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The Tsallis Distribution at the LHC

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Abstract. The Tsallis distribution has been used widely in high energy physics to describe the transverse momentum distributions of particles. In this note we show that the use of a thermodynamically consistent form of this distribution leads to a description of identified particles with the same values of the temperature T and the parameter q.

There exists a rich and wide variety of distributions covering a large range of applications [1, 2]. Those having a power law behaviour have attracted considerable attention in physics in recent years but there is a a long history in other fields such as biology and economics [3].

In high energy physics power law distributions have been applied by a very large number of scientists [4–8] to the description of transverse momenta of secondary particles produced in $p-p$ collisions. Indeed the available range of transverse momenta has expanded considerably with the advent of the Large Hadron Collider (LHC). Collider energies up to 8 TeV are now available in $p-p$ collisions and transverse momenta of hundreds of GeV are a common occurrence. In this presentation the focus will be on various forms of distributions first proposed by C. Tsallis about twenty-five years ago [9].

In the analysis of the new data, a Tsallis-like distribution gives excellent fits to the transverse momentum distributions as shown by the by the ALICE [6], ATLAS [7] and CMS [8] collaborations at the LHC and by the STAR [4] and PHENIX [5] collaborations at RHIC. In this paper we review the parameterization used by these groups and propose a slightly different one which has a more consistent interpretation and has the bonus of being thermodynamically consistent.

For high energy physics a consistent form of Tsallis statistics (see e.g. [10] and references therein) for the particle number, energy density and pressure is given by the expressions given below

$$N = gV\int\frac{d^3p}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \tag{1}$$

$$\epsilon = g\int\frac{d^3p}{(2\pi)^3}E\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \tag{2}$$

$$P = g\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}. \tag{3}$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor. As is well-known the Tsallis distribution [9, 11] introduces a new parameter q which for transverse momentum spectra is always close to 1, typical values for the parameter q obtained are in the range 1.1 to 1.2. In the remainder of this paper we will always assume $q > 1$.

The expressions (1), (2) and (3) are thermodynamically consistent, e.g. it can be easily shown [10] that relations of the type

$$N = V \left.\frac{\partial P}{\partial \mu}\right|_T , \tag{4}$$

are satisfied [10, 12]. Note that without the extra power of q in the equations (1),(2), (3) the thermodynamic consistency would not be achieved.
It follows from (1) that the momentum distribution is given by,

$$\frac{d^3N}{d^3p} = \frac{gV}{(2\pi)^3}\left[1 + (q-1)\frac{E-\mu}{T}\right]^{-q/(q-1)} , \tag{5}$$

or, expressed in terms of transverse momentum, p_T, transverse mass, m_T, and rapidity y

$$\frac{d^2N}{dp_T\,dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2}\left[1 + (q-1)\frac{m_T\cosh y - \mu}{T}\right]^{-q/(q-1)} , \tag{6}$$

At mid-rapidity $y = 0$ and for zero chemical potential $\mu = 0$ this reduces to

$$\left.\frac{d^2N}{dp_T\,dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2}\left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)} . \tag{7}$$

This is the expression used in [10, 12] to fit the LHC transverse momentum spectra.
It is well-known since 1988 [9] that in the limit where the parameter q goes to 1 Eq. (6) reduces to the standard Boltzmann distribution:

$$\lim_{q\to1}\frac{d^2N}{dp_T\,dy} = gV\frac{p_T m_T\cosh y}{(2\pi)^2}\exp\left(-\frac{m_T\cosh y - \mu}{T}\right). \tag{8}$$

The parameterization given in Eq. (7) is close to the one used (but different) e.g. by the ALICE [6], ATLAS [7], CMS [8], STAR [4] and PHENIX [5] collaborations where the following form is used :

$$\frac{d^2N}{dp_T\,dy} = p_T\frac{dN}{dy}\frac{(n-1)(n-2)}{nC(nC + m_0(n-2))}\left[1 + \frac{m_T - m_0}{nC}\right]^{-n} \tag{9}$$

where n, C and m_0 are fit parameters. Indeed, after substituting

$$n \to \frac{q}{q-1} \tag{10}$$

and

$$nC \to \frac{T + m_0(q-1)}{q-1}. \tag{11}$$

The Eq. (9) becomes

$$\begin{aligned}\frac{d^2N}{dp_T\,dy} = \quad & p_T\frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nC(nC + m_0(n-2))}\\ & \left[\frac{T}{T + m_0(q-1)}\right]^{-q/(q-1)}\\ & \left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)} .\end{aligned} \tag{12}$$

