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(Pseudo)goldstinos, SUSY fluids, Dirac gravitino and gauginos

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Abstract. We review the emergence and fate of goldstinos in different frameworks. First, we consider a super-Higgs mechanism when supersymmetry breaking is induced by neither an F-term nor a D-term but related to a more general stress energy-momentum tensor. This allows us to build a novel Lagrangian that describes the propagation of a spin-$\frac{3}{2}$ state in a fluid. Then we briefly review the ubiquitous pseudo-goldstinos when breaking supersymmetry in an extra dimension. We remind that the fermion (gravitino or gaugino) soft masses can be tuned to be of Dirac-type. Finally, we briefly connect the latter to the study of models with Dirac-type gaugino masses and stress the advantage of having both an F and a D-term sizable contributions for the hierarchies of soft-terms as well as for minimizing R-symmetry breaking.

1 Introduction

Interest in supersymmetry (SUSY) can be motivated through different arguments among which its role as an essential ingredient of the fundamental theory unifying gravity with all the other interactions in a consistent quantum theory. At experimentally probed energies, supersymmetry is not manifest and we would like to think of that as a consequence of its spontaneous breaking at a higher energy scale. The spontaneous breaking gives rise in the global limit to a *massless* Goldstone fermion, the *goldstino* [1, 2]. Once gravitational interactions are taken into account, this state is absorbed by the gravitino to become the spin-$\frac{1}{2}$ component [3, 4] of the massive spin-$\frac{3}{2}$ particle. The corresponding dynamics is described by the Rarita-Schwinger Lagrangian [5] which appears supplemented with appropriate constraints. We shall review here a few aspects when departing from the minimal set-up.

In the first part, we shall construct a Lagrangian that allows to describe the propagation of a spin-$\frac{3}{2}$ state in a fluid [6].. This is obtained as the result of super-Higgs mechanism when supersymmetry is broken by a non-vanishing energy-momentum tensor. The modification to the Rarita-Schwinger Lagrangian appears as a deformation of the quadratic mass term. This allows to describe different propagation velocities of the different gravitino helicities. The second part reviews the omnipresence of pseudo-goldstinos in models where supersymmetry is broken in different sectors of models with one extra dimension [7]. We restrict to the simplest case of at most two branes while the more general case with an arbitrary number of supersymmetry breaking sectors and more dimensions can be found in [8]. Finally, the last part discusses models Dirac gaugino masses (see [9] for a review) where the soft terms are induced by gauge mediation. In particular, we focus on the advantages of having sizable

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F and D-term supersymmetry breaking and point out how this can help to keep R-symmetry unbroken by generating a Dirac gravitino mass.

2 A Lagrangian for a spin-$\frac{3}{2}$ propagating in a fluid

We start by a brief review of the well known non-linear realization of supersymmetry and super-Higgs effect.

2.1 The Rarita-Schwinger Lagrangian from the super-Higgs mechanism

2.1.1 The Volkov-Akulov Lagrangian

We consider a global supersymmetric theory in flat space time. Supersymmetry is broken spontaneously when the vacuum has non-zero energy which, as we will see below, is not the case in local supersymmetric theory. If one insists on preserving Lorentz invariance, this is accomplished for $N = 1$ supersymmetry by giving a vev to an auxiliary field in a chiral multiplet (F-term) or in a vector multiplet (D-terms). Without loss of generality, we shall chose to focus on the F-term case in this section. As a consequence of Goldstone theorem, the low energy spectrum contains a fermionic massless mode, known as the *goldstino* for each broken supersymmetry.

The goldstino is a spin $\frac{1}{2}$ field $(G_\alpha, \bar{G}_{\dot{\alpha}})$ in the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation of the Lorentz group[1]. It has a mass dimension of $\frac{3}{2}$. Supersymmetry is non-linearly realized on the field G through

$$(\epsilon Q + \bar{\epsilon}\bar{Q}) \times G_\alpha(x) = \sqrt{2} F \epsilon_\alpha - i \frac{1}{\sqrt{2}F} \left[G(x)\sigma^\mu \bar{\epsilon} - \epsilon \sigma^\mu \bar{G}(x) \right] \partial_\mu G_\alpha(x). \tag{1}$$

where the F-term F is taken to be real and has a mass dimension 2, and plays the role of the order parameter of supersymmetry breaking.

The invariant (up to a divergence) non-linear Lagrangian for G is given by the Volkov–Akulov Lagrangian [2]

$$\mathcal{L}_{\text{A-V}} = -F^2 \det\left(\delta^\nu_\mu + i \frac{1}{F^2} \bar{G}\bar{\sigma}^\nu \partial_\mu G \right) \tag{2}$$

$$= -F^2 - i\bar{G}\bar{\sigma}^\mu \partial_\mu G + \cdots , \tag{3}$$

where the dots refer to higher order terms that we do not discuss here. This canonically normalized goldstino field satisfies the Dirac equation

$$\bar{\sigma}^\mu \partial_\mu G = 0, \quad \sigma^\mu \partial_\mu \bar{G} = 0 \,. \tag{4}$$

2.1.2 The Rarita-Schwinger Lagrangian for a massless gravitino

We are interested by theories with $N = 1$ local supersymmetry. The supersymmetric partner of the graviton is a gravitino field $(\psi_{\mu\alpha}, \bar{\psi}_{\mu\dot{\alpha}})$ of spin $\frac{3}{2}$ and mass dimension $\frac{3}{2}$. Following Fierz and Pauli, the irreducible spin $\frac{3}{2}$ representation is obtained from $\psi_{\mu\alpha}$ in the $(\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, 0) = (1, \frac{1}{2}) \oplus (0, \frac{1}{2})$

[1] We will work in 4 dimensions and we use Wess and Bagger [10] notations. $\eta_{\mu\nu} = diag(-,+,+,+)$, $\epsilon^{12} = -\epsilon^{21} = 1$. ζ_α is a left Weyl spinor in the $(\frac{1}{2}, 0)$ representation. $\bar{\zeta}_{\dot{\alpha}}$ is a right Weyl spinor in the $(0, \frac{1}{2})$ representation. Complex conjugation exchanges $SU(2)_L$ and $SU(2)_R$. The complex conjugate of a left Weyl spinor is a right Weyl spinor. We use $\hbar = 1$, $c = 1$ and denote by M_P the reduced Planck mass: $M_P = \sqrt{\frac{\hbar c}{8\pi \mathbf{G}}} \simeq 2.435 \times 10^{18}$ GeV with $\mathbf{G}$ the Newton constant.

representation, and $\bar{\psi}_{\mu\dot{\alpha}}$ in the $(\frac{1}{2},\frac{1}{2}) \otimes (0,\frac{1}{2}) = (\frac{1}{2},1) \oplus (\frac{1}{2},0)$ representation by imposing constraints that project out the additional spin $\frac{1}{2}$ components. The $(0,\frac{1}{2})$ and $(\frac{1}{2},0)$ parts in the decomposition of $(\psi_{\mu\alpha},\bar{\psi}_{\mu\dot{\alpha}})$ are removed by imposing

$$\bar{\sigma}^{\mu}\psi_{\mu} = 0, \quad \sigma^{\mu}\bar{\psi}_{\mu} = 0 . \tag{5}$$

The representations $(1,\frac{1}{2})$ and $(\frac{1}{2},1)$ have dimension six each. In order to reduce the number of degrees of freedom to four we impose

$$\partial^{\mu}\psi_{\mu\alpha} = 0, \quad \partial^{\mu}\bar{\psi}_{\mu\dot{\alpha}} = 0 . \tag{6}$$

One can get this structure of equations and constraints from a Lagrangian. The massless gravitino Rarita-Schwinger Lagrangian is:

$$\mathcal{L}_{\psi} = \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\bar{\sigma}_{\nu}\partial_{\rho}\psi_{\sigma} . \tag{7}$$

The field equations are

$$\epsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\nu}\partial_{\rho}\psi_{\sigma} = 0, \quad \epsilon^{\mu\nu\rho\sigma}\sigma_{\nu}\partial_{\rho}\bar{\psi}_{\sigma} = 0 . \tag{8}$$

By imposing on this equation the condition (5) we get

$$\bar{\sigma}^{\rho}\partial_{\rho}\psi_{\sigma} = 0, \quad \sigma^{\rho}\partial_{\rho}\bar{\psi}_{\sigma} = 0 . \tag{9}$$

It is easy to see that (9) and (5) imply (6).

