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Primordial scalar perturbations via conformal mechanisms: statistical anisotropy

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Abstract. We review theoretical and phenomenological properties of conformal rolling scenario, the novel model of primordial scalar perturbations creation. We focus on the particular prediction of this model, statistical anisotropy. We search for the corresponding signal in the CMB data, and establish the constraint on the unique parameter h^2 of the conformal rolling scenario.

1 Introduction

Nowadays inflation is the best established candidate to the role of the early Universe theory. Provided there is a period of the rapid accelerated expansion prior to the conventional hot epoch, horizon and flatness problems obtain an elegant solution [1]. Still, the major merit of inflation is that it gives rise to primordial scalar perturbations [2], the basic ingredient of the future structure formation. Remarkably, these seeds are characterized by the nearly flat spectrum. This property is a consequence of the approximate de Sitter symmetry during the rapid expansion, and is in a wonderful agreement with experimental data. Though tremendous success of the inflation, one should be careful when treating it as the true theory of the early Universe. With the Planck data available, inflation will become the subject of rigorous analysis. Thus, it is of particular importance to think of the crucial tests, which could rule it in or rule it out.

In this regard, detection of statistical anisotropy would be a powerful "anti-smoking" gun for a row of early Universe models. Indeed, statistical isotropy is favored by inflation. This follows from the fact that the quasi-de Sitter metric is highly isotropic during the inflationary stage. However, several proposals exist, where the statistical anisotropy is achieved by extending the field content of inflation in terms of vector fields. Historically the first example has been given by Ackerman, Carroll and Wise [3], who considered the model of the space-like massive vector field with the fixed norm in the exponentially expanding Universe. The dynamical attractor of the cosmological evolution becomes slightly anisotropic in this case. Furthermore, inflaton fluctuations evolving on this "hairy"

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background acquire the direction dependent power spectrum,

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k)(1 + g(\mathbf{dn}_k)^2) \ . \quad (1)$$

This direction dependence is of the special quadrupole type, i.e. it is characterized by the unique vector $\mathbf{d}$ and the amplitude g. For comparison, direction-dependence of the general quadrupole type has the form

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_0(k)\left[1 + a(k)\sum_{LM} q_{LM} Y_{LM}(\hat{\mathbf{k}})\right] \ . \quad (2)$$

Here $Y_{LM}(\hat{\mathbf{k}})$ are the spherical harmonics, $\hat{\mathbf{k}} = \mathbf{k}/k$ is the direction associated with the momentum $\mathbf{k}$ of the cosmological mode; $a(k)$ is the amplitude of statistical anisotropy, and q_{LM} are some coefficients, which we assume independent of the wavenumber k. The formula (2) matches the prediction of the ACW model described by the Eq. (1) provided that only one coefficient, q_{20}, is non-zero. Short afterwards is has been understood that the model of the Ref. [3] suffers from ghost instabilities. Moreover, it has been shown that similar problems arise in all vector inflationary models with broken $U(1)$ invariance [4]. Nowadays, the unique ghost free model of the inflation with vector fields predicting statistical anisotropy is one of the Ref. [5] . Still, the dependence of the power spectrum on the direction predicted there is of the special quadrupole type and parametrized by the formula (1).

Violation of statistical isotropy is hard in inflation, but it may be the natural consequence from the alternative frameworks. In this letter, we consider the concept of the (pseudo)-conformal Universe developed recently in the Refs. [6–8]. The main assumption behind this novel cosmological framework is that the Universe has started from or passed through the conformally invariant state with effectively flat geometry prior to the conventional hot Big Bang. This state is unstable and conformal symmetry $SO(4, 2)$ gets broken down to $SO(4, 1)$ by a time-dependent (rolling) field. During this conformal rolling stage, another field of zero conformal field developes perturbations with flat power spectrum.

Conformal rolling scenario is the simplest realization of this idea [7]. Its main ingredient is the massless complex scalar field rolling down the negative quartic potential. While rolling, it developes perturbations in the radial direction and in the orthogonal one. The latter, phase perturbation, plays the role of the zero conformal weight field, which acquires flat spectrum at rather late times.

