EPJ Web of Conferences **70**, 00078 (2014)
DOI: 10.1051/epjconf/20147000078


# CP violation and electroweak baryogenesis in the Standard Model

Tomáš Brauner

[1] *Faculty of Physics, University of Bielefeld, Germany*
*Department of Theoretical Physics, Nuclear Physics Institute ASCR, Řež, Czech Republic*
*e-mail: tbrauner@physik.uni-bielefeld.de*

**Abstract.** One of the major unresolved problems in current physics is understanding the origin of the observed asymmetry between matter and antimatter in the Universe. It has become a common lore to claim that the Standard Model of particle physics cannot produce sufficient asymmetry to explain the observation. Our results suggest that this conclusion can be alleviated in the so-called cold electroweak baryogenesis scenario. On the Standard Model side, we continue the program initiated by Smit eight years ago; one derives the effective CP-violating action for the Standard Model bosons and uses the resulting effective theory in numerical simulations. We address a disagreement between two previous computations performed effectively at zero temperature, and demonstrate that it is very important to include temperature effects properly. Our conclusion is that the cold electroweak baryogenesis scenario within the Standard Model is tightly constrained, yet producing enough baryon asymmetry using just known physics still seems possible.

## 1 Introduction

It is a matter of experimental evidence that the Universe is asymmetric, being composed mostly of matter and virtually no antimatter. The currently observed balance between the matter and antimatter density, $n_{\mathrm{B}}$ and $n_{\bar{\mathrm{B}}}$, is usually expressed in terms of the density of photons of the cosmic microwave background,

$$\frac{n_{\mathrm{B}} - n_{\bar{\mathrm{B}}}}{n_\gamma} \approx 6 \times 10^{-10}. \tag{1}$$

Naively, there are three generic ways to explain this fact. First, the observed asymmetry might be merely spurious; the Universe could consist of large domains of matter or antimatter only and the total, overall baryon number would be zero. Second, the asymmetry might be encoded in the initial conditions from which the Universe expanded. However, both these scenarios are practically ruled out: the former by absence of observation of annihilation radiation coming from collisions of the matter and antimatter domains, the latter by the inflation paradigm. Thus, the most plausible explanation, both from the experimental and the theoretical point of view, is that the asymmetry arose as a result of nontrivial dynamics during the expansion of the Universe, hence the term baryo*genesis*.

It has been known for decades thanks to work of Sakharov that any candidate theory of baryogenesis must incorporate: (i) baryon number violation; (ii) C and CP violation; (iii) departure from chemical equilibrium during the period in which the first two conditions are satisfied. In the Standard Model, the widely accepted and experimentally vigorously tested theory of all fundamental forces except of gravity, the first Sakharov condition is satisfied in a very nontrivial manner. Namely, while baryon number is a symmetry of the Standard Model Lagrangian, it is violated on the quantum level by the chiral anomaly. As we shall see later, this tightly constrains the possible mechanisms for baryogenesis basing on the Standard Model. The weak subnuclear interactions are known to be purely left-handed, thus violating parity and hence charge conjugation maximally. The CP violation as demanded by the second Sakharov condition is more subtle. It also stems from the electroweak sector and requires the existence of (at least) three fermion families, being encoded in the complex phase(s) of the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix. As to the third Sakharov condition, the mechanism for, and the strength of, the departure from chemical equilibrium is strongly model-dependent. It will be discussed to some extent in section 1.2.