2.1.3 The super-Higgs mechanism and the massive gravitino

We would like to promote the non-linear realization of supersymmetry above from a global to a local realization. The graviton degrees of freedom are described by the vierbein fields e^a_{μ} where a is a tangent space index. We define $e \equiv \det(e^a_{\mu})$. The spontaneous F-term supersymmetry breaking is associated with a stress-energy tensor with a vev $T^{\mu\nu}_S = -F^2\eta^{\mu\nu}$, where F is the vev of the auxiliary field as defined above.

Promoting ϵ to a local parameter $\epsilon(x)$, at leading order, the supersymmetry transformation read

$$\begin{aligned} \delta e^a_{\mu} &= -\frac{1}{M_P}\left(i\bar{\epsilon}\bar{\sigma}^a\psi_{\mu} - i\epsilon\sigma^a\bar{\psi}_{\mu}\right), \\ \delta\psi_{\mu} &= -M_p 2\partial_{\mu}\epsilon, \quad \delta G = \sqrt{2}F\epsilon , \\ \delta\bar{\psi}_{\mu} &= -M_p 2\partial_{\mu}\bar{\epsilon}, \quad \delta\bar{G} = \sqrt{2}F\bar{\epsilon} . \end{aligned} \tag{10}$$

and, up to a divergence, the supersymmetric Lagrangian is:

$$\mathcal{L} = -\frac{1}{2M_P^2}eR - \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\bar{\sigma}_{\nu}\partial_{\rho}\psi_{\sigma} - F^2 e - i\bar{G}\bar{\sigma}^{\mu}\partial_{\mu}G - i\frac{F}{\sqrt{2}M_p}\left(\psi_{\mu}\sigma_{\nu}\bar{G} + \bar{\psi}_{\mu}\bar{\sigma}_{\nu}G\right) + \cdots \tag{11}$$

where one sees that the term F^4 represents now a cosmological constant. This is problematic as we wish to work in a flat background. This issue is solved by adding a combination of a canceling contribution to the cosmological constant and a gravitino mass term (where the an anti-symmetric structure $\sigma^{\mu\nu}$ is necessary to avoid the appearance of a pathological fermionic term of the form $(\partial G)^2$) :

$$\Delta\mathcal{L} = F^2 e - m_{\frac{3}{2}}\psi_{\mu}\sigma^{\mu\nu}\psi_{\nu} - m^*_{\frac{3}{2}}\bar{\psi}_{\mu}\sigma^{\mu\nu}\bar{\psi}_{\nu} - m_{\frac{3}{2}}GG - m^*_{\frac{3}{2}}\bar{G}\bar{G} . \tag{12}$$

and the total Lagrangian is invariant under supersymmetry variation

$$\delta e^a_\mu = -\frac{1}{M_P}\left(i\bar{\epsilon}\bar{\sigma}^a\psi_\mu - i\epsilon\sigma^a\bar{\psi}_\mu\right),$$
$$\delta\psi_\mu = -M_p\left(2\partial_\mu\epsilon - im^*_{\frac{3}{2}}\sigma_\mu\bar{\epsilon}\right), \quad \delta G = \sqrt{2}F\epsilon\ ,$$
$$\delta\bar{\psi}_\mu = -M_p\left(2\partial_\mu\bar{\epsilon} + im_{\frac{3}{2}}\bar{\sigma}_\mu\epsilon\right), \quad \delta\bar{G} = \sqrt{2}F\bar{\epsilon}\ . \tag{13}$$

only if:

$$m_{\frac{3}{2}} = \frac{F}{\sqrt{3}M_p}\ . \tag{14}$$

Finally, we can go to the unitary gauge by performing the transformation

$$\psi_{\mu\alpha} \to \psi_{\mu\alpha} + \frac{\sqrt{2}M_P}{F}\partial_\mu G_\alpha + i\frac{1}{\sqrt{6}}\sigma_{\mu\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}}\ . \tag{15}$$

and derive the Rarita-Schwinger Lagrangian for a massive gravitino

$$\mathcal{L}_g = \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma - m_{\frac{3}{2}}\psi_\mu\sigma^{\mu\nu}\psi_\nu - m^*_{\frac{3}{2}}\bar{\psi}_\mu\sigma^{\mu\nu}\bar{\psi}_\nu\ . \tag{16}$$

2.2 Modified Rarita-Schwinger Lagrangian from the super-Higgs mechanism in fluids

In this section we will derive a Generalized Rarita-Schwinger Lagrangian from the study the super-Higgs mechanism in fluids. More precisely, we generalize the previous section as now supersymmetry is broken spontaneously by the vacuum expectation value of the stress-energy tensor $T_{\mu\nu}$ which also, in general, breaks spontaneously the Lorentz symmetry.

2.2.1 The goldstino in supersymmetric fluids: the phonino

Consider for simplicity a supersymmetric field theory in thermal equilibrium described by a background stress-energy tensor taken to be a perfect fluid:

$$T^{\mu\nu} = \mathrm{diag}\,(\varepsilon, p, p, p)\ . \tag{17}$$

where p is the pressure and ε is the energy density. This expectation value of the stress-energy tensor (17) breaks spontaneously supersymmetry and Lorentz symmetry but keeps rotational invariance.

Based on the study of the supersymmetric Ward-Takahashi identity, it was argued that the associated spontaneous breaking of supersymmetry implies a massless fermionic field in the spectrum, the goldstino called here a phonino [11]. In fact, the supersymmetric Ward-Takahashi identity for the supercurrent two-point function:

$$\partial_\mu\langle T\{S^\mu(x)\bar{S}^\nu(y)\}\rangle \sim \delta^{(4)}(x-y)\langle T^{\nu\rho}\rangle\sigma_\rho\ . \tag{18}$$

shows that the correlator has to have a singularity when $k \to 0$ when going to momentum space and assuming a constant energy-momentum tensor the correlator. Note that without Lorentz invariance it is possible to have a singularity without having a massless particle. While this is known to happen for instance in a free theory, in a generic interacting system it is expected that the massless mode is present (see for example [12]). Here, we will consider such a situation.

The field equations of the phonino take the form

$$T^{\mu\nu}\bar{\sigma}_\mu\partial_\nu G = 0, \quad T^{\mu\nu}\sigma_\mu\partial_\nu\bar{G} = 0 \,. \tag{19}$$

which can be obtained from the Lagrangian

$$\mathcal{L}_G = -\frac{i}{\mathcal{T}^4}T^{\mu\nu}\bar{G}\bar{\sigma}_\mu\partial_\nu G \,, \tag{20}$$

where $\mathcal{T} = |\,\mathrm{Tr}\,\langle T^{\mu\nu}\rangle|^{\frac{1}{4}}$ has dimension of mass. Note that for $T^{\mu\nu} = -F^2\eta^{\mu\nu}$ the Lagrangian (20) reduces to the previous section and the propagator of the phonino becomes that of the usual goldstino. Note that the gravitino and the goldstino remain massless in a CFT fluid.

2.2.2 Modified Rarita-Schwinger Lagrangian

In the following we will be working at the quadratic order, dropping in particular the four-fermion interaction in supergravity, and keep the lowest order of an expansion in powers of the dimensionless parameter $\frac{\mathcal{T}}{M_p}$.