$p-p$ 900 GeV		
Particle	q	T
π^+	1.154 ±0.036	0.0682 ±0.0026
π^-	1.146 ±0.036	0.0704 ± 0.0027
K^+	1.158 ±0.142	0.0690 ±0.0223
K^-	1.157 ±0.139	0.0681 ± 0.0217
K_S^0	1.134 ±0.079	0.0923 ±0.0139
p	1.107 ±0.147	0.0730 ± 0.0425
$\bar{p}$	1.106 ±0.158	0.0764 ±0.0464
Λ	1.114 ±0.047	0.0698 ± 0.0148
Ξ^-	1.110 ±0.218	0.0440 ± 0.0752

Table 1. Fitted values of the T and q parameters measured in $p-p$ collisions by the ALICE and CMS collaborations using the Tsallis form (7) for the momentum distribution.

Which, at mid-rapidity $y = 0$ and zero chemical potential, has the same dependence on the transverse momentum as (7) apart from an additional factor m_T on the right-hand. It has to be pointed out explicitly that the inclusion of the rest mass in the substitution Eq. (11) is not in agreement with the Tsallis distribution as it breaks m_T scaling which is present in the Tsallis form (6) but not in Eq. (9). The inclusions of the factor m_T leads to a more consistent interpretation of the variables q and T [10, 12].

The distribution (7) has been used to fit the data for identified particles, π, K and p for the ALICE [6] collaboration and K_s^0, Λ and Ξ for the CMS [8] collaboration in $p-p$ collisions at 900 GeV [10, 12]. The results are shown in Table 1 for the parameters T and q. The coresponding transverse momentum distributions for the ALICE [6] are shown in Fig. (1). For all identified particles the results are consistent with having a system at a Tsallis freeze-out temperature of about

$$T \approx 70\,\text{MeV} \tag{13}$$

and a value for the q parameter of about

$$q \approx 1.15 \tag{14}$$

These values are comparable to the ones obtained recently in [13, 14] where the original proposal of Hagedorn [15] was extended to a Hagedorn-Tsallis distribution.

The consistency of the values of q is shown in Fig. (2).

In conclusion we can say that the use of the Tsallis parameterization presented in (1) leads to a good description of identified particles in $p-p$ collisions at 900 GeV with a consistent set of parameters.