It is assumed that the conformal invariance is explicitly broken at sufficiently large field values, and the rolling stops. Since this point on phase perturbations behave like the free massless scalar field minimally coupled to gravity. If the cosmologically interesting modes are superhorizon by the end of the roll, they remain frozen out until the beginning of the hot Big Bang, when the conversion into the standard adiabatic perturbations occurs by one or another mechanism [13, 14]. It is assumed that primordial perturbations literally inherit the properties of the phase (up to the possible non-Gaussianities of the local type). Predictions of this particular version of the conformal rolling scenario are the non-Gaussianity at the level of the four-point function and the quadrupole statistical anisotropy [9, 10]. Conformal rolling scenario with superhorizon modes is natural in the dynamical picture. Namely, the conformal scalar field with negative quartic potential may drive the evolution of the Universe, which undergoes the stage of the slow contraction in that case [6]. Though several distinctions, predictions of the dynamical model about the non-Gaussianity and the statistical anisotropy coincide with ones of the spectator model [11]

If cosmologically interesting modes are still subhorizon by the end of the roll, relevant phase perturbations proceed to evolve at the intermediate stage, which takes place between the end of the conformal rolling and the beginning of the hot epoch [12]. Provided this evolution is long enough, one results with fairly non-trivial predictions for the properties of primordial scalar perturbations: negative scalar tilt, non-Gaussianity of the peculiar shape and the statistical anisotropy of all even multipoles starting from the quadrupole of the general type, i.e. all the coefficients q_{LM} are non zero in the Eq. (2).

Remarkably, conformal rolling scenario is just the particular case in the myriad of models representing the idea of the conformal Universe [6]. Galilean Genesis is the other example [8]. Though the latter relies on the drastically different Lagrangian, the result is again the flat spectrum of the zero weight conformal field. This similarity in predictions of two models is not a coincidence. Both they rely on the symmetry breaking pattern: $SO(4,2) \to SO(4,1)$ created by the background value of the non-zero conformal weight field: radius in case of conformal rolling scenario and Galileon field in case of Galilean Genesis. Moreover, this symmetry breaking pattern fixes the phenomenogical consequences of the Galilean Genesis: these are ones of the conformal rolling scenario with superhorizon perturbations [10].

2 Statistical anisotropy from conformal rolling scenario

Let us discuss dynamics of the fields at the conformal rolling stage in more details. At this stage, the theory is described by the action

$$S = S_{G+M} + S_\phi ,$$

where S_{G+M} is the action for gravity and some matter that dominates the evolution of the Universe, and S_ϕ is the action of the complex massless scalar field conformally coupled to gravity and rolling down the negative quartic potential,

$$S_\phi = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi + \frac{R}{6} |\phi|^2 + h^2 |\phi|^4 \right] .$$

For our purposes, it will be convenient to perform the field ϕ in terms of the radius and phase, i.e. $\phi = \rho e^{i\theta/\sqrt{2}}$, and write the action as follows

$$S_\phi = S_\rho + S_\theta ,$$

where

$$S_\rho = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \rho \partial_\nu \rho + \frac{R}{6} \rho^2 + h^2 \rho^4 \right]$$

and

$$S_\theta = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \rho^2 \partial_\mu \theta \partial_\nu \theta \right] \qquad (3)$$

From the group theoretical point of view, ρ and θ are two separate fields characterized by conformal weights $\Delta = 1$ and $\Delta = 0$, respectively. The action (3) then represents the minimal way of conformal coupling these two fields. In the particular treatment of the conformal rolling scenario this minimality is protected by the $U(1)$ invariance.

Let us follow the classical evolution of the fields ρ and θ. Assume their background values are homogeneous. Due to the conformal coupling of the field ρ to gravity, the dynamics of the field $\chi = a(\eta)\rho$ is the same as in flat space-time. As it rolls down its potential $V(\chi) = -h^2|\chi|^4$, it approaches the late-time attractor,

$$\chi_c = \frac{1}{h(\eta_* - \eta)} . \qquad (4)$$

Here η_* is the constant of integration, which has the meaning of the end-of-roll time. As the field χ_c grows, the phase θ freezes out rapidly, i.e. it tends to some constant value θ_c. With no loss of generality, one chooses the zero value for the background phase, i.e. $\theta_c = 0$. This is the consequence of the $U(1)$ symmetry. The point of the Ref. [7] is that the behaviour of the phase perturbations $\delta\theta$ in the background (4) is very similar to what happens at inflation to the fluctuations of a massless scalar field minimally coupled to gravity (e.g., inflaton itself). The phase perturbations $\delta\theta$ start off as vacuum fluctuations and eventually freeze out. To the leading order in h, resulting phase perturbations are Gaussian and have flat power spectrum

