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Honoring Epimenides of Crete (±Δx): From Quantum Paradoxes, through Weak Measurements, to the Nature of Time

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Abstract. Quantum temporal peculiarities, involving ordinary and weak measurements, are explored. We introduce the foundations of weak measurement and outline some novel theoretical and experimental predictions derived from it. We then show how weak values, which explicitly depend on both forward and backward evolving state-vectors, can serve as important tools for gaining new insights into the nature of time.

1 Introduction

Thankful for this invitation to present our work in the stunning island of Crete, we take the liberty of indulging in some historical reflections as an introduction.

Crete was the birthplace of Epimenides (7th/6th Century BC), author of the famous paradox based on the self-contradicting statement "all Cretans are liars." The Quantum Liar Paradox [1] described below is a physical manifestation of that ancient millstone, suggesting that Nature herself is capable of creating self-contradictions.

Second, not far from this place is the village Milatos, which in ancient times gave its name to the more famous town Miletus, birthplace of the first scientist known to history. Thales (624–546 BC) was the discoverer of electricity and magnetism, and it so happened that another work of ours presented in this conference [2,3] deals with the very nature of the electric and magnetic fields.

And of course there were many other giants in the neighboring islands and shores during that golden age. Archimedes (287–212 BC), long before the advent of calculus, recognized the importance of infinitesimals. The "slope of a line segment as short as a point" sounds just as absurd as Epimenides' "this statement is false," yet it eventually turned out to be one of mathematics' most powerful tools. Archimedes was inspired by the earlier legendary dispute between Parmenides (5th century BC) and Heraclitus (535–475 BC), about the nature of time. For Parmenides, change was an illusion of the senses, reality being eternal and

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immutable. Heraclitus, in contrast, held change to be reality's main attribute. Parmenides' disciple, Zeno (490–430 BC), has derived one of the most famous paradoxes concerning the alleged atomicity of time, a challenge that provoked not only Archimedes' infinitesimals but also, in 20th Century, the quantum-mechanical realization [4].

All these thinkers belonged to the small genius nation of which Crete was part, whose scholars have first raised the great questions of science and philosophy with utmost clarity and acuity. Our research aspires to resolve quantum paradoxes as well as problems about the nature of space-time, guided by the intuition that these two realms are closely related. Presenting this work in this ancient cradle of science is therefore another source of awe and inspiration.

2 Quantum Temporal Paradoxes

The most paradoxical effects displayed by quantum measurements involve spatial and temporal anomalies, e.g., respectively, the EPR [5] and the delayed-choice [6] experiments. Because time is the most elusive and unique dimension of space-time, we shall focus on some novel quantum effects that strongly strain common-sense intuitions about time.

2.1 Motivation

Several years ago, during a class on quantum mechanics, one of us (AE) was presented with a question from an inquisitive student. Why, she asked, should one think that Schrodinger's cat was superposed before the box's opening? This possibility can be ruled out by allowing a certain time interval, say, three days, pass between the potentially lethal event and the opening. Then, if the cat is found to be dead, it would be also decomposed, whereas if it is alive it would be also lean and starved. In both cases, it should be clear that it has never been superposed!

It takes some reflection in order to realize why this clever reasoning does not rule out that superposition has prevailed within the box all along: Quantum observation determines not only the cat's state at that moment, but also its entire history since the lethal event's (non)occurrence!

This example indicates that some kind of retrocausality is inherent in nearly all quantum-mechanical paradoxes, thereby giving additional credence to models that take this peculiarity as their cornerstone. The most daring attempt of this kind is Aharonov's two state-vector formalism [7,8], which furthermore derives many surprising predictions, some of which are being verified nowadays. Cramer's [9] transactional interpretation also invokes this kind of peculiar causality, yielding an interpretation which is elegant and parsimonious. Novel elaborations of this model, attempting to accommodate it to recently-discovered quantum phenomena, merit further interest [10-15].

2.2 IFM

While the first quantum experiment presents a spatial oddity, later variations of it were equally opposed to classical notions of time and causality, hence we introduce it first.