### 1.1 CP violation in the CKM matrix

In order to proceed, it is necessary to recall some basic notions related to the CKM matrix as a source of CP violation. Any physical observable must be invariant with respect to arbitrary, and independent, change of phases of the rows and columns of the matrix, which amounts to changing the (unobservable) phases of the quark Dirac fields. The simplest observable constructed out of the CKM matrix $V_{ij}$ which is sensitive to CP violation is the Jarlskog invariant, $J$, defined by the identity (no summation over repeated indices implied)

$$\mathrm{Im}(V_{ij}V^{-1}_{jk}V_{kl}V^{-1}_{li}) \equiv J\epsilon_{ik}\epsilon_{jl}, \tag{2}$$

where $\epsilon_{ij} \equiv \sum_k \epsilon_{ijk}$. Using the experimental values for the CKM matrix elements, one obtains $J \approx 2.96\times10^{-5}$ [1]. Should we desire an observable that is perturbative in the Yukawa couplings and hence in the quark masses, the simplest combination can be constructed using the non-diagonal complex mass matrices of the $u$-type and $d$-type quarks, $M_u$ and $M_d$, and reads

$$2J\Delta \equiv \mathrm{Im}\det[M_uM_u^\dagger, M_dM_d^\dagger] = 2J(m_u^2-m_c^2)(m_c^2-m_t^2)(m_t^2-m_u^2)(m_d^2-m_s^2)(m_s^2-m_b^2)(m_b^2-m_d^2), \tag{3}$$

with obvious notation for the quark masses. This expression elucidates several properties of the CKM matrix such as that the complex phase can be removed by a redefinition of the quark fields and thus there is no CP violation, if there is some “horizontal” degeneracy among the quark masses, that is, the masses of two $u$-type or two $d$-type quarks are equal.

### 1.2 (Cold) electroweak baryogenesis

As stressed above, the Standard Model in principle fulfills all Sakharov conditions and thus is capable of generating baryon asymmetry [2]. However, one has to ensure that sufficient amount of baryon matter can be produced. To that end, we must inspect concrete mechanisms to reach the the far-off-equilibrium environment. This is most naturally achieved if the Universe during its expansion and cooling down passes through a thermal phase transition. For a large departure off the equilibrium, the transition should be strongly first order. However, it is by now well known that the electroweak phase transition is of first order only for Higgs boson masses below roughly 80 GeV [3], which is ruled out by experiment.

Another potential problem with the Standard Model is the quantitative amount of CP violation. Provided baryon asymmetry is active at temperatures around, or above, the electroweak scale, the Yukawa couplings can be treated perturbatively and the amount of CP violation characterized by the dimensionless ratio $\Delta/v^{12} \simeq 10^{-24}$ where $v \approx 246$ GeV is the Higgs field vacuum expectation value. This simple order-of-magnitude estimate together with the absence of a strongly first-order thermal phase transition resulted in the common lore that Standard Model CP violation is not sufficient to explain the observed baryon asymmetry [4]. This was historically one of the most important motives for exploring physics beyond the Standard Model.

However, amending the Standard Model is not the only logical possibility to resolve the problem. An interesting alternative was put forward by García-Bellido et al. [5] and by Krauss and Trodden [6]. In this so-called cold electroweak baryogenesis scenario one modifies the cosmology instead. By augmenting the Standard Model with an additional scalar field which may, but need not, be the inflaton, electroweak transition during the expansion of the Universe can be postponed down to temperatures much below the electroweak scale. With some engineering, the effective temperature of the electroweak transition can be pushed below 1 GeV [7]. The transition then proceeds via a tachyonic instability which drives the system far off equilibrium as required.

Since the baryon number generation through the chiral anomaly in an off-equilibrium environment is an intrinsically nonperturbative process, the resulting net baryon number density cannot be calculated analytically. One instead resorts to numerical simulations. Nevertheless, due to the technical issues associated with the lattice implementation of chiral fermions, the strategy is to represent the CP violation coming from the Yukawa sector by one or more effective bosonic operators, and subsequently simulate such a bosonic effective theory only. Since large occupation numbers of low-momentum modes are generated by the tachyonic instability, classical simulations are fully appropriate to get a quantitatively correct picture. Initially, the effective CP-violating operator was merely guessed and its coupling was treated as as unknown parameter in the simulation. The first attempt to derive such effective operator(s) from the Standard Model was made in Ref. [8]. Subsequent simulations [9] produced the asymmetry $(n_\mathrm{B}-n_{\bar{\mathrm{B}}})/n_\gamma \approx 4\times10^{-6}$, that is, four orders of magnitude more than the observed value (1). Although this result has several caveats, some of which will be discussed below, and thus should be treated with caution, it at least provides a very strong motivation to investigate the cold electroweak baryogenesis scenario further in greater detail.