$$\mathcal{P}_{\delta\theta} = \frac{h^2}{(2\pi)^2} \ .$$

This result does not necessarily rely on the existence of the field with the negative quartic potential. Important is the existence of conformal symmetries at early stages of cosmological evolution. Furthermore, one requires that the time-dependent background of the field with $\Delta \neq 0$ (*a la* radius of the conformal rolling scenario) spontaneously breaks down the conformal group $SO(4,2)$ down to the de Sitter group $SO(4,1)$. Then, the zero weight conformal field (*a la* phase) evolves on this emergent de Sitter background and acquires the flat spectrum [6]. These assumptions are not particularly restrictive, and the speech is about the whole class of models, of which conformal rolling scenario is just the concrete example. Galilean Genesis is the other example [8]. There the potential term is absent at all, while the non-trivial dynamics is guaranteed by the high-derivative interactions of the field (Galileon) with $\Delta = 1$. For the sake of concreteness, we proceed with the study of the conformal rolling scenario.

Interaction of phase perturbations with radial ones at the conformal rolling stage is the source of non-trivial phenomenology in this model. In particular, it leads to the statistical anisotropy, i.e. the direction-dependence of the power spectrum. Radial perturbations obey the linearized field equation, which in the momentum representation reads

$$(\delta\chi_1)'' + p^2\delta\chi_1 - 6h^2\chi_c^2\delta\chi_1 = 0 \ . \tag{5}$$

The properly normalized solution to this equation is

$$\delta\chi_1 = \frac{1}{4\pi}\sqrt{\frac{\eta_* - \eta}{2}}H^{(1)}_{5/2}[p(\eta_* - \eta)]B_{\mathbf{p}} + \text{h.c.} \ .$$

where $H^{(1)}_{5/2}$ is the Hankel function of the order $5/2$, and $B_{\mathbf{p}}$ is the annihilation operator. At late times, i.e. in the regime $p(\eta_* - \eta) \ll 1$, the solution approaches the asymptotics

$$\delta\chi_1 = \frac{3}{4\pi^{3/2}}\frac{1}{p^{5/2}(\eta_* - \eta)^2}B_{\mathbf{p}} + \text{h.c.} \ .$$

The late-time perturbation $\delta\chi_1$ of the field χ_1 can be absorbed into the redefinition of the end-of-roll time η_*, so that we can write the radial solution as follows,

$$\chi_c(\eta, \mathbf{x}) = \frac{1}{h[\eta_*(\mathbf{x}) - \eta]} \tag{6}$$

where

$$\eta_*(\mathbf{x}) = \eta_* + \delta\eta_*(\mathbf{x}) \ ,$$

and the shift $\delta\eta_*(\mathbf{x})$ is the random Gaussian field characterized by modes with the red spectrum, i.e.

$$\delta\eta_*(\mathbf{p}) = -\frac{3h}{4\sqrt{2}\pi^{3/2}p^{5/2}}B_{\mathbf{p}} + \text{h.c.} \ .$$

This can be viewed as the simple book-keeping tool: to the first non-trivial order in h the right hand side of the Eq. (6) is equivalent to $\chi_c + \delta\chi_1$. Clearly, the homogeneous shift of the end-of-roll time is irrelevant from the physical point of view. Interesting effects appear, once we consider the spatial variation of $\eta_*(\mathbf{x})$. It is convenient to introduce the notation

$$v_i = -\partial_i \eta_*(\mathbf{x}) \ ,$$

while keeping the standard notation for the second order derivative, $\partial_i\partial_j\eta_*$. To account for the interaction with long ranged radial perturbations, one should study the evolution of the phase in the inhomogeneous background (6). This calculation has been made in the Ref. [9] up to the corrections of the order $\partial_i\partial_j\eta_*/k$ and v^2. The result reads

$$\delta\theta(\mathbf{x},\eta) = \int \frac{d^3k}{\sqrt{k}} \frac{h}{4\pi^{3/2}\gamma(k+\mathbf{kv})} e^{i\mathbf{kx}-ik\eta_*(\mathbf{x})} \left(1 - \frac{\pi}{2k}\frac{k_i k_j}{k^2}\partial_i\partial_j\eta_* + \frac{\pi}{6k}\partial_i\partial_j\eta_*\right) A_{\mathbf{k}} + h.c. \,. \tag{7}$$