Consider a super-sensitive bomb with which even the slightest interaction possible leads to its explosion. Can one detect the bomb's presence at a certain location without destroying it? Elitzur and Vaidman [16] posed this question with a new answer in the positive. Their solution was based on the device known as Mach-Zehnder Interferometer (MZI), shown in Fig. 1. A single photon impinges on the first beam splitter, the transmission coefficient of which is 50%. The transmitted and reflected parts of the wave-function are then reflected by the two solid mirrors and then reunited by a second beam splitter with the same transmission coefficient. Two detectors are positioned to detect the photon after it passes through the second beam splitter. The positions of the beam splitters and the mirrors are arranged in such a way that (due to destructive and constructive interference) the photon is never detected by detector D, but always by C. In order to test the bomb, let it be placed on one of the MZI's routes (v) and let a single photon pass through the system. Three outcomes of this trial are now possible:

• The bomb explodes,
• Detector C clicks,
• Detector D clicks.

If detector D clicks (the probability for which being 1/4), the goal is achieved: we know that interference has been disturbed, ergo, the bomb is inside the interferometer. Yet, it did not explode.

The problem can be formulated in an even more intriguing way: Can one test whether the supersensitive bomb is "good" (better say: "bad") without bringing about its explosion? Again, all one should do is to place the bomb on one of the MZI's routes such that, if the photon passes on that route, the bomb's sensitive part can be triggered by absorbing only some of the photon's energy. Here too, the bomb constitutes a "which way" detector: Just as its explosion would indicates that the photon took the bomb's route, its silence indicates that it took the other route.

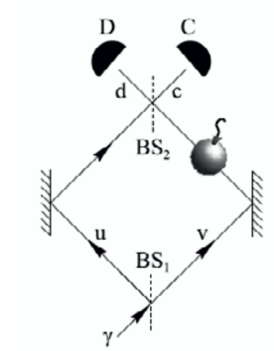

Fig. 1. Interaction Free Measurement. BS_1 and BS_2 are beam splitters. In the absence of the obstructing bomb, there will be constructive interference at path c (the detector C will click) and destructive interference on path d (detector D never clicks).

And again, interference is destroyed by the bomb's mere non-explosion, indicating that the bomb is explosive. Since the EV paper, numerous works, experimental and theoretical, have elaborated it and expanded its scope. Zeilinger et al. [17] refined it so as to save nearly 100% of the bombs. Other applications of IFM range from quantum computation [18] to imaging [19].

2.3 Partial Measurements - Hybridizing IFM with EPR

Apart from its technological applications, IFM is extremely efficient for experiments that aim to give better understanding of the nature f the wave-function. One such an experiment has been proposed [20] for studying the EPR effect. Consider a particle split not only to two parts, as in the ordinary MZI, but to 100. Then measure one of the wave-function's parts. In most cases, no detection would occur. This is a weak IFM that changes the wave-function only slightly. Rather than the abrupt transition from superposition to position, the likelihood of the particle to be in a certain state has increased or decreased. This is partial measurement. Next consider an EPR pair whose particles undergo partial measurements. Here, some intriguing effects occur:

1. Partial measurement on one particle yields a partial nonlocal effect on the other particle;

2. The other particle can then undergo another partial measurement and exert its own slight effect back on the first.

3. Partial measurement can be totally time-reversed, returning the wave-function to its original superposition, giving rise to a new kind of quantum erasure.

4. This erasure nonlocally erases the previous partial nonlocal effect on the distant particle.

5. This way, the particles may keep "talking" to one another for a long time, unlike the ordinary EPR in which they become disentangled after one measurement. This method, and the ones describe below, have this feature in common. Quantum measurement is ill-understood and abrupt. If one makes it gradual, some novel features of the measuring process emerge.

2.4 MAKING IFM MUTUAL: SUPERPOSED PARTICLES MEASURE ONE ANOTHER

Next we study more advance variants. To understand their intriguing nature, recall that the uniqueness of IFM lies in an exchange of roles: The quantum object, rather than being the subject of measurement, becomes the measuring apparatus itself, whereas the macroscopic detector is the object to be measured. In their original paper, Elitzur and Vaidman mentioned the possibility of an IFM in which both objects, the measuring one as well as the one being measured, are single particles, in which case even more intriguing effects can appear. This proposition was taken up in a seminal work by Hardy [21]. He considered an EV device (Fig. 1) similar to that described in Section 2.1, but with a more delicate "bomb," henceforth named a "Hardy atom". This atom's state is as follows. Let a spin-1/2 atom be prepared in an "up" spin-x state (X^+) and then split by a non-uniform magnetic field B into its z components. The two components are carefully put into two boxes Z^+ and Z^- while keeping their superposition state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\gamma\rangle(i|Z^+\rangle + |Z^-\rangle) \quad (1)$$