### 1.3 Gradient expansion of the Standard Model effective action

The effective action for the Standard Model bosons is derived by integrating out the fermions. Since the action is bilinear in the fermionic fields, this can in principle be done exactly by calculating the determinant of the fermion Dirac operator in presence of background gauge and Higgs fields. Unfortunately, this cannot be done in a closed form for arbitrary spacetime-dependent background fields. In order to obtain a controlled analytic result, one therefore resorts to the (covariant) gradient expansion, which orders the various contributing operators by an increasing number of Lorentz indices. Gauge fields are treated on the same footing as gradients which ensures that gauge invariance is preserved.

As shown by Smit [10], there is no CP violation in the effective bosonic action of the Standard Model up to fourth order in the gradient expansion. The next, sixth order at zero temperature was calculated independently by two groups, obtaining qualitatively different results [8, 11]. While Hernandez et al. [8] found operators that break P and conserve C, all operators reported by García-Recio and Salcedo conserve P and break C. It should be stressed that the two groups used different methods, namely the worldline formalism and the method of symbols, to evaluate the same object, the CP-violating effective action in the Standard Model at order six. The discrepancy might therefore in principle indicate an error in some of the algebraic manipulations as well as a deeper problem rooted in one of the methods.

This state of affairs provided the initial motivation for our project. Our goals were twofold: (i) to resolve the discrepancy between the existing results at zero temperature; (ii) to extend the calculation to finite temperature. The latter point stems from the fact that, as argued in the introduction, at temperatures around or above the electroweak scale, effects of CP violation are extremely strongly suppressed. On the other hand, as pointed out above, using the zero-temperature effective action yields baryon asymmetry in excess of four orders of magnitude over the observed value. The detailed knowledge of temperature dependence of CP violation is therefore clearly crucial in order to constrain possible scenarios of electroweak baryogenesis.

## 2 Results

In this section we report all the results obtained at zero as well as nonzero temperature at the leading and next-to-leading order in the gradient expansion. The calculations were carried out using the following strategy. The trace of the logarithm of the Dirac operator is first formally expanded in powers of the $W^{\pm}_{\mu}$ fields, leading to a series of monomials in the gauge fields and (pseudo)differential operators containing the covariant derivatives. The trace of each contribution is evaluated using the method of covariant symbols [11] and subsequently re-expanded up to the desired order in the gradients. The actual manipulations were performed using the Mathematica package Feyncalc [12]. The results shown here were published for the first time in Ref. [13]. Details of the computations are given in ref. [14]. All effective actions presented below are given in Euclidean spacetime.

### 2.1 Order six at finite temperature

In the Standard Model, nontrivial CP-violating contributions first appear at the sixth order of the gradient expansion of the effective bosonic action. As stressed above, our goal here was to resolve the discrepancy between the two existing calculations [8, 11], and to extend their results to finite temperature. The CP-violating part of the effective action takes the form

$$\Gamma^{4+2}_{\text{CP-odd}} = -\frac{\mathrm{i}}{2} N_{\mathrm{c}} J G_{\mathrm{F}} \kappa^{4+2}_{\mathrm{CP}} \int \mathrm{d}^4 x \left(\frac{v}{\phi}\right)^2 (O_0 + O_1 + O_2), \tag{4}$$

where $N_{\mathrm{c}} = 3$ is the number of colors, $G_{\mathrm{F}} = 1/(\sqrt{2}v^2)$ is the Fermi coupling, and the coefficient $\kappa^{4+2}_{\mathrm{CP}}$ is defined by

$$\kappa^{4+2}_{\mathrm{CP}} \equiv \frac{\Delta}{G_{\mathrm{F}}} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{(p^2)^3}{\prod_{q=1}^{6} (p^2 + m_q^2)^2} \approx 309. \tag{5}$$