Here $\gamma = (1-v^2)^{-1/2}$ is the standard Lorentz boost factor, and the linearization in v^2 is understood; $A_{\mathbf{k}}$ is the annihilation operator for phase perturbations. As it follows, phase perturbations do not depend on the time. So, they remain frozen out until the end of conformal rolling. After conformal symmetries get broken explicitly and the radial field takes the condensate value, the phase behaves like free massless field minimally coupled to gravity. There are two options depending on the cosmological evolution at these times: phase perturbations are already superhorizon by the end of the roll or they are still subhorizon. First, we focus on the opportunity with superhorizon modes. In that case phase perturbations remain frozen out until the beginning of the hot epoch. Deep in the radiation dominated era, they must be converted into the adiabatic perturbations of the standard matter. We assume that the latter literally inherit the properties of phase perturbations. In particular, this concerns the power spectrum, which reads

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k)\,[1 + Q_1(\mathbf{k}) + Q_2(\mathbf{k})] \ .$$

Functions $Q_1(\mathbf{k})$ and $Q_2(\mathbf{k})$ here encode the information about the statistical anisotropy appearing in the leading and in the subleading orders in constant h, respectively. Their form directly follows from the Eq. (7) and reads

$$Q_1(\mathbf{k}) = -\frac{\pi}{k}\hat{k}_i\hat{k}_j\left(\partial_i\partial_j\eta_* - \frac{1}{3}\delta_{ij}\partial_k\partial_k\eta_*\right) \ , \tag{8}$$

$$Q_2(\mathbf{k}) = -\frac{3}{2}(\hat{\mathbf{k}}\mathbf{v})^2 \ , \tag{9}$$

The leading order contribution, $Q_1(\mathbf{k})$, describes the direction dependence of the general quadrupole type. We expand the Eq. (8) into spherical harmonics,

$$Q_1(\mathbf{k}) = a(k)\sum_{LM} q_{2M} Y_{2M}(\hat{\mathbf{k}}) \ ,$$

where

$$a(k) = H_0 k^{-1} \ .$$

Coefficients q_{2M} here are independent Gaussian quantities, since these are the properties of the field $\partial_i\partial_j\eta_*$. They have zero mean values and variances given by

$$\langle q_{2M} q^*_{2M'}\rangle = \frac{\pi h^2}{25}\delta_{MM'} \ .$$

Though coefficients q_{LM} are non-zero in the first order in the coupling constant h, the statistical anisotropy of the general quadrupole is additionally suppressed by the amplitude $a(k)$. Indeed, the latter decreases with the wavenumber k. This fact is crucial from the viewpoint of CMB observations. Namely, the contribution of the general quadrupole falls down with the CMB multipole number $l \sim kH_0^{-1}$, which results into the effectively low statistics of multipoles useful in the analysis. It turns out that the subleading order contribution, $Q_2(\mathbf{k})$, is in fact more relevant. This corresponds to the statistical anisotropy of the special quadrupole type. Note that coefficients q_{2M} are not Gaussian in that case, since the quantity v_i enters non-linearly into the Eq. (9). They are given by

$$q_{2M} = -\frac{4\pi v^2}{5} Y^*_{2M}(\hat{\mathbf{v}}) \,, \tag{10}$$

where $\hat{\mathbf{v}} = \mathbf{v}/v$ is the unit vector in the direction of the "velocity". In what follows, we will also need the expression for the dispersion of the "velocity",

$$\langle v_i^2 \rangle = \frac{3h^2}{8\pi^2} \ln \frac{H_0}{\Lambda} \ . \tag{11}$$

Here Λ plays the role of the infrared cut off for momenta p. Its value is dictated by the physics beyond conformal rolling scenario.

