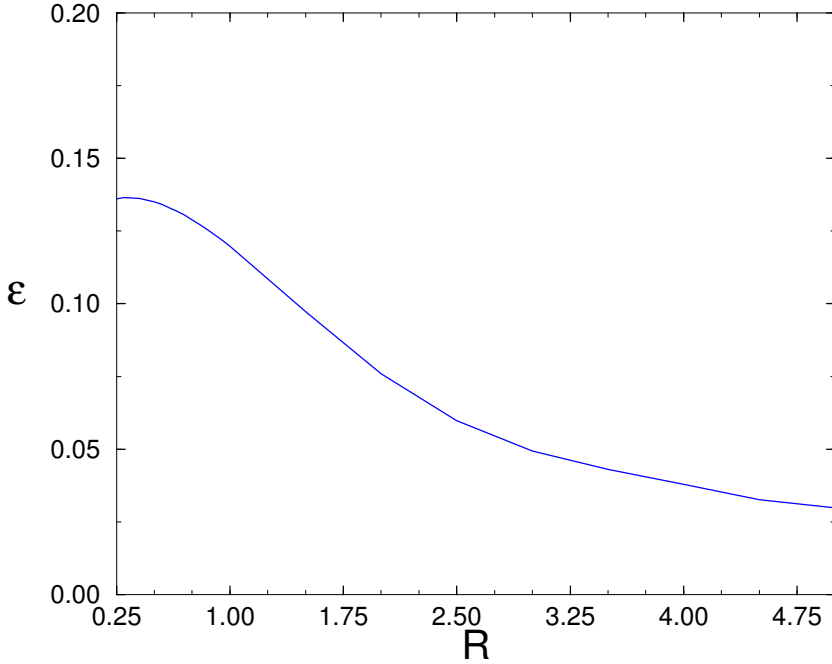
and the field theory result is finite and can be explicitly computed. As a result of the Taylor expansion around  $a = 0$ , we are able to extract the one-loop contribution to the coefficient of the term of the potential quadratic in the Higgs field. It is given by a loop factor times the compactification scale [17]. One thus obtains  $\mu^2 \sim g^2/R^2$  up to a proportionality constant which is calculable in the effective field theory. On the other hand, if we consider  $R \rightarrow 0$ , which by  $T$ -duality corresponds to taking the extra

dimension as transverse and very large, the one-loop effective potential receives contributions from the whole tower of string oscillators as appearing in  $f_s(l)$ , leading to squared masses given by a loop factor times  $M_s^2$ , according to eq. (5).

More precisely, from the expression (7), one finds:

$$\varepsilon^2(R) = \frac{1}{2\pi^2} \int_0^\infty \frac{dl}{(2l)^{5/2}} \frac{\theta_2^4}{4\eta^{12}} \left( il + \frac{1}{2} \right) R^3 \sum_n n^2 e^{-2\pi n^2 R^2 l}, \quad (9)$$

which is plotted in Fig. 5. For the asymptotic value  $R \rightarrow 0$  (corresponding upon T-duality to a large



**Figure 5.** The coefficient  $\varepsilon$  of the one loop Higgs mass (5).

transverse dimension of radius  $1/R$ ),  $\varepsilon(0) \simeq 0.14$ , and the effective cut-off for the mass term is  $M_s$ , as can be seen from Eq. (5). At large  $R$ ,  $\mu^2(R)$  falls off as  $1/R^2$ , which is the effective cut-off in the limit  $R \rightarrow \infty$ , as we argued above, in agreement with field theory results in the presence of a compactified extra dimension [18, 19]. In fact, in the limit  $R \rightarrow \infty$ , an analytic approximation to  $\varepsilon(R)$  gives:

$$\varepsilon(R) \simeq \frac{\varepsilon_\infty}{M_s R}, \quad \varepsilon_\infty^2 = \frac{3 \zeta(5)}{4 \pi^4} \simeq 0.008. \quad (10)$$

The potential (4) has the usual minimum, given by the VEV of the neutral component of the Higgs doublet  $v = \sqrt{-\mu^2/\lambda}$ . Using the relation of  $v$  with the  $Z$  gauge boson mass,  $M_Z^2 = (g_2^2 + g'^2)v^2/4$ , and the expression of the quartic coupling  $\lambda$ , one obtains for the Higgs mass a prediction which is the Minimal Supersymmetric Standard Model (MSSM) value for  $\tan\beta \rightarrow \infty$  and  $m_A \rightarrow \infty$ :  $m_H = M_Z$ .



The tree level Higgs mass is known to receive important radiative corrections from the top-quark sector and rises to values around 120 GeV. Furthermore, from (5), one can compute  $M_s$  in terms of the Higgs mass  $m_H^2 = -2\mu^2$ :

$$M_s = \frac{m_H}{\sqrt{2} g_E}, \quad (11)$$

yielding naturally values in the TeV range.

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