The boxes are transparent for the photon but opaque for the atom. Now let the atom's Z^- box be positioned across the photon's v path in such a way that the photon can pass through the box and interact with the atom inside in a 100% efficiency.
Next let the photon be transmitted by BS_1:

$$|\psi\rangle = \frac{1}{2}(i|u\rangle + |v\rangle)(i|Z^+\rangle + |Z^-\rangle) \quad (2)$$

Discarding all these cases of the photon's absorption by the atom (25% of the experiments)**:**

$$|\psi\rangle = \frac{1}{2}(-|u\rangle|Z^+\rangle + i|u\rangle|Z^-\rangle + i|v\rangle|Z^+\rangle) \quad (3)$$

Next, reunite the photon by BS_2:

$$|v\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|d\rangle + i|c\rangle)$$
$$|u\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|d\rangle - i|c\rangle) \quad (4)$$

So that:

$$|\psi\rangle = \frac{i}{\sqrt[3]{2}}[|c\rangle(i|Z^+\rangle + 2|Z^-\rangle) - |d\rangle|Z^+\rangle] \quad (5)$$

After the photon reaches one of the detectors, the atom's Z boxes are joined and a reverse magnetic field –B is applied to bring it to its final state. Measuring this state's spin yields:

$$|\psi\rangle = \frac{1}{4}|d\rangle(-i|X^+\rangle + |X^-\rangle) + \frac{1}{4}|c\rangle(-3|X^+\rangle + i|X^-\rangle) \quad (6)$$

In 1/16 of the cases, the photon hits detector D, while the atom is found in a final spin state of X^- rather than its initial state X^+. In every such a case, both particles performed IFM on one another; they both destroyed each other's interference. Nevertheless, the photon has not been absorbed by the atom, so no interaction seems to have taken place.

Hardy's analysis stressed a striking aspect of this result: The atom can be regarded as EV's "bomb" as long as it is in superposition, and its interaction with the photon can end up with one out of three consequences:

• The atom absorbs the photon – this is analogous to the explosion in EV's original device.

• The atom remains superposed – this is analogous to the no-explosion outcome.

• The atom does not absorb the photon but loses its superposition – a third possibility that does not exist with the classical bomb and amounts to a delicate form of explosion.

Hence, when the last case occurs, it appears that the photon has traversed the u arm of the MZI, while still affecting the atom on the other arm by forcing it to assume (as measurement will indeed reveal) a Z^+ spin!

2.5 EPR Effects between Particles that never interacted in the past

Another elegant experiment by Hardy [21] brings together nearly all the famous quantum experiments, such as the double-slit, the delayed choice, EPR and IFM – all in one simple setup (Fig. 2).

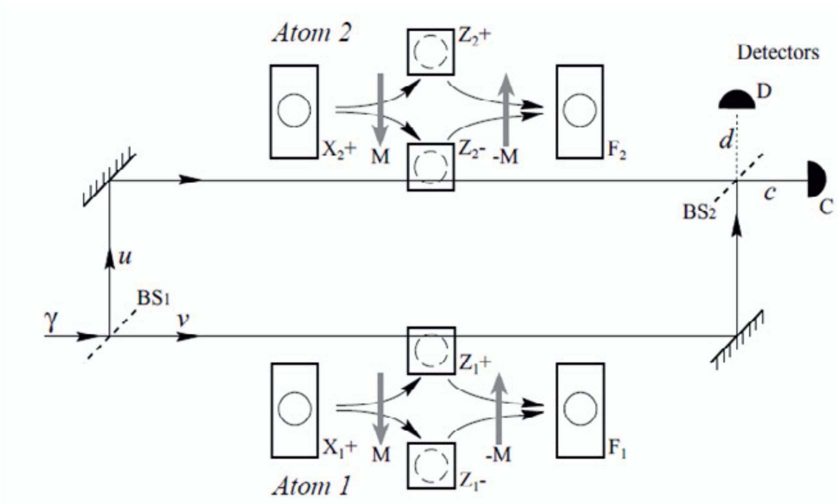

Fig. 2. Entangling two atoms that never interact.