If cosmologically interesting modes are subhorizon by the end of the roll, they proceed to evolve at the intermediate stage [12]. Barring fine tuning, we claim that this evolution is long enough, i.e. $r \equiv \eta_1 - \eta_* \gg k^{-1}$, where η_1 is the time, when phase perturbations get frozen out (in the conventional sense). Second, the cosmological evolution at the intermediate stage must be described by the nearly Minkowski metric. Otherwise, the flat spectrum of phase perturbations would be grossly modified. Dynamics of the phase at the intermediate stage is fairly non-trivial, and the final expression for phase perturbations is quite complicated. Let us simply write down the answer for the power spectrum of primordial scalar perturbations,

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) \left[1 + \mathbf{n}_k(\mathbf{v}(\mathbf{n}_k r) - \mathbf{v}(-\mathbf{n}_k r))\right] \ .$$

The direction-dependence here encodes the statistical anisotropy of all even multipoles starting from the quadrupole of the general type, i.e. all the coefficients q_{LM} with even L are non-zero in the Eq. (2). They are random Gaussian variables with zero means and variances given by

$$\langle q_{LM} q^*_{L'M'} \rangle = \frac{3}{\pi} \frac{h^2}{(L-1)(L+2)} \ . \tag{12}$$

Remarkably, the amplitude $a(k)$ does not depend on the wavenumber k, i.e.

$$a(k) = 1 \ .$$

This prediction of the conformal rolling scenario is in sharp contrast to inflationary ones. Thus, detection of corresponding signatures in the CMB sky would place the inflation in a very uncomfortable corner.

Before we proceed with testing conformal rolling scenario in CMB data, let us make one important remark. Though we are focused on the concrete example of the conformal rolling scenario, similar phenomenology follows from a much broader class of models relying on conformal symmetries. Indeed, the form of Eqs. (4) and (5) is dictated by the symmetry breaking pattern $SO(4,2) \to SO(4,1)$, rather any features of the microscopic physics. Consequently, the form of the Eq. (7) for the zero weight conformal field is generic one. Hence, predictions about the statistical anisotropy.

3 Testing conformal rolling scenario in CMB data

Direction-dependence of the primordial power spectrum makes a very clear imprint on the properties of cosmic microwave background. This leaves us with the hope that predictions of the conformal rolling scenario can be tested by making use of the CMB data.

Given the temperature anisotropies $\delta T(\mathbf{n})$ in the CMB sky, one defines coefficients

$$a_{lm} = \int d\Omega \delta T(\mathbf{n}) Y^*_{lm}(\mathbf{n})$$

Theoretical covariance of CMB coefficients defined as $S_{lm;l'm'} = \langle a_{lm} a^*_{l'm'} \rangle$ is diagonal provided primordial power spectrum does not depend on the direction of the wavevector $\mathbf{k}$. In the opposite case, diagonality of the theoretical covariance breaks down, and one writes

$$\mathbf{S} = \mathbf{S}^i + \delta\mathbf{S} \ . \tag{13}$$

The first term on the right hand side is the diagonal part of the covariance,

$$S^i_{lm;l'm'} = C_l \delta_{ll'} \delta_{mm'} \ ,$$

where C_l's represent the standard CMB angular spectrum. The information about the statistical anisotropy is encoded in the second term on the right hand side of the Eq. (13). It reads

$$\delta S_{lm;l'm'} = i^{l'-l} C_{ll'} \sum_{LM} q_{LM} \int d\Omega_{\mathbf{k}} Y^*_{lm}(\hat{\mathbf{k}}) Y_{l'm'}(\hat{\mathbf{k}}) Y_{LM}(\hat{\mathbf{k}}) \ ,$$

where

$$C_{ll'} = 4\pi \int d\ln k \Delta_l(k) \Delta_{l'}(k) a(k) \mathcal{P}_\zeta(k) \ , \tag{14}$$

and coefficients q_{LM} here are precisely ones parametrizing the direction-dependence of primordial scalar spectrum, see the Eq. (2).