In the numerical evaluation, we used the quark masses provided by the Particle Data Group [1], $m_u = 2.3$ MeV, $m_d = 4.8$ MeV, $m_c = 1.275$ GeV, $m_s = 95$ MeV, $m_t = 173.5$ GeV, $m_b = 4.18$ GeV. Thanks to the hierarchy of quark masses, one can obtain a good analytic approximation, $\kappa^{4+2}_{\mathrm{CP}} \approx 1/(16\pi^2 G_{\mathrm{F}} m_c^2) \approx 334$.

The operators $O_n$ are composed of the electroweak gauge fields $W^{\pm}_{\mu}$, $Z_{\mu}$, and the derivative of the Higgs field $\phi$, $\varphi_{\mu} \equiv (\partial_{\mu}\phi)/\phi$. The index $n$ counts the number of $Z_{\mu}$ and $\varphi_{\mu}$. At zero temperature, all contributing operators must be Lorentz invariant. However, this constraint is lost at nonzero temperature. We can nevertheless make all expressions formally covariant by introducing the timelike vector $u_{\mu} \equiv \delta_{\mu 0}$ defining the thermal bath rest frame. We list here all contributions which are

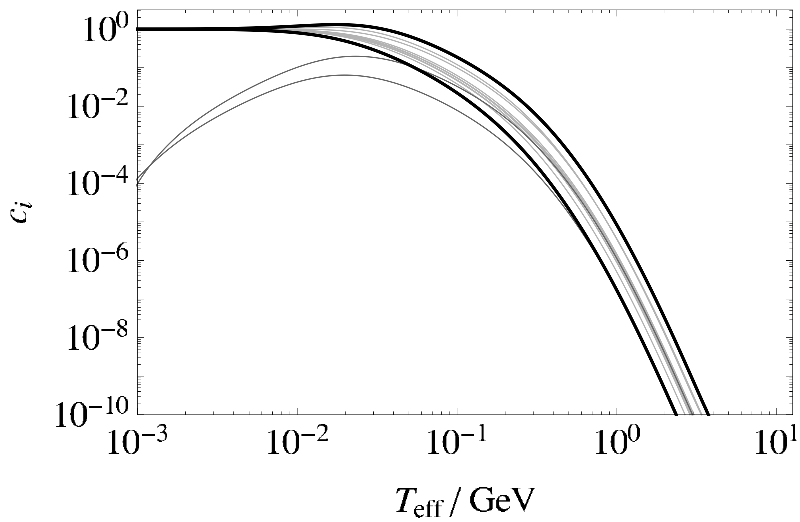


**Figure 1.** The coefficients $c_1$–$c_{13}$ plotted as functions of the effective temperature $T_{\text{eff}} = Tv/\phi$. The thick lines correspond to the smallest and the largest of the couplings that survive at zero temperature, that is, $c_1$ and $c_{10}$. This figure was previously published in Ref. [13].