The goldstino variation needs to modified :

$$\delta G_\alpha(x) = \sqrt{2}\frac{1}{\mathcal{T}^2}g_{\mu\nu}T^{\mu\nu}\epsilon_\alpha + \cdots \,, \tag{21}$$

as well as the Lagrangian

$$\mathcal{L} = -\mathcal{T}^4 - i\frac{T^{\mu\nu}}{\mathcal{T}^4}\bar{G}\bar{\sigma}_\mu\partial_\nu G + \cdots \,, \tag{22}$$

As in the usual case, promoting the supersymmerty transformation to local ones requires dealing with the contribution of the goldstino energy density to the energy-momentum stress energy tensor. This requires adding a canceling cosmological constant term and a gravitino quadratic "mass" term. However, as the dispersion relation for the phonino is no more Lorentz invariant, we need to allow these quadratic terms to be non-Lorentz invariant. It is straightforward to see that the the Lagrangian:

$$\begin{aligned}
\mathcal{L} \;&=\; \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma + \frac{i}{4}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\gamma}\bar{\psi}_\mu\bar{\sigma}_\rho\sigma^\gamma\bar{\psi}_\nu - \frac{i}{4}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\gamma}\psi_\mu\sigma_\rho\bar{\sigma}^\gamma\psi_\nu \\
&-\; \frac{i}{\sqrt{2}}\frac{\mathcal{T}^2}{M_P}\frac{T^{\mu\nu}}{\mathcal{T}^4}(\bar{\psi}_\mu\bar{\sigma}_\nu G + \psi_\mu\sigma_\nu\bar{G}) \\
&+\; i\frac{T^{\mu\nu}}{\mathcal{T}^4}\bar{G}\bar{\sigma}_\mu\partial_\nu G + \frac{1}{4}\frac{T^{\mu\nu}n_{\mu\nu}}{\mathcal{T}^4}GG + \frac{1}{4}\frac{T^{\mu\nu}n_{\mu\nu}}{\mathcal{T}^4}\bar{G}\bar{G}\,.
\end{aligned}$$

is invariant under the modified supersymmetry transformations with Lorentz violating coefficients:

$$\begin{aligned}
\delta G^\alpha &= \sqrt{2}\mathcal{T}^2\varepsilon^\alpha \,, \\
\delta\psi_{\mu\alpha} &= -M_P(2\partial_\mu\varepsilon_\alpha + in_{\mu\nu}\sigma^\nu_{\alpha\dot{\alpha}}\bar{\varepsilon}^{\dot{\alpha}}) \,, \\
\delta\bar{\psi}_{\mu\dot{\alpha}} &= -M_P(2\partial_\mu\bar{\varepsilon}_{\dot{\alpha}} - in^*_{\mu\nu}\varepsilon^\alpha\sigma^\nu_{\alpha\dot{\alpha}}) \,.
\end{aligned} \tag{23}$$

and leads to the equation of motion for the goldstino. For $n_{\mu\nu}$ real, the requirement of supersymmetry in flat space implies that it should satisfy

$$-\frac{1}{2}\epsilon^{\mu\nu\sigma\rho}\epsilon_\rho{}^{\lambda\gamma\kappa}n_{\nu\lambda}n_{\sigma\gamma} = \frac{T^{\mu\kappa}}{M_P^2} \,. \tag{24}$$

The unitary gauge is obtained by making a supersymmetry transformation to set $G = 0$:

$$\psi_{\mu\alpha} \to \psi_{\mu\alpha} + \frac{\sqrt{2}M_P}{\mathcal{T}^2}\partial_\mu G_\alpha + i\frac{M_P}{\sqrt{2}\mathcal{T}^2} n_{\mu\nu}\sigma^\nu_{\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}} \,. \tag{25}$$

As a result we obtain the Generalized Rarita-Schwinger Lagrangian

$$\mathcal{L} = \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma - \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_\sigma^{\ \gamma}\bar{\psi}_\mu\bar{\sigma}_{\rho\gamma}\bar{\psi}_\nu + \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_\sigma^{\ \gamma}\psi_\mu\sigma_{\rho\gamma}\psi_\nu \,. \tag{26}$$

and the corresponding equation of motion is

$$\epsilon^{\mu\nu\rho\sigma}\bar{\sigma}_\nu\partial_\rho\psi_\sigma - \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\gamma}\bar{\sigma}_\rho\sigma^\gamma\bar{\psi}_\nu = 0 \,. \tag{27}$$

As for the usual Rarita-Schwinger case, two constraints are necessary in order to reduce the number of degrees of freedom of ψ_μ to the four that describe a massive gravitino. The first is obtained by acting on the equation of motion by $n_{\mu\lambda}\sigma^\lambda$ which gives

$$-\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_{\mu\lambda}n_{\sigma\gamma}\sigma^\lambda\bar{\sigma}_\rho\sigma^\gamma\bar{\psi}_\nu = 0 \,. \tag{28}$$

Using the symmetry of $n_{\mu\lambda}$, this can be put in the form:

$$T^{\mu\nu}\sigma_\mu\bar{\psi}_\nu = 0 \,, \tag{29}$$

This replaces the standard F-term breaking constraint $\bar{\sigma}^\mu\psi^\mu = 0$ of the gravitino. A procedure similar to the case of curved space-time [13], allows to get a second constraint by taking the component $\mu = 0$ of (27).

We shall illustrate how to apply these constraint in the next subsection.

2.2.3 Explicit formulae for a perfect fluid

Here will show how the gravitino mass can be expressed as a function of the fluid variables. We will consider relativistic ideal fluids with stress-energy tensor

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p\eta^{\mu\nu} \,, \tag{30}$$

where u^μ is the fluid four-velocity $u^\mu u_\mu = -1$. In order to solve (24) we parametrize the solution $n_{\mu\nu}$ as

$$n_{\mu\nu} = (n_T - n_L)u_\mu u_\nu + n_T\eta_{\mu\nu} \,. \tag{31}$$

Plugging $n_{\mu\nu}$ and $T_{\mu\nu}$ and solving for n_T and n_L we get

$$n_T^2 = \frac{\varepsilon}{3M_P^2} \,, \qquad -n_T(n_T + 2n_L) = \frac{p}{M_P^2} \,, \tag{32}$$

hence

$$n_L = -n_T\left(\frac{\varepsilon + 3p}{2\varepsilon}\right) \,. \tag{33}$$

Note that for F-term breaking, $\varepsilon = -p, n_L = n_T$.

For constant energy density and pressure, in the fluid rest frame $n_\mu^\nu = diag(n_L, n_T, n_T, n_T)$. We introduce the notation

$$\not{D} = \sigma^\mu \partial_\mu, \quad \not{\partial} = \sigma^i \partial_i \,, \tag{34}$$

and

$$\Psi = \bar{\sigma}^\mu \psi_\mu, \quad \bar{\psi}_{\frac{1}{2}} = \bar{\sigma}^i \psi_i, \quad \psi_{\frac{1}{2}} = \sigma^i \bar{\psi}_i \,. \tag{35}$$

The constraint (29) can be used to solve for one of the components

$$\psi_0 \quad = \quad -v\,\sigma_0 \psi_{\frac{1}{2}} \,, \tag{36}$$

where $v = \left|\frac{p}{\epsilon}\right|$ is the phonino velocity. The component $\mu = 0$ of equation of motion gives the constraint

$$\not{\partial}\bar{\psi}_{\frac{1}{2}} - i n_T \psi_{\frac{1}{2}} + \partial \cdot \psi \quad = \quad 0 \,. \tag{37}$$

Putting all the constraints together leads to

$$(\bar{\sigma}^0 \partial_0 + v\bar{\not{\partial}})\psi_{\frac{1}{2}} - i\hat{m}\bar{\psi}_{\frac{1}{2}} \quad = \quad 0 \,. \tag{38}$$

This is the Dirac equation satisfied by the longitudinal spin-1/2 mode with mass

$$\hat{m} = \frac{n_L + n_T}{2} = \frac{n_T}{4}\,|(1-3v)| = \frac{\sqrt{3}}{4M_P}\left|\frac{p - \frac{\varepsilon}{3}}{\sqrt{\varepsilon}}\right| . \tag{39}$$

where as the eqs. (32) determine n_L, n_T only up to a sign, we have used this freedom in the last equation to have a positive mass $\hat{m}$.

The projector on the transverse part of the spinor is

$$\psi_j \quad = \quad \psi_j^T - \left(\frac{1}{2}\sigma_j - \frac{k_j \not{k}}{2k^2}\right)\psi_{\frac{1}{2}} + \left(\frac{3k_j}{2k^2} + \frac{1}{2}\frac{\sigma_j \bar{\not{k}}}{k^2}\right) k \cdot \psi \,. \tag{40}$$

and the transverse part satisfies then the decoupled equation

$$(\bar{\sigma}^0 \partial_0 + \bar{\not{\partial}})\psi_j^T + i\hat{m}\,\bar{\psi}_j^T = 0 \,. \tag{41}$$

In the fluid, the gravitino has two distinct propagating modes, the longitudinal and the transverse, with the same mass but different dispersion relations.