To estimate coefficients q_{LM} from the CMB data, one makes use of the likelihood methods. For this purpose, we introduce the log-likelihood of the observed CMB $\hat{\mathbf{a}}$,

$$-\mathcal{L}(\hat{\mathbf{a}}|\mathbf{q}) = \frac{1}{2} \hat{\mathbf{a}}^\dagger \mathbf{C}^{-1} \hat{\mathbf{a}} + \frac{1}{2} \ln \det \mathbf{C} \tag{15}$$

Here $\mathbf{C}$ is the real covariance, which incorporates the instrumental noise, i.e. $\mathbf{C} = \mathbf{S} + \mathbf{N}$. Formally, we can estimate coefficients q_{LM} from the condition,

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}^\dagger} = 0 \ . \tag{16}$$

In practice, the things are not so simple for the reason that the real covariance is not sparse. First, the noise covariance is not diagonal in the harmonic space. Second, the theoretical covariance also carries off-diagonal elements for we assume violation of statistical isotropy. Thus, direct implication of the condition (16) is numerically challenging and one should take care of the appropriate approximation to work in. It is natural to assume that coefficients q_{LM}'s are small enough, i.e. $q_{LM} \ll 1$, and expand the log-likelihood up to the second order in the latter [15],

$$\mathcal{L} = \mathcal{L}_0 + \left.\frac{\partial \mathcal{L}}{\partial \mathbf{q}}\right|_0 \mathbf{q} + \frac{1}{2} \mathbf{q}^\dagger \left.\frac{\partial^2 \mathcal{L}}{\partial \mathbf{q}^\dagger \partial \mathbf{q}}\right|_0 \mathbf{q} \ .$$

The subscript "0" here implies that the derivatives of the log-likelihood are calculated under the assumption of the statistical isotropy. We replace the second derivative of the log-likelihood by its expectation value,

$$\langle \frac{\partial^2 \mathcal{L}}{\partial \mathbf{q} \partial \mathbf{q}^\dagger} \rangle = -\langle \frac{\partial \mathcal{L}}{\partial \mathbf{q}} \frac{\partial \mathcal{L}}{\partial \mathbf{q}^\dagger} \rangle = -\mathbf{F} ,$$

where $\mathbf{F}$ is the Fisher matrix. The first equality here follows from the normalization condition

$$\int \exp(\mathcal{L}) d\hat{\mathbf{a}} = 1 .$$

We substitute the quadratic likelihood into the Eq. (16), and derive estimators for coefficients q_{LM} [15],

$$q_{LM} = \sum_{L'M'} F^{-1}_{LM;L'M'} \frac{\partial \mathcal{L}}{\partial q_{L'M'}} .$$

Note that quantities on the right hand side are calculated in the absense of the statistical anisotropy. Now, the things become much simpler–mostly thanks to the multigrid preconditioner of the Ref. [16], which allows for the fast computation of the log-likelihood derivative. Following the Ref. [15], we calculate the Fisher matrix in the full sky homogeneous noise approxiamtion. In this situation, it takes the simple diagonal form

$$F_{LM;L'M'} = F_L \delta_{LL'} \delta_{MM'} .$$

Referring the interested reader to articles [15] and [17] for more details, we plot results of the model-independent analysis in Fig. 1. This we do for quantities

$$C^q_L = \frac{1}{2L+1} \sum_M |q_{LM}|^2 ,$$

reconstructed from V- and W-channels of the seven-year WMAP data. Analogous results but for the five-year WMAP data, have been obtained in the Ref. [15]. Interestingly, the quadrupole of the statistical anisotropy is not consistent with the isotropic hypothesis at more than 3σ confidence level in V-channel. The anomalous signal is even more prominent in the W-channel, which is in agreement with the exact results of the Refs. [18]. This frequency dependence of the signal indicates the non-cosmological origin of the statistical anisotropy detected. Moreover, it has been shown in the Refs. [15, 18] that the signal is characterized by the unique direction aligned with poles of the ecliptic plane. On this basis we conclude with the systematical origin of the anomalous quadrupole.

Remind that coefficients q_{LM} are not parameters of the model, but rather random quantities, which obey the Gaussian statistics in the first order in the constant h. Consequently, in the linear order their distribution is described by the probability density

$$\mathcal{W}(\mathbf{q}|h^2) \sim \frac{1}{\sqrt{\det \mathbf{Q}}} \exp\left(-\frac{1}{2} \mathbf{q}^\dagger \mathbf{Q}^{-1} \mathbf{q} \right) .$$

The likelihood of the observed CMB with respect to the coupling constant h^2 is then given by

$$\mathcal{W}(\hat{\mathbf{a}}|h^2) = \int \mathcal{W}(\hat{\mathbf{a}}|\mathbf{q}) \mathcal{W}(\mathbf{q}|h^2) d\mathbf{q} ,$$

here $\mathcal{W}(\hat{\mathbf{a}}|\mathbf{q}) = \exp[\mathcal{L}(\mathbf{a}|\mathbf{q})]$. Once again we make use of the quadratic likelihood appromation, i.e. we expand the Eq. (15) up to the quadratic order in $\mathbf{q}$. Then, the integral takes the Gaussian form and