Lorentz-invariant in the sense that they do not contain any factors of $u_\mu$,

$$\begin{aligned}
O_0 =& -\frac{c_1}{3}(W^+)^2 W^-_{\mu\mu} W^-_{\nu\nu} + \frac{5c_2}{3}(W^+)^2 W^-_{\mu\nu} W^-_{\mu\nu} - \frac{c_1}{3}(W^+)^2 W^-_{\mu\nu} W^-_{\nu\mu} + \frac{4c_3}{3} W^+_\mu W^+_\nu W^-_{\mu\nu} W^-_{\alpha\alpha} \\
&- \frac{2c_1}{3} W^+_\mu W^+_\nu W^-_{\mu\alpha} W^-_{\nu\alpha} - 2c_4 W^+_\mu W^+_\nu W^-_{\alpha\mu} W^-_{\alpha\nu} + \frac{4c_3}{3} W^+_\mu W^+_\nu W^-_{\mu\alpha} W^-_{\alpha\nu} - \text{c.c.}, \\
O_1 =& \frac{8}{3}(Z_\mu + \varphi_\mu)[c_5 (W^+)^2 W^-_\mu W^-_{\nu\nu} - c_6 (W^+)^2 W^-_\nu W^-_{\mu\nu} - c_6 (W^+)^2 W^-_\nu W^-_{\nu\mu} \\
&- c_3 (W^+ \cdot W^-) W^+_\mu W^-_{\nu\nu} + c_7 (W^+ \cdot W^-) W^+_\nu W^-_{\mu\nu} + c_7 W^+_\mu W^+_\nu W^-_\alpha W^-_{\alpha\nu} \\
&- c_{12} (W^+ \cdot W^-) W^+_\nu W^-_{\nu\mu} - c_{12} W^+_\mu W^+_\nu W^-_\alpha W^-_{\nu\alpha} + c_{13} W^-_\mu W^+_\nu W^+_\alpha W^-_{\nu\alpha}] - \text{c.c.}, \\
O_2 =& 4(Z_\mu Z_\nu + \varphi_\mu \varphi_\nu)[c_8 (W^+)^2 W^-_\mu W^-_\nu - c_8 (W^-)^2 W^+_\mu W^+_\nu] - \frac{16}{3}(Z \cdot \varphi)[c_9 (W^+ \cdot W^-)^2 - 2c_6 (W^+)^2 (W^-)^2] \\
&+ \frac{4}{3}(Z_\mu \varphi_\nu + Z_\nu \varphi_\mu)[c_{10} (W^+)^2 W^-_\mu W^-_\nu + c_{10} (W^-)^2 W^+_\mu W^+_\nu - 2c_{11} (W^+ \cdot W^-)(W^+_\mu W^-_\nu + W^+_\nu W^-_\mu)],
\end{aligned} \tag{6}$$

where "c.c." stands for complex conjugation. The covariant derivative of the charged weak gauge boson fields is defined as $W^\pm_{\mu\nu} \equiv (\partial_\mu \pm B_\mu) W^\pm_\nu$, where $B_\mu$ is the hypercharge gauge field. In order to simplify the notation, we use rescaled fields, related to the canonically normalized (Euclidean) gauge fields, denoted by a tilde, via $W^\pm_\mu = (g/\sqrt{2})\tilde{W}^\pm_\mu$, $Z_\mu = [g/(2\cos\theta_{\text{W}})]\tilde{Z}_\mu$, and $B_\mu = g'\tilde{B}_\mu$, where $g, g'$ are the isospin and hypercharge gauge couplings and $\theta_{\text{W}}$ is the Weinberg angle.

We have checked that the Lorentz-noninvariant part of the effective action vanishes in the limit of zero temperature as it must. Note that all operators displayed above conserve P and break C. At zero temperature, our result reduces to that of Ref. [11], thus independently verifying its conclusions.

All temperature dependence of the effective action is encoded in the coefficients $c_i$. These depend, apart from the quark masses, only on the temperature $T$ and the local Higgs field in the combination $T_{\text{eff}} \equiv Tv/\phi$. The coefficients $c_1$–$c_{11}$ correspond to operators that appear in the zero-temperature action of Ref. [11]; they are normalized so that they approach unity in the zero-temperature limit. In addition, several new operators appear at nonzero temperature with the couplings $c_{12}$ and $c_{13}$. The temperature dependence of the coefficients $c_i$ is shown in Fig. 1. We would like to point out that the coefficients $c_{12}$ and $c_{13}$ reach roughly the same size as the rest at temperatures around 300 MeV. This demonstrates convincingly that their vanishing at zero temperature is a result of accidental cancellations rather than some hidden symmetries or regularities in the effective action. At high temperatures, the coefficients drop extremely fast, thus interpolating between the low-temperature regime and the perturbative estimate of Ref. [4].