3 Pseudo-goldstinos and Dirac gravitinos from extra dimensions

In the previous section, we have discussed the presence of a goldstino associated breaking supersymmetry in the global limit. We then proceeded to coupling it to gravity in order to obtain the massive gravitino Lagrangian. We shall now discuss a different case where the supersymmetry breaking is intimately related with the gravity sector: the goldstino is given by a gravitino component along an internal dimension. Only in the limit of comparable sizes of supersymmetry breaking and compactification scales are the extra dimensions relevant and we shall therefore focus on such case. For simplicity, we will illustrate these in the very simple case of one single extra-dimension. We will also mention the possibility to tune the parameters to give Dirac type mass for the gravitino.

3.1 Minimal supergravity in five dimension

For the sake of establishing our notations and to illustrate the main ideas, we shall start the discussion with the simplest case of a five-dimensional space parametrized by coordinates (x^μ, x^5) with $\mu = 0, \cdots, 3$ and $x^5 \equiv y$ parametrizing the interval $S^1/\mathbb{Z}_2$. The latter can be constructed as an orbifold from the circle of length $2\pi R$ ($y \sim y + 2\pi R$) through the identification $y \sim -y$. We take the theory in the bulk to be the five-dimensional supergravity with the minimal on-shell content: the fünfbein e_M^A, the gravitino Ψ_{MI} and the graviphoton B_M. We focus on the Lagrangian part involving the gravitino and drop the terms involving B_M as well as the spin connection as we will consider for simplicity only the case of a flat extra-dimension.

The on-shell Lagrangian is given by[2] [14]:

$$\mathcal{L}_{SUGRA} = e_5 \Big\{ -\frac{1}{2} R(\omega) + \frac{i}{2} \check{\Psi}^I_M \Gamma^{MNP} \partial_N \Psi_{PI} + \cdots \Big\} \tag{42}$$

and the on-shell supersymmetry transformations are :

$$\begin{aligned} \delta e_M^A &= i \check{\Xi}^I \Gamma^A \Psi_{MI} \\ \delta \Psi_{MI} &= 2 \partial_M \Xi_I + \cdots \end{aligned} \tag{43}$$

where Ξ is the supersymmetry transformation parameter. The five-dimensional spinors Ψ_{MI} and Ξ_I are symplectic Majorana spinors. The five-dimensional gravitino Ψ_{MI} will be written using the two-component Weyl spinors ψ_{MI} as:

$$\Psi_{M1} = \begin{pmatrix} \psi_{M1} \\ \overline{\psi}_{M2} \end{pmatrix}, \qquad \Psi_{M2} = \begin{pmatrix} -\psi_{M2} \\ \overline{\psi}_{M1} \end{pmatrix} \tag{44}$$

and the supersymmetry transformation parameter as:

$$\begin{aligned} \Xi_1 = -\Xi^2 &= \begin{pmatrix} \epsilon_1 \\ \overline{\epsilon}_2 \end{pmatrix}, \qquad & \Xi_2 = \Xi^1 &= \begin{pmatrix} -\epsilon_2 \\ \overline{\epsilon}_1 \end{pmatrix} \\ \check{\Xi}_1 = -\check{\Xi}^2 &= \begin{pmatrix} -\epsilon_1, & \overline{\epsilon}_2 \end{pmatrix}, \qquad & \check{\Xi}_2 = \check{\Xi}^1 &= \begin{pmatrix} \epsilon_2, & \overline{\epsilon}_1 \end{pmatrix}. \end{aligned} \tag{45}$$

The on-shell supersymmetry transformations in two-component spinor notation are given by:

$$\begin{aligned} \delta e_M^a &= i \left(\epsilon_1 \sigma^a \overline{\psi}_{M1} + \epsilon_2 \sigma^a \overline{\psi}_{M2} \right) + h.c. \\ \delta e_M^{\hat{5}} &= \epsilon_2 \psi_{M1} - \epsilon_1 \psi_{M2} + h.c. \\ \delta \psi_{1M} &= 2 \partial_M \epsilon_1 + \cdots \\ \delta \psi_{2M} &= 2 \partial_M \epsilon_2 + \cdots \end{aligned} \tag{46}$$

The fermionic part of the bulk Lagrangian expressed in two-component spinor notation reads now:

$$\begin{aligned} \mathcal{L}_{Fermi} = e_5 \Big\{ & \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} \left(\overline{\psi}_{1\mu} \overline{\sigma}_\nu \partial_\rho \psi_{1\lambda} + \overline{\psi}_{2\mu} \overline{\sigma}_\nu \partial_\rho \psi_{2\lambda} \right) + e_{\hat{5}}^5 \left(\psi_{1\mu} \sigma^{\mu\nu} \partial_5 \psi_{2\nu} - \psi_{\mu 2} \sigma^{\mu\nu} \partial_5 \psi_{1\nu} \right) \\ & - e_{\hat{5}}^5 \left(\psi_{15} \sigma^{\mu\nu} \partial_\mu \psi_{\nu 2} - \psi_{25} \sigma^{\mu\nu} \partial_\mu \psi_{1\nu} + \psi_{1\mu} \sigma^{\mu\nu} \partial_\nu \psi_{25} - \psi_{2\mu} \sigma^{\mu\nu} \partial_\nu \psi_{15} \right) + h.c. + \cdots \Big\} \end{aligned} \tag{47}$$

where the five-dimensional covariant derivatives expressed have been replaced by partial derivatives as we work in a flat metric.

[2] We recall that we use the approximation of dropping in the Lagrangian the four-fermions terms and in the supersymmetry transformations the three and four-fermions terms.

3.2 SUSY breaking through twisted boundary conditions

We will perform our study in the simplest case with no branes in the bulk other than the boundary ones at $y = 0$ and $y = \pi R$, as it contains all the qualitative features.

3.2.1 The twisted boundary conditions fields basis

Every generic field φ has a well defined $\mathbb{Z}_2$ transformation:

$$\mathbb{Z}_2 : \quad \varphi(y) \to \mathcal{P}_0 \varphi(-y) \tag{48}$$

that allows us to define the orbifold $S^1/\mathbb{Z}_2$ from the original five-dimensional compactification on S^1. Here $\mathcal{P}_0$ is the parity of the field φ which obeys $\mathcal{P}_0^2 = 1$.

The Lagrangian (42) and supersymmetry transformations (43) must be invariant under the action of the mapping (48). A possible choice of parity assignments is

$$\psi_{1\mu}(-y) = +\psi_{1\mu}(y). \tag{49}$$

At the point $y = 0$, the other fields parity transformations are determined from invariance of supersymmetry transformations under the mapping (48). We must assign a parity $\mathcal{P}_\pi$ for each generic field φ at the point $y = \pi R$ which keeps the Lagrangian and the supersymmetry transformations invariant

$$\varphi(\pi R + y) = \mathcal{P}_\pi \varphi(\pi R - y). \tag{50}$$

We also need to impose periodicity condition, we choose to be:

$$\begin{pmatrix} \psi_{M1}(y + 2\pi R) \\ \psi_{M2}(y + 2\pi R) \end{pmatrix} = \begin{pmatrix} \cos(2\pi\omega) & \sin(2\pi\omega) \\ -\sin(2\pi\omega) & \cos(2\pi\omega) \end{pmatrix} \begin{pmatrix} \psi_{M1}(y) \\ \psi_{M2}(y) \end{pmatrix} \tag{51}$$

which correspond for $\omega \neq 0$ to implement a Scherk-Schwarz supersymmetry breaking in the bulk [15]. Then, invariance of the supersymmetry transformations under the $\mathbb{Z}_2$ mapping (50) determines the parities of all fields. The result is given in table 1,

Table 1. Parity assignments for bulk fields at $y = 0$ and $y = \pi R$.