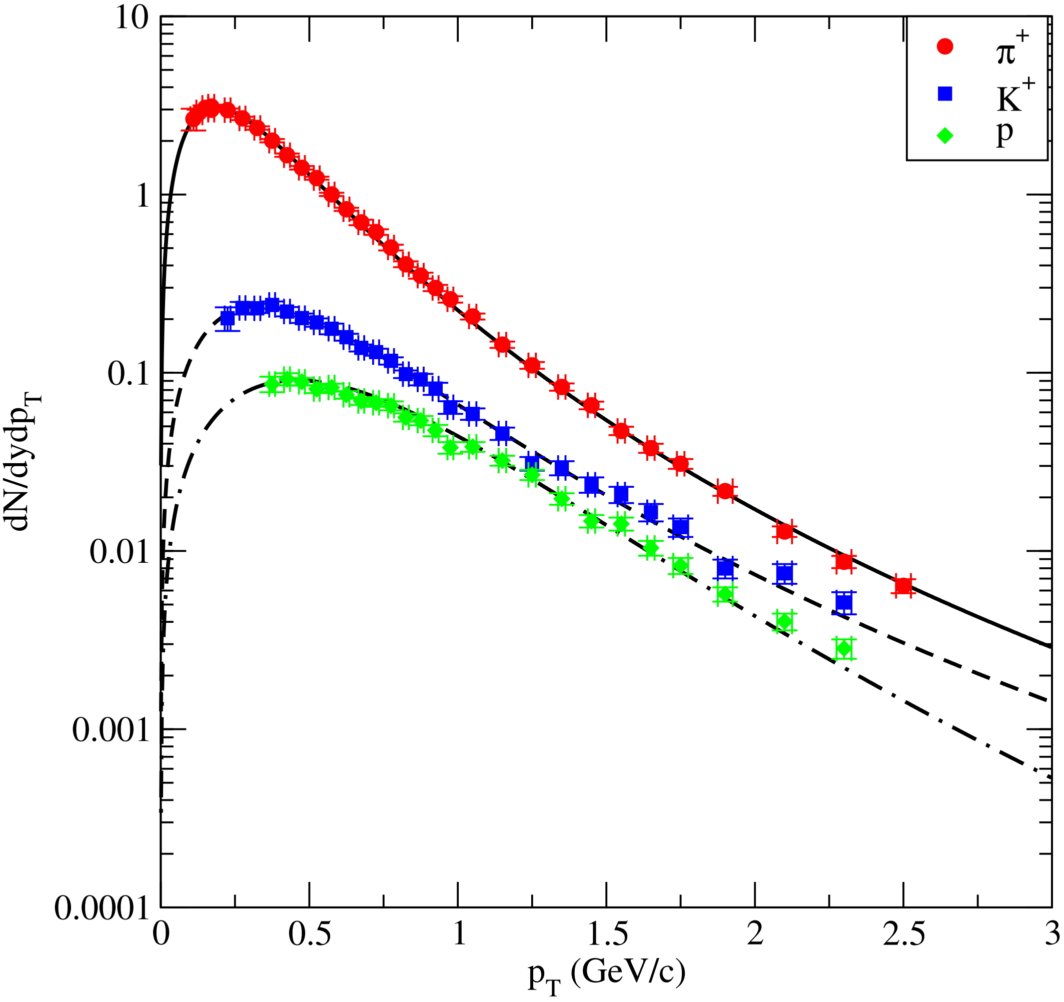

Figure 1. Fit to the π, K, p transverse momentum distributions in $p-p$ collisions as measured by the ALICE collaboration [6] using the Tsallis distribution function as given by (7).

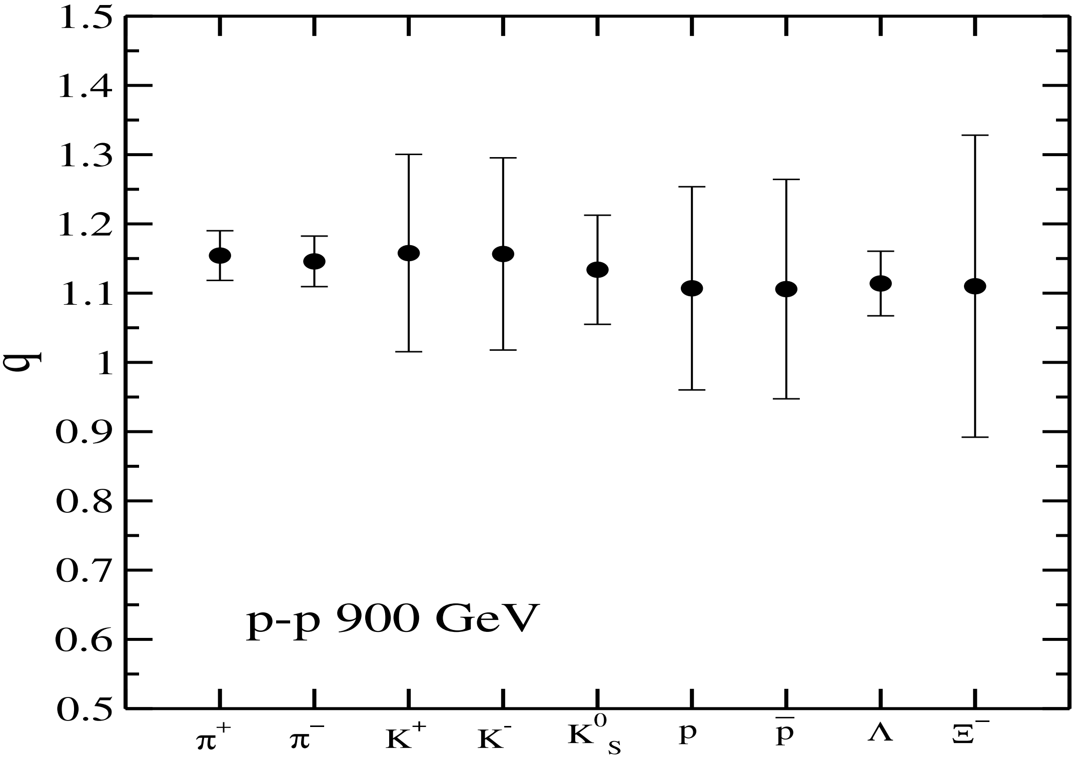

Figure 2. Values of the Tsallis parameter q for different species of hadrons.

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