The fact that the effective couplings only depend on the combination $T_{\text{eff}}$ has important implications for the generation of baryon asymmetry. At fixed (physical) temperature, the couplings drop fast as the value of the Higgs field decreases. As a consequence, the thermal suppression of CP-violating effects can be compensated, or at least alleviated, by larger values of $\phi$. In the previous simulations [9] using the zero-temperature action of Ref. [8], the gradient expansion was invalidated in regions where the Higgs field was close to zero as a result of the overall prefactor $(v/\phi)^2$ in Eq. (4), which necessitated the use of an unphysical cutoff. Our result shows that the limits of zero temperature $T$ and zero Higgs field $\phi$ do not commute, and at any fixed finite temperature, the gradient expansion is well behaved for any value of the Higgs field.

### 2.2 Order eight at zero temperature

The actual convergence of the gradient expansion can be investigated more closely by inspecting contributions to the effective action at the next, eighth order. In addition, as observed in Ref. [11] and confirmed by us also at nonzero temperature, all CP-violating operators at the sixth order of the gradient expansion preserve parity. As shown by Salcedo [15], parity-odd CP-violating operators first appear at order eight. The CP-violating operators at order eight come in two types, containing either four or six factors of the $W^{\pm}_{\mu}$ fields. We have evaluated all operators of the latter type; the resulting Euclidean action at zero temperature reads

$$\Gamma^{6+2}_{\text{CP-odd}} = \mathrm{i}N_c J G_F^2 \kappa^{6+2}_{\text{CP}} \int \mathrm{d}^4x \left(\frac{v}{\phi}\right)^4 (O_0 + O_1 + O_2)\,, \tag{7}$$

where the various contributions, using the same notation as in the case of order six, are

$$\begin{aligned}
O_0 =&(W^+)^2(W^+\cdot W^-)(W^-_{\mu\mu}W^-_{\nu\nu}-4W^-_{\mu\nu}W^-_{\mu\nu}+W^-_{\mu\nu}W^-_{\nu\mu})+2(W^+)^2W^+_\mu W^-_\nu(W^-_{\mu\nu}W^-_{\alpha\alpha}+W^-_{\nu\mu}W^-_{\alpha\alpha}+W^-_{\mu\alpha}W^-_{\nu\alpha}\\
&-4W^-_{\alpha\mu}W^-_{\alpha\nu}+W^-_{\mu\alpha}W^-_{\alpha\nu}+W^-_{\alpha\mu}W^-_{\nu\alpha})+6(W^+\cdot W^-)W^+_\mu W^+_\nu(-W^-_{\mu\nu}W^-_{\alpha\alpha}+\tfrac{1}{3}W^-_{\mu\alpha}W^-_{\nu\alpha}+2W^-_{\alpha\mu}W^-_{\alpha\nu}\\
&-W^-_{\mu\alpha}W^-_{\alpha\nu})+4W^+_\mu W^+_\nu W^+_\alpha W^-_\beta W^-_{\mu\nu}W^-_{\alpha\beta}-6W^+_\mu W^+_\nu W^+_\alpha W^-_\beta W^-_{\mu\nu}W^-_{\beta\alpha}-\text{c.c.},\\
O_1 =&2(Z_\mu+\varphi_\mu)[-2(W^+)^2(W^-)^2W^+_\mu W^-_{\nu\nu}+3(W^+)^2(W^-)^2W^+_\nu W^-_{\mu\nu}-2(W^+)^2(W^-)^2W^+_\nu W^-_{\nu\mu}\\
&-4(W^+)^2(W^+\cdot W^-)W^-_\mu W^-_{\nu\nu}+6(W^+)^2(W^+\cdot W^-)W^-_\nu W^-_{\mu\nu}+(W^+)^2(W^+\cdot W^-)W^-_\nu W^-_{\nu\mu}\\
&+6(W^+\cdot W^-)^2W^+_\mu W^-_{\nu\nu}-9(W^+\cdot W^-)^2W^+_\nu W^-_{\mu\nu}+(W^+\cdot W^-)^2W^+_\nu W^-_{\nu\mu}+(W^+)^2W^+_\mu W^-_\nu W^-_\alpha W^-_{\nu\alpha}\\
&+(W^+)^2W^-_\mu W^+_\nu W^-_\alpha W^-_{\nu\alpha}+(W^+)^2W^-_\mu W^-_\nu W^+_\alpha W^-_{\nu\alpha}+(W^-)^2W^+_\mu W^+_\nu W^+_\alpha W^-_{\nu\alpha}\\
&-3(W^+\cdot W^-)W^+_\mu W^+_\nu W^-_\alpha W^-_{\nu\alpha}-3(W^+\cdot W^-)W^+_\mu W^-_\nu W^+_\alpha W^-_{\nu\alpha}+2(W^+\cdot W^-)W^-_\mu W^+_\nu W^+_\alpha W^-_{\nu\alpha}]-\text{c.c.},\\
O_2 =&10(Z_\mu Z_\nu+\varphi_\mu\varphi_\nu)(W^+\cdot W^-)[(W^-)^2W^+_\mu W^+_\nu-(W^+)^2W^-_\mu W^-_\nu]+32(Z\cdot\varphi)(W^+\cdot W^-)[(W^+\cdot W^-)^2\\
&-(W^+)^2(W^-)^2]+2(Z_\mu\varphi_\nu+Z_\nu\varphi_\mu)\{-(W^+)^2(W^+\cdot W^-)W^-_\mu W^-_\nu-(W^-)^2(W^+\cdot W^-)W^+_\mu W^+_\nu\\
&+[4(W^+\cdot W^-)^2-3(W^+)^2(W^-)^2](W^+_\mu W^-_\nu+W^+_\nu W^-_\mu)\}.
\end{aligned} \tag{8}$$