$\mathcal{P}_0 = +1$	e_μ^a	$e_5^{\hat{5}}$	$\psi_{1\mu}$	ψ_{25}	ϵ_1
$\mathcal{P}_0 = -1$	e_5^a	$e_\mu^{\hat{5}}$	$\psi_{2\mu}$	ψ_{15}	ϵ_2
$\mathcal{P}_\pi = +1$	e_μ^a	$e_5^{\hat{5}}$	$\psi_{\mu+}$	ψ_{5+}	ϵ_+
$\mathcal{P}_\pi = -1$	e_5^a	$e_\mu^{\hat{5}}$	$\psi_{\mu-}$	ψ_{5-}	ϵ_-

where the following definitions have been introduced:

$$\begin{aligned}
\psi_{\mu+} &= \cos(\pi\omega)\psi_{\mu 1} - \sin(\pi\omega)\psi_{\mu 2} \\
\psi_{\mu-} &= \sin(\pi\omega)\psi_{\mu 1} + \cos(\pi\omega)\psi_{\mu 2} \\
\psi_{5+} &= \sin(\pi\omega)\psi_{51} + \cos(\pi\omega)\psi_{52} \\
\psi_{5-} &= \cos(\pi\omega)\psi_{51} - \sin(\pi\omega)\psi_{52} \\
\epsilon_+ &= \cos(\pi\omega)\epsilon_1 - \sin(\pi\omega)\epsilon_2 \\
\epsilon_- &= \sin(\pi\omega)\epsilon_1 + \cos(\pi\omega)\epsilon_2
\end{aligned} \tag{52}$$

3.2.2 The Periodic fields basis

It is often useful to work in a basis of periodic fields $\tilde{\psi}_{MI}$ (ie. $\tilde{\psi}_{MI}(x, y+2\pi R) = \tilde{\psi}_{MI}(x,y)$) in contrast to the multi-valued ψ_{MI} used up to now. These are related by the rotation:

$$\begin{pmatrix}\psi_{M1}\\ \psi_{M2}\end{pmatrix} = \begin{pmatrix}\cos[f(y)] & \sin[f(y)]\\ -\sin[f(y)] & \cos[f(y)]\end{pmatrix}\begin{pmatrix}\tilde{\psi}_{M1}\\ \tilde{\psi}_{M2}\end{pmatrix}, \qquad f(y) = \frac{\omega}{R}y \tag{53}$$

The supersymmetry breaking mass terms for the gravitinos is then manifest as we perform this fields transformation in the kinetic terms of the Lagrangian to give the action:

$$\begin{aligned} S_{Kinetic} &= \int_0^{2\pi R} dy \int d^4x \Big\{ \frac{1}{2} e_5 \Big[\frac{1}{2} \epsilon^{\mu\nu\rho\lambda} \left(\bar{\tilde{\psi}}_{\mu 1}\overline{\sigma}_\nu \partial_\rho \tilde{\psi}_{\lambda 1} + \bar{\tilde{\psi}}_{\mu 2}\overline{\sigma}_\nu \partial_\rho \tilde{\psi}_{\lambda 2}\right) \\ &\quad + e^5_{\hat{5}} \left(\tilde{\psi}_{\mu 1}\sigma^{\mu\nu}\partial_5 \tilde{\psi}_{\nu 2} - \tilde{\psi}_{\mu 2}\sigma^{\mu\nu}\partial_5\tilde{\psi}_{\nu 1}\right) - 2e^5_{\hat{5}}\left(\tilde{\psi}_{51}\sigma^{\mu\nu}\partial_\mu \tilde{\psi}_{\nu 2} - \tilde{\psi}_{52}\sigma^{\mu\nu}\partial_\mu\tilde{\psi}_{\nu 1}\right) \\ &\quad - \left(\frac{\omega}{R}\right) e^5_{\hat{5}} \left(\tilde{\psi}_{\mu 1}\sigma^{\mu\nu}\tilde{\psi}_{\nu 1} + \tilde{\psi}_{\mu 2}\sigma^{\mu\nu}\tilde{\psi}_{\nu 2}\right)\Big] + h.c.\Big\}. \end{aligned} \tag{54}$$

with the fields now being periodic.

Going to the new basis requires then the following redefinition for the supersymmetry transformation parameters,

$$\begin{pmatrix}\epsilon_1\\ \epsilon_2\end{pmatrix} = \begin{pmatrix}\cos[f(y)] & \sin[f(y)]\\ -\sin[f(y)] & \cos[f(y)]\end{pmatrix}\begin{pmatrix}\tilde{\epsilon}_1\\ \tilde{\epsilon}_2\end{pmatrix}, \tag{55}$$

and the supersymmetry transformations take now the form:

$$\begin{aligned} \delta\tilde{\psi}_{\mu 1} &= 2\partial_\mu \tilde{\epsilon}_1 + \cdots \\ \delta\tilde{\psi}_{\mu 2} &= 2\partial_\mu \tilde{\epsilon}_2 + + \cdots \\ \delta\tilde{\psi}_{51} &= 2\partial_5 \tilde{\epsilon}_1 + 2\frac{df}{dy}\tilde{\epsilon}_2 + \cdots \\ \delta\tilde{\psi}_{52} &= 2\partial_5 \tilde{\epsilon}_2 - 2\frac{df}{dy}\tilde{\epsilon}_1 + \cdots \end{aligned} \tag{56}$$

where $\cdots$ stand for higher order terms and terms proportional to F^{MN}.

It is important to note that the fields $\tilde{\psi}_{51}$ and $\tilde{\psi}_{52}$ transforms non linearly under supersymmetry transformations: *they are the Goldstino fields associated with the supersymmetry breaking in the bulk* and the supersymmetry breaking is measured by the $\frac{df}{dy}$ therefore by the transformation between the two basis or stated differently, by the change of the gravitino component preserved at each point of the extra dimension.

3.2.3 The super-Higgs mechanism

From now on we drop the ˜ from the fermion symbols and use the periodic basis unless stated differently.

Equations (56) show that four fields ψ_{15} and ψ_{25} transform non linearly under supersymmetry transformations. These are the goldstinos associated with breaking of supersymmetry in the bulk that shall be a“absorbed” by the two gravitinos for the super-Higgs mechanism.

In order to study further this effect we will concentrate on the bi-linear terms of the fermionic fields: $\psi_{1\mu}$, $\psi_{2\mu}$, ψ_{15}, ψ_{25}. Some field redefinition are necessary to obtain standard kinetic terms for the fields ψ_{5I}:

$$\begin{aligned}
\psi_{1\mu} &\rightarrow \psi_{1\mu} + \frac{i}{\sqrt{6}}\sigma_\mu \overline{\psi}_{25} \\
\psi_{2\mu} &\rightarrow \psi_{2\mu} - \frac{i}{\sqrt{6}}\sigma_\mu \overline{\psi}_{15} \\
\psi_{15} &\rightarrow \frac{2}{\sqrt{6}}\psi_{15} \\
\psi_{25} &\rightarrow \frac{2}{\sqrt{6}}\psi_{25}.
\end{aligned} \tag{57}$$

This leads to the Lagrangian density:

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2}\Big\{\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\left(\overline{\psi}_{1\mu}\overline{\sigma}_\nu\partial_\rho\psi_{1\lambda} + \overline{\psi}_{2\mu}\overline{\sigma}_\nu\partial_\rho\psi_{2\lambda}\right) + \psi_{1\mu}\sigma^{\mu\nu}\partial_5\psi_{2\nu} - \psi_{\mu 2}\sigma^{\mu\nu}\partial_5\psi_{1\nu} \\
& -\frac{i}{2}\left(\overline{\psi}_{15}\overline{\sigma}^\mu\partial_\mu\psi_{15} + \overline{\psi}_{25}\overline{\sigma}^\mu\partial_\mu\psi_{25}\right) + \psi_{15}\partial_5\psi_{25} - \psi_{25}\partial_5\psi_{15} \\
& -\frac{\omega}{R}\left(\psi_{1\mu}\sigma^{\mu\nu}\psi_{1\nu} + \psi_{2\mu}\sigma^{\mu\nu}\psi_{2\nu} + \psi_{15}\psi_{15} + \psi_{25}\psi_{25}\right) \\
& -i\frac{\sqrt{6}}{2}\left[\partial_5\overline{\psi}_{15}\overline{\sigma}^\mu\psi_{1\mu} + \partial_5\overline{\psi}_{25}\overline{\sigma}^\mu\psi_{2\mu} + \frac{\omega}{R}\left(\overline{\psi}_{25}\overline{\sigma}^\mu\psi_{1\mu} - \overline{\psi}_{15}\overline{\sigma}^\mu\psi_{2\mu}\right)\right]\Big\} + h.c.
\end{aligned} \tag{58}$$