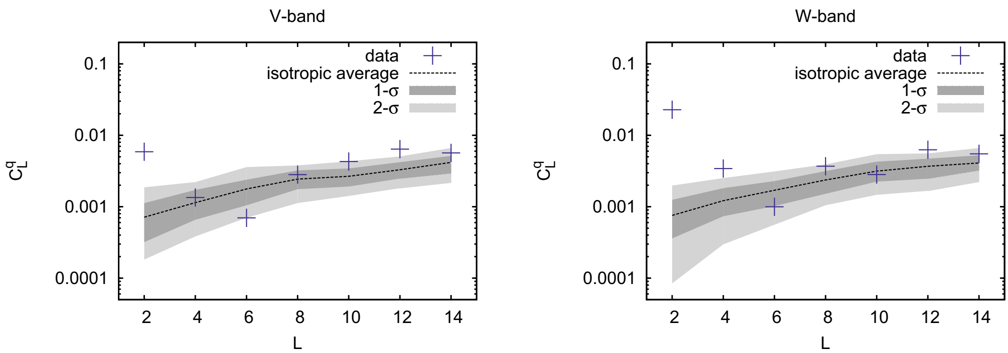

Figure 1. C_L^q of the q_{LM} reconstruction for the V (left) and W (right) bands of the seven-year WMAP data. This analysis assumes $a(k) = 1$ in Eq. (2). The 1σ (dark grey) and 2σ (light grey) confidence intervals are calculated using MC simulated statistically isotropic maps. The analysis is performed with the WMAP temperature analysis mask and $l_{max} = 400$.

can be evaluated in the straightforward manner,

$$\mathcal{W} \sim \frac{1}{\sqrt{\det(\mathbf{FQ}+\mathbf{I})}} \det\left(\frac{1}{2}\frac{\partial\mathcal{L}}{\partial\mathbf{q}^\dagger}(\mathbf{F}+\mathbf{Q}^{-1})^{-1}\frac{\partial\mathcal{L}}{\partial\mathbf{q}}\right) .$$

Next, we set the derivative of the likelihood to zero and obtain the estimator of the parameter h^2,

$$h^2 \sum_L \frac{(2L+1)F_L^2\tilde{Q}_L^2}{(1+F_L\tilde{Q}_L h^2)^2} = \sum_L \frac{(2L+1)F_L\tilde{Q}_L}{(1+F_L\tilde{Q}_L h^2)^2}(F_L C_L^q - 1) .$$

Once again we make use of the full sky and homogeneous noise approximation to calculate the Fisher matrix. Coefficients $\tilde{Q}_L$ are defined from

$$Q_{LM;L'M'} \equiv \langle q_{LM} q^*_{L'M'} \rangle = \tilde{Q}_L h^2 \delta_{LL'} \delta_{MM'} .$$

Next, we apply the estimator constructed to the seven-year WMAP data. We start with the analysis of the conformal rolling scenario with the intermediate stage. We choose to work with the V-band as one less suffering from the systematics. First, we estimate the parameter h^2 using the range of multipoles $L = [2, 14]$, which includes the anomalous quadrupole. As it follows from the Fig. 2, the estimator of the parameter h^2 is highly inconsistent with expectations of the isotropic hypothesis. In particular, it reads $h^2 = 0.015$ at $L_{max} = 14$. Once again, we assume that this large value is dictated by the systematics rather than any non-trivial cosmology, and establish the upper limit on the parameter h^2. The constraining procedure is as follows. Using the Eq. (12), we generate many sets of coefficients $\{q_{LM}\}$ out of some given value of the parameter h^2. Having these sets, we generate anisotropic maps. We claim that the parameter h^2 estimated from these anisotropic maps should not exceed the value observed in seven-year WMAP data. In this way, we establish the upper constraint, which reads [17]

$$h^2 < 0.045 \tag{17}$$

at the 95% confidence level.