The dimensionless coefficient $\kappa^{6+2}_{\rm CP}$ is again expressed in terms of the Jarlskog determinant (3), quark masses and the CKM matrix elements,

$$\kappa^{6+2}_{\rm CP} \equiv \frac{4}{15}\frac{\Delta}{G_{\rm F}^2}\int\frac{{\rm d}^4p}{(2\pi)^4}\frac{(p^2)^4}{\prod_{q=1}^6(p^2+m_q^2)^2}\sum_{i,j}\frac{|V_{ij}|^2}{(p^2+m^2_{u,i})(p^2+m^2_{d,j})}\approx 2.65\times 10^9. \tag{9}$$

The notation for the masses of $u$-type and $d$-type quarks, $m_{u,i}$ and $m_{d,i}$, is self-explanatory. Let us remark that, owing to the higher order in the gradient expansion, the momentum integral is strongly sensitive to the masses of the light quarks. In fact, 99% of its value come from the $i = j = 1$ term in the sum. As in the case of $\kappa^{4+2}_{\rm CP}$, a very good approximate analytic expression can be found by factorizing out all the heavy quark masses,

$$\kappa^{6+2}_{\rm CP} \approx \frac{|V_{ud}|^2}{30\pi^2G_{\rm F}^2m_c^2m_s^2}\left(\log\frac{m_s}{\bar m}-\frac{197}{120}\right)\approx 2.64\times 10^9, \tag{10}$$

where $\bar m \equiv (m_u+m_d)/2$.

Note that all operators presented in Eq. (8) preserve parity, analogously to the finding at order six. A selected class of operators breaking parity, of the type $(W^+W^-)^2Z^3D$, was identified by Salcedo [15]. All these operators contains just four factors of the $W^\pm_\mu$ fields. We have not evaluated any CP-violating operators of this type. The main reason is the vast number of contributing operators; by mere combinatorics one can estimate that there will be several thousands of different operators. It is very unlikely that the coefficients of a majority of them would vanish by accidental cancellations, and thus the result will almost certainly be of little practical use.