The first line represents the five-dimensional kinetic term for the four-dimensional gravitinos, the second line corresponds to mass terms coming from the propagation in the fifth dimension, the third line

$$\begin{aligned}
\psi_{1\mu\alpha} &\rightarrow \psi_{1\mu\alpha} + \sqrt{\frac{2}{3}}\frac{R}{\omega}\partial_\mu(\psi_{25\alpha} + \frac{R}{\omega}\partial_5\psi_{15\alpha}) + i\frac{1}{\sqrt{6}}\sigma_{\mu\alpha\dot{\alpha}}(\bar{\psi}^{\dot{\alpha}}_{25} + \frac{R}{\omega}\partial_5\bar{\psi}^{\dot{\alpha}}_{15})\,. \\
\psi_{2\mu\alpha} &\rightarrow \psi_{2\mu\alpha} + \sqrt{\frac{2}{3}}\frac{R}{\omega}\partial_\mu(\psi_{15\alpha} - \frac{R}{\omega}\partial_5\psi_{25\alpha}) + i\frac{1}{\sqrt{6}}\sigma_{\mu\alpha\dot{\alpha}}(\bar{\psi}^{\dot{\alpha}}_{15} - \frac{R}{\omega}\partial_5\bar{\psi}^{\dot{\alpha}}_{25})\,.
\end{aligned} \tag{59}$$

It is straightforward to check that this gauge fixing term provides the cancellation of mixing terms between gravitino and goldstino fields, which is the aim of our gauge choice :

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2}\Big\{\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\left(\overline{\psi}_{1\mu}\overline{\sigma}_\nu\partial_\rho\psi_{1\lambda} + \overline{\psi}_{2\mu}\overline{\sigma}_\nu\partial_\rho\psi_{2\lambda}\right) + \psi_{1\mu}\sigma^{\mu\nu}\partial_5\psi_{2\nu} - \psi_{\mu 2}\sigma^{\mu\nu}\partial_5\psi_{1\nu} \\
& -\frac{\omega}{R}\left(\psi_{1\mu}\sigma^{\mu\nu}\psi_{1\nu} + \psi_{2\mu}\sigma^{\mu\nu}\psi_{2\nu}\right)\Big\} + h.c.
\end{aligned} \tag{60}$$

In this gauge, the gravitino propagators have poles at their physical mass and the degrees of freedom of would-be goldstinos are eliminated, traded for the longitudinal components for the gravitinos, through the super-Higgs mechanism.

The equations of motion for the gravitinos $\psi_{\mu I}(y)$ in the unitary gauge can be extracted from the Lagrangian (60):

$$\begin{aligned}
-\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\sigma_\nu\partial_\rho\overline{\psi}_{1\lambda} + \sigma^{\mu\nu}\partial_5\psi_{2\nu} - \frac{\omega}{R}\sigma^{\mu\nu}\psi_{1\nu} &= 0 \\
-\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\sigma_\nu\partial_\rho\overline{\psi}_{2\lambda} - \sigma^{\mu\nu}\partial_5\psi_{1\nu} - \frac{\omega}{R}\sigma^{\mu\nu}\psi_{2\nu} &= 0
\end{aligned} \tag{61}$$

Assuming the gravitinos have a *four-dimensional mass* $m_{3/2}$:

$$\epsilon^{\mu\nu\rho\lambda}\sigma_\nu\partial_\rho\overline{\psi}_{\lambda I} = -2m_{3/2}\sigma^{\mu\nu}\psi_{\nu I} \tag{62}$$

their equations of motion can take the form:

$$\begin{aligned} \partial_5\psi_{2\mu} + \left(m_{3/2} - \frac{\omega}{R}\right)\psi_{1\mu} &= 0 \\ \partial_5\psi_{1\mu} - \left(m_{3/2} - \frac{\omega}{R}\right)\psi_{2\mu} &= 0 \end{aligned} \tag{63}$$

A solution for the equations (63) in the interval $0 < y < \pi R$ satisfying the first condition in (71):

$$\begin{aligned} \psi_{1\mu}(y) &= \cos\left[\left(m_{3/2} - \frac{\omega}{R}\right)y\right]\psi_{1\mu}(0) \\ \psi_{2\mu}(y) &= -\sin\left[\left(m_{3/2} - \frac{\omega}{R}\right)y\right]\psi_{1\mu}(0). \end{aligned} \tag{64}$$

The second condition in (71) is then used to determine the gravitino mass:

$$m_{3/2} = \frac{\omega}{R} + \frac{n}{R}, \quad n \in \mathbb{Z} \tag{65}$$

3.3 Pseudo-goldstinos and brane localized gravitino mass terms

3.3.1 Gravitino mass

Matter fields live on branes localized for instance at particular points $y = y_n$. Here, we will consider the simplest case where the branes are localized on the boundaries $y_b = 0, \pi R$, as it contains all the qualitative features. The generalization can be found in . On each of these branes supersymmetry can be locally broken by the matter scalar potential and a “would-be-goldstino“ appears localized. The corresponding action can be written as:

$$S = \int_0^{2\pi R} dy \int d^4x \left\{\frac{1}{2}\mathcal{L}_{BULK} + \mathcal{L}_0\delta(y) + \mathcal{L}_\pi\delta(y-\pi R)\right\}. \tag{66}$$

There are four fields $\psi_{15}, \psi_{25}, \chi_0$ and χ_π transform non linearly under supersymmetry transformations. These are the *“local would be goldstinos”* associated with breaking of supersymmetry in the bulk and in the two branes respectively. As we have two gravitinos then two *local would be goldstinos* will be absorbed in the super-Higgs effect to give mass to the gravitino fields $\psi_{1\mu}$ and $\psi_{2\mu}$, while two linear combination of the fields $\psi_{15}, \psi_{25}, \chi_0$ and χ_π remain as *pseudo-goldstinos*.

The additional bi-linear terms of the fermionic fields: $\psi_{\mu 1}, \psi_{2\mu}, \psi_{15}, \psi_{25}, \chi_0$ and χ_π take the form:

$$\begin{aligned} \Delta\mathcal{L} = \;& \delta(y)\left\{-\frac{i}{2}\overline{\chi}_0\overline{\sigma}^\mu\partial_\mu\chi_0 - M_0\left[\psi_{1\mu}\sigma^{\mu\nu}\psi_{1\nu} + i\frac{\sqrt{6}}{2}\left(\overline{\chi}_0 + \overline{\psi}_{25}\right)\overline{\sigma}^\mu\psi_{1\mu}\right.\right. \\ & \left.\left. + (\chi_0 + \psi_{25})(\chi_0 + \psi_{25})\right]\right\} + \delta(y-\pi R)\left\{-\frac{i}{2}\overline{\chi}_\pi\overline{\sigma}^\mu\partial_\mu\chi_\pi\right. \\ & \left. -M_\pi\left[\psi_{1\mu}\sigma^{\mu\nu}\psi_{1\nu} + i\frac{\sqrt{6}}{2}\left(\overline{\chi}_\pi + \overline{\psi}_{25}\right)\overline{\sigma}^\mu\psi_{1\mu} + (\chi_\pi + \psi_{25})(\chi_\pi + \psi_{25})\right]\right\} + h.c. \end{aligned} \tag{67}$$