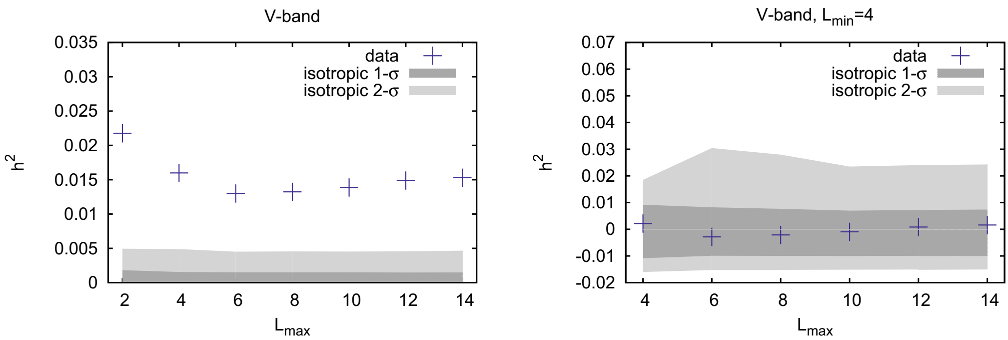

Figure 2. Parameter h^2 of the conformal rolling scenario with intermediate stage estimated from the multipole range $L = [2, 14]$ (left) and $L = [4, 14]$ (right). V band of the seven-year WMAP data has been used. The 1σ (dark grey) and 2σ (light grey) confidence intervals obtained from MC simulations are also shown.

Let us drop the anomalous quardrupole and estimate the parameter h^2 using the range of multipoles $L = [4, 14]$. This is allowed mostly for the reason that anisotropy coefficients q_{LM} with different L's do not correlate with each other. The picture looks much healthier in that case (see the right plot of the Fig. 2). Indeed, the estimates of the parameter h^2 are consistent with expectations of the isotropic hypothesis at the 1σ confidence level. The disadvantage is that the sensitivity of the CMB data to the higher multipoles of the statistical anisotropy is lower. Moreover, the statistical anisotropy predicted by the conformal rolling scenario with the intermediate stage falls down with the number of multipole L. As a result, the upper constraint on the parameter h^2 does not improve essentially. It reads [17]

$$h^2 < 0.040 \tag{18}$$

at the 95% confidence level.

We conclude with the analysis of the statistical anisotropy predicted from the alternative version of the conformal rolling scenario, i.e. one which does not involve the long intermediate stage. Remind that the statistical anisotropy of the general quadrupole type predicted there decreases with the wavenumber k. In turn this implies the suppression by the CMB multipole number. Indeed, the integral in the Eq. (14) is roughly saturated at $k \sim lH_0$. Hence, the suppression. Not surprisingly that the resulting constraint on the parameter h^2 is very weak. It reads

$$h^2 < 190$$

at the 95% confidence level. To improve this constraint, one turns to the statistical anisotropy of the special quadrupole type. Note that the latter is described by the non-Gaussian quantities q_{2M}. Formally, it means that the techniques applied so far does not work anymore. This is, however, not a sort of worry for us, since one can estimate the parameter h^2 directly from the quantity C_2^q in the case of the quadrupole statistical anisotropy. Furthermore, one makes use ot the Eqs. (11) and (10) to generate anisotropic maps. We compare values of the quantity C_2^q estimated from these maps with one obtained from the seven-year WMAP data. The result reads [17]

$$h^2 \ln \frac{H_0}{\Lambda} < 7 \tag{19}$$

at the 95% confidence level. Avoiding speculations about the value of the parameter Λ, we conclude that this constraint is still very weak.

4 Conclusions

It turns out that the statistical anisotropy is a very promising signature of the conformal rolling scenario with intermediate stage. Hopefully, the quadrupole anomaly will be absent in the Planck data. We also expect the significant increase in the number of CMB multipoles useful for the analysis. If this is the case, the constraint (17) will be improved by about two orders of magnitude, or, in the most favorable case, the indication of the signal will be found.

On the other hand, statistical anisotropy is a very weak signature in the alternative version of the conformal rolling scenario/Galilean Genesis. We do not expect the significant improvement of the constraint (19) with the advent of the Planck data. Fortunately, there is the other prediction of these models: non-Gaussianity at the level of the four-point function, which is argued to be in sharp contrast to the analogous predictions of the single-field inflation [10]. We leave for the future search for corresponding signals in the CMB data.

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