## 3 Conclusions

We have calculated the temperature dependence of the CP-violating effective action for Standard Model bosons to the leading nontrivial, sixth order in the covariant gradient expansion. The zero-temperature limit of our result coincides with the action derived in Ref. [11], and thus casts doubt on the result of Ref. [8]. While we followed the same method as García-Recio and Salcedo and thus cannot point out an explicit error in the computation of Hernandez et al., it is clear that their derivation requires a careful inspection. The steep temperature dependence of the effective couplings is compatible with the limiting cases of zero [11] and high [4] temperature, and sets tight constraints on possible scenarios for electroweak baryogenesis. Although detailed numerical simulations of the obtained effective bosonic theory are yet to be done, a simple estimate allowed us to conclude that Standard Model CP violation may be the source of the cosmological baryon asymmetry provided that the electroweak phase transition happens at low enough temperature, of the order of 1 GeV [13]. This implies that while "hot" electroweak baryogenesis with the Standard Model as the only source of CP violation is firmly ruled out as has been known for many years, other cosmological scenarios such as the cold electroweak baryogenesis might provide a viable solution to the baryon asymmetry problem.

Several points require further study. The immediate follow-up research project is to redo the simulations of Ref. [9] using the correct CP-violating action with the appropriate temperature-dependent effective couplings. On the conceptual side, the issue of gauge invariance of the gradient expansion at nonzero temperature has to be carefully dealt with. The correct gauge-invariant effective action is then necessarily nonlocal; here we avoided this problem by including only the Lorentz-invariant operators in Eq. (6). Finally, it would be very interesting to apply the same computational method to extensions of the Standard Model. This would in particular allow us to study the effects of additional sources of CP violation. All the above points will be addressed in our future work.

## Acknowledgements

The presented results were obtained in collaboration with Olli Taanila, Anders Tranberg and Aleksi Vuorinen. Financial support from the Sofja Kovalevskaja program of the Alexander von Humboldt Foundation and from NordForsk is gratefully acknowledged.

## References

[1] J. Beringer et al. (Particle Data Group), Phys. Rev. D **86**, 010001 (2012)
[2] V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B **155**, 36 (1985)
[3] K. Kajantie, M. Laine, K. Rummukainen, M.E. Shaposhnikov, Phys. Rev. Lett. **77**, 2887 (1996)
[4] M.E. Shaposhnikov, Nucl. Phys. B **299**, 797 (1988)
[5] J. García-Bellido, D.Y. Grigoriev, A. Kusenko, M.E. Shaposhnikov, Phys. Rev. D **60**, 123504 (1999)
[6] L.M. Krauss, M. Trodden, Phys. Rev. Lett. **83**, 1502 (1999)
[7] K. Enqvist, P. Stephens, O. Taanila, A. Tranberg, JCAP **09**, 019 (2010)
[8] A. Hernandez, T. Konstandin, M.G. Schmidt, Nucl. Phys. B **812**, 290 (2009)
[9] A. Tranberg, A. Hernandez, T. Konstandin, M.G. Schmidt, Phys. Lett. B **690**, 207 (2010)
[10] J. Smit, JHEP **09**, 067 (2004)
[11] C. García-Recio, L.L. Salcedo, JHEP **07**, 015 (2009)
[12] R. Mertig, M. Böhm, A. Denner, Comput. Phys. Commun. **64**, 345 (1991)
[13] T. Brauner, O. Taanila, A. Tranberg, A. Vuorinen, Phys. Rev. Lett. **108**, 041601 (2012)
[14] T. Brauner, O. Taanila, A. Tranberg, A. Vuorinen (2012), `arXiv:1208.5609 [hep-ph]`
[15] L.L. Salcedo, Phys. Lett. B **700**, 331 (2011)