The modification necessary to fix the unitary gauge is straightforward and two would-be goldstinos are eliminated, absorbed to provide the longitudinal components for the gravitinos, through the super-Higgs mechanism while two remain in the spectrum with masses and fields content explicitly given in . The gravitino equations of motion in the bulk-branes system. The equations of motion for the gravitinos $\psi_{\mu I}(y)$ are then given by:

$$\begin{aligned} -\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\sigma_\nu\partial_\rho\overline{\psi}_{1\lambda} + \sigma^{\mu\nu}\partial_5\psi_{2\nu} - \frac{\omega}{R}\sigma^{\mu\nu}\psi_{1\nu} &= 2M_0\sigma^{\mu\nu}\psi_{1\nu}\delta(y) + 2M_\pi\sigma^{\mu\nu}\psi_{1\nu}\delta(y-\pi R) \\ -\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\sigma_\nu\partial_\rho\overline{\psi}_{2\lambda} - \sigma^{\mu\nu}\partial_5\psi_{1\nu} - \frac{\omega}{R}\sigma^{\mu\nu}\psi_{2\nu} &= 0 \end{aligned} \tag{68}$$

Again, assuming the gravitinos have a *four-dimensional mass* $m_{3/2}$:

$$\epsilon^{\mu\nu\rho\lambda}\sigma_\nu\partial_\rho\overline{\psi}_{\lambda I} = -2m_{3/2}\sigma^{\mu\nu}\psi_{\nu I} \tag{69}$$

the equations of motion become:

$$\begin{aligned} \partial_5\psi_{2\mu} + \left(m_{3/2} - \frac{\omega}{R}\right)\psi_{1\mu} &= 2M_0\psi_{1\mu}\delta(y) + 2M_\pi\psi_{1\mu}\delta(y-\pi R) \\ \partial_5\psi_{1\mu} - \left(m_{3/2} - \frac{\omega}{R}\right)\psi_{2\mu} &= 0 \end{aligned} \tag{70}$$

Integration of the equations (70) near the points $y = 0$ and $y = \pi R$, taking into account the parity assumptions, leads to the following expressions for the discontinuities of the odd gravitino fields:

$$\begin{aligned} \psi_{2\mu}(0^+) &= M_0\ \psi_{1\mu}(0) = -\psi_{2\mu}(0^-) \\ \psi_{2\mu}(\pi R^-) &= -M_\pi\ \psi_{1\mu}(\pi R) = -\psi_{\mu 2}(\pi R^+). \end{aligned} \tag{71}$$

A solution for the equations (70) in the interval $0 < y < \pi R$ satisfying the first condition in (71):

$$\begin{aligned} \psi_{1\mu}(y) &= \left\{\cos\left[\left(m_{3/2} - \frac{\omega}{R}\right)y\right] + M_0\sin\left[\left(m_{3/2} - \frac{\omega}{R}\right)y\right]\right\}\psi_{1\mu}(0) \\ \psi_{2\mu}(y) &= \left\{M_0\cos\left[\left(m_{3/2} - \frac{\omega}{R}\right)y\right] - \sin\left[\left(m_{3/2} - \frac{\omega}{R}\right)y\right]\right\}\psi_{1\mu}(0). \end{aligned} \tag{72}$$

and the second condition in (71) is then used to determine the gravitino mass:

$$m_{3/2} = \frac{\omega}{R} + \frac{1}{\pi R}\left[\arctan(M_0) + \arctan(M_\pi)\right] + \frac{n}{R}, \quad n \in \mathbb{Z} \tag{73}$$

3.3.2 Pseudo-goldstinos

We will concentrate now on the would-be goldstino fields $\psi_{15}(y)$, $\psi_{25}(y)$, χ_0 and χ_π. In the unitary gauge, a stationary action (in order to derive of the equations of motion) is possible if:

$$\begin{aligned} \partial_5\psi_{15} + \frac{\omega}{R}\psi_{25} &= -2\delta(y)M_0(\chi_0 + \psi_{25}) - 2\delta(y-\pi R)M_\pi(\chi_\pi + \psi_{25}) \\ \partial_5\psi_{25} - \frac{\omega}{R}\psi_{15} &= 0. \end{aligned} \tag{74}$$

which imply that the fields $\psi_{5I}(y)$, in the interval $0 < y < \pi R$ can be written as:

$$\begin{aligned} \psi_{15}(y) &= \frac{1}{\sqrt{\pi R}}\left[\cos\left(\frac{\omega}{R}y + \theta\right)\chi_1 + \sin\left(\frac{\omega}{R}y + \theta\right)\chi_2\right] \\ \psi_{25}(y) &= \frac{1}{\sqrt{\pi R}}\left[\sin\left(\frac{\omega}{R}y + \theta\right)\chi_1 - \cos\left(\frac{\omega}{R}y + \theta\right)\chi_2\right] \end{aligned} \tag{75}$$

where χ_1 and χ_2 are y independent 4d spinors and θ is a constant which corresponds to a choice of basis for χ_1 and χ_2.

Integrating the equations (74) near $y = 0$ and $y = \pi$ we find:

$$\begin{aligned} \psi_{15}(0^+) + M_0\left[\chi_0 + \psi_{25}(0)\right] &= 0 \\ \psi_{15}(\pi R^-) - M_\pi\left[\chi_\pi + \psi_{25}(\pi R)\right] &= 0 \end{aligned} \tag{76}$$

which implies (for $M_\pi \neq 0$ and $M_0 \neq 0$):

$$\begin{aligned} \chi_\pi &= \frac{1}{\sqrt{\pi R}}\left[-\sin(\omega\pi + \theta) + \frac{1}{M_\pi}\cos(\omega\pi + \theta)\right]\chi_1 + \frac{1}{\sqrt{\pi R}}\left[\cos(\omega\pi + \theta) + \frac{1}{M_\pi}\sin(\omega\pi + \theta)\right]\chi_2 \\ \chi_0 &= -\frac{1}{\sqrt{\pi R}}\left[\sin(\theta) + \frac{1}{M_0}\cos(\theta)\right]\chi_1 + \frac{1}{\sqrt{\pi R}}\left[\cos(\theta) - \frac{1}{M_0}\sin(\theta)\right]\chi_2. \end{aligned} \tag{77}$$

Here we see that there are two equations for four fermion fields, and we see how the super Higgs mechanism operates: from the original two 5d and two 4d degrees of freedom ($\psi_{15}(y)$, $\psi_{25}(y)$, χ_0 and χ_π), an infinity of Kaluza-Klein modes is absorbed to give mass to the fields $\psi_{1\mu}(y)$ and $\psi_{2\mu}(y)$ and only two degrees of freedom remain in the unitary gauge: the pseudo-goldstinos χ_1 and χ_2.

3.4 Dirac gravitino and R-symmetry

We will restore the explicit dependence on the (reduced) five-dimensional Planck mass $M_5 = \kappa^{-1}$. It is related to the four-dimensional Planck mass M_P by

$$\pi R M_5^3 = M_P^2. \tag{78}$$

The four-dimensional gravitino mass can be read from (73):

$$m_{3/2} = \frac{\omega}{R} + \frac{1}{\pi R}\left[\arctan(\kappa M_0) + \arctan(\kappa M_\pi)\right] + \frac{n}{R}, \tag{79}$$

First consider the case with $M_0 = M_\pi = 0$. It is well known [15, 16, 20] that the Scherk-Schwarz m mechanism described above, leads for $\omega = 1/2$ to a tower of Dirac-type Kaluza-Klein excitation of fermions the bulk fermions. The two modes with $n = 0$ and $n = -1$ lead to two degenerate Majorana fermions with mass $1/2R$. The two states correspond to the two orthogonal supersymmetry charges of $N = 2$ supergravity. One couples to the boundary at $y = 0$ and the other one couples to the one at $y = \pi R$.

The original $N = 2$ Lagrangian is invariant under an $SU(2)_\mathcal{R}$ R-symmetry, under which the gravitinos ψ_{M1} and ψ_{M2} transform in the representation **2** of $SU(2)_\mathcal{R}$. For $\omega = 1/2$ there is a remanent $U(1)_R$ symmetry left with the Dirac gravitino charged under it. This R-symmetry corresponds to the exchange of the two boundaries $y = 0 \leftrightarrow y = \pi R$. For generic $\omega \neq 0, 1/2$ the R-symmetry is totally broken

Consider the case where there are also contribution the gravitino mass from potential on the boundary branes. Looking at the mass formula:

$$m_{3/2} = \frac{n+\omega}{R} + \frac{1}{\pi R}\left[\arctan(\kappa M_0) + \arctan(\kappa M_\pi)\right], \tag{80}$$

As it stands the localized masses can shift the value of $\frac{\omega}{\pi}$ to any desired value, it could for example be canceled or shifted to $\omega = 1/2$, by the appropriate choice of values of M_0 and M_π. However, we are mainly interested in the case when these localized masses arise from dynamics on the boundary branes and can have a four-dimensional description. Then, two remarks are in order.

First, the Scherk-Schwarz twist can not compensate the effects of supersymmetry breaking due to F or D-term dynamics as explicitly shown in [7].

Second, the mass formula can be approximated by:

$$m_{3/2} \simeq \frac{n+\omega}{R} + \frac{\kappa}{\pi R}\left(M_0 + M_\pi\right). \tag{81}$$

and as $\kappa M_b << 1$ they cannot have a sizable effect on the numerical value of ω if this is not already small. For small values of ω, the lightest gravitino lies far below the Kaluza-Klein tower and a four-dimensional approximation can be used to study the system within four-dimensional supergravity.

4 Dirac gauginos

A few features of the goldstinos have been exhibited in the previous section: i) there might be more than one sector breaking supersymmetry (the bulk and all the branes located inside it or at the boundaries). ii) only a global description of the model allows to identify which of the linear combination of the would-be-goldstinos is the (true) goldstino while the rest, the pseudo-goldstinos, remain as matter fermions iii) if one starts with an extended R-symmetry due to the presence of of an extended supersymmetry in the gravitational sector, it is possible to build models where the breaking parameters can be tuned to keep a part of this R-symmetry unbroken with the gravitino having a Dirac mass.

While previous sections focused mainly on the gravitational sector, we would like to discuss the important role played by similar features in the case of models with Dirac gauginos [21, 22].

Let us first start with the first point: why would we need two or more would-be-goldstinos?

A gaugino λ_a acquire Dirac masses $m_D(\lambda_a\psi_a)$ by coupling to other chiral fermions ψ_a in the adjoint representation. This gaugino mass is soft and can be seen as originating either from $U(1)_a$ non-vanishing D-terms or F-term F_b. In a gauge mediation type scenario, new states are introduced at a mass scale M_m (we consider a single scale for simplicity) to serve as mediators of the breaking, and they couple to the visible and secluded sector through gauge couplings of strengths g, $g_{mD}a$ and g_{mFb}, respectively (see for example [23] for discussion on gravity mediation). The exchange of loops of these messengers induces soft masses that scale parametrically as:

$$m_D = \frac{g}{\sqrt{2}}\left(\sum_a c_{Da\frac{1}{2}}\frac{g_{mDa}}{8\pi^2}\frac{D_a}{M_m} + \sum_b c_{Fb\frac{1}{2}}\frac{g_{mFb}}{16\pi^2}\frac{F_b^2}{6M_m^3}\right) \tag{82}$$

where the coefficients $c_{Da\frac{1}{2}}$ and $c_{Fb\frac{1}{2}}$ are calculable model dependent coefficients that take into account summation on other quantum numbers of the messengers.

The chiral adjoints, pairing up with the gauginos, have scalar superpartners Σ in the adjoint representation of the gauge group. The soft induced masses are parametrized as $m_\Sigma^2\,\mathrm{tr}(\Sigma^\dagger\Sigma) + \frac{1}{2}B_\Sigma\,\mathrm{tr}(\Sigma^2 + (\Sigma^\dagger)^2)$ and are given by

$$\begin{aligned} m_\Sigma^2 &= \sum_a g_{mDa}^2\left(c_{Da\Sigma 2}\frac{1}{96\pi^2}\frac{D_a^2}{M_m^2} + c_{Da\Sigma 1}\frac{3D_a}{64\pi^2}\right) + \sum_b g_{mFb}^2 c_{Fb\Sigma 2}\frac{1}{16\pi^2}\frac{F_b^2}{M_m^2} \\ B_\Sigma &= -2\sum_a c_{Da\Sigma 3}\frac{g_{mDa}^2}{96\pi^2}\frac{D_a^2}{M_m^2} - 2\sum_b c_{Fb\Sigma 3}\frac{g_{mFb}^2}{16\pi^2}\frac{F_b^2}{M_m^2} \end{aligned} \tag{83}$$

As the B_Σ contribution tends to make tachyonic the mass of one of the component of the adjoint scalars, the generation of viable soft mass for the adjoint scalars turns out to be not totally trivial.

The simplest interactions between the DG-adjoints and the messengers, as the Yukawa couplings descending from $N = 2$ lead to tachyonic masses as the B_Σ contribution. Historically, this was the main reason for abandoning the Dirac gaugino scenario in [21]. To avoid such a result, the required forms of the adjoint-messengers interactions have been fully classified in [24, 25] (see also [26]) .

The scalar partners of the chiral fermions in the visible sector get leading order contribution in D/M_m^2 and F/M_m^2 to their soft masses at three-loop from D-term and two-loop from the F-term:

$$m_{\tilde{f}}^2 = \sum_i C_{\tilde{f}}^i \frac{(m_{iD})^2 \alpha_i}{\pi} \log\left(\frac{m_{\Sigma_P}^{(i)}}{m_{bD}}\right)^2 + 2c_{F0} \sum_{i,b} C_{\tilde{f}}^i \left(\frac{\alpha_i}{4\pi}\right)^2 \frac{F_b^2}{M_m^2} \tag{84}$$

where $C_{\tilde{f}}^i$ is the quadratic Casimir of the field f under group i.

To allow Dirac gaugino masses generated at the leading order in the supersymmetry breaking parameter, and sufficiently heavy selectrons, we shall consider a combination of D- and F-term breaking, with both D- and F-terms comparable. This will generate a spectrum with masses of generic order of magnitude (a) gaugino masses $\sim \frac{g_{mD} g}{16\pi^2} \frac{D}{M_m}$ (b) sfermion masses $\sim \frac{g^2}{16\pi^2} \frac{F}{M_m}$ (c) adjoint scalar masses $\sim \frac{g_{mD}}{4\pi} \frac{D}{M_m}, \frac{g_{mF}}{4\pi} \frac{F}{M_m}$. and we expect the adjoint scalars to be the most massive states. Explicit models can be found, for example, in [25].

The requirement of using both an F-term and a D-term means that the goldstino will be a linear combination of the $U(1)_D$ gaugino λ_{Da} and of the chiral fermion χ_b associated with the F-term,

$$G = \frac{\sum_a D_a \lambda_{Da} + \sum_b F_b \chi_b}{\sqrt{\sum_a D_a^2 + \sum_b F_b^2}}. \tag{85}$$

The goldstino is "absorbed" through the super-Higgs mechanism as explained in the previous sections. Its coupling to matter is easy to obtain. More interesting is the fate of the fermions orthogonal to G. The obvious possibility is that all become pseudo-goldstinos, therefore remains as one of the massive matter fermions. However another possibility is one of them will be absorbed as a true goldstino by another gravitino.

As λ_D and χ are not visible sector fields, there is no obstruction to make them part of an $N = 2$ sector that will also include the gravitinos [27]. For instance, in the case of just two would be goldstinos, associated with one F and one D-term, one can use the orthogonal combination to G to be absorbed by the the second gravitino. This can be tuned as the couplings of the two gravitinos to these fields can appear different, even with opposite sign, as seen in the previous section. In this case, the second gravitino will have the same mass as the visible sector one. The two degenerate Majorana spin-3/2 states will combine to give rise to a Dirac gravitino that preserves an R-symmetry. Of course, the Higgs sector still breaks R-symmetry as this seems necessary to obtain the right size of the Higgs mass. But the induced Majorana mass for the gauginos can be kept very small. In summary:

- A generic Dirac gaugino model build for phenomenological purpose would involve at least two different supersymmetry breaking sources corresponding to non-vanishing D-term and R-preserving F-term.
- Each sector of the breaking gives rise to a would-be-goldstino and therefore there are at least two of them.
- The gravitino is chosen to to originate in an extended $N = 2$ supersymmetric structure. The two gravitinos could eat two linear combination of the would-be-goldstinos giving rise to degenerate Majorana masses that combine in an R-preserving Dirac mass.

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