Let again a single photon traverse a MZI. Now let two Hardy atoms be prepared as in Sec. 2.3, each atom superposed in two boxes that are transparent for the photon but opaque for the atom. Then let the two atoms be positioned on the MZI's two arms such that atom 1's Z_+ box lies across the photon's v path and 2's Z_{-2} box is positioned across the photon's u path. On both arms, the photon can pass through the box and interact with the atom inside in 100% efficiency. Now let the photon be transmitted by BS_1:

$$|\psi\rangle = \frac{1}{\sqrt[3]{2}}(i|u\rangle + |v\rangle)(i|Z^+\rangle_1 + |Z^-\rangle_1)(i|Z^+\rangle_2 + |Z^-\rangle_2) \qquad (7)$$

Once the photon was allowed to interact with the atoms, we discard the cases in which absorption occurred (50%), to get:

$$|\psi\rangle = \frac{1}{\sqrt[3]{2}}(-i|u\rangle|Z^+\rangle_1|Z^+\rangle_2 - |u\rangle|Z^-\rangle_1|Z^+\rangle_2 + i|v\rangle|Z^-\rangle_1|Z^+\rangle_2 + |v\rangle|Z^-\rangle_1|Z^-\rangle_2) \qquad (8)$$

Now, let photon parts u and v pass through BS_2:

$$\begin{aligned} |v\rangle &\xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|d\rangle + i|c\rangle) \\ |u\rangle &\xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|c\rangle + i|d\rangle) \end{aligned} \qquad (9)$$

giving

$$|\psi\rangle=\frac{1}{4}(|d\rangle|Z^+\rangle_1|Z^+\rangle_2+|d\rangle|Z^-\rangle_1|Z^-\rangle_2-i|c\rangle|Z^-\rangle_1|Z^+\rangle_2-2|c\rangle|Z^-\rangle_1|Z^-\rangle_2) \tag{10}$$

If we If we now post-select only the experiments in which the photon was surely disrupted on one of its two paths, thereby hitting detector D, we get:

$$|\psi\rangle=\frac{1}{4}|d\rangle(|Z^+\rangle_1|Z^+\rangle_2+|Z^-\rangle_1|Z^-\rangle_2) \tag{11}$$

Consequently, the atoms, which never met in the past, become entangled in an EPR-like relation. In other words, they would violate Bell's inequality [22]. Unlike the ordinary EPR, where the two particles have interacted earlier or emerged from the same source, here the only common event in the two atoms' past is the single photon that has "visited" both of them.

2.6 EPR Upside-down

Hardy's abovementioned experiment [21] inspired Elitzur and Dolev propose a simpler version [23] that constitutes an inverse EPR. Let two coherent photon beams be emitted from two distant sources as in Fig. 3. Let the sources be of sufficiently low intensity such that, on average, one photon is emitted during a given time interval. Let the beams be directed towards an equidistant BS. Again, two detectors are positioned next to the BS:

$$\begin{aligned}
|\phi\rangle_{\gamma u}&=p|1\rangle_u+q|0\rangle_u\\
|\phi\rangle_{\gamma v}&=p|1\rangle_v+q|0\rangle_v\\
|\psi\rangle_{A1}&=\frac{1}{\sqrt{2}}(i|Z^-\rangle_1+|Z^+\rangle_1)\\
|\psi\rangle_{A2}&=\frac{1}{\sqrt{2}}(i|Z^-\rangle_2+|Z^+\rangle_2)
\end{aligned} \tag{12}$$

where $|1\rangle$ denotes a photon state (with probability p^2), $|0\rangle$ denotes a state of no photon (with probability q^2), $p \ll 1$, and $p^2 + q^2 = 1$.

Since the two sources' radiation is of equal wavelength, a static interference pattern will be manifested by different detection probabilities in each detector. Adjusting the lengths of the photons' paths v and u will modify these probabilities, allowing a state where one detector, D, is always silent due to destructive interference, while all the clicks occur at the other detector, C, due to constructive interference. Notice that each single photon obeys these detection probabilities only if both paths u and v, coming from the two distant sources, are open. We shall also presume that the time during which the two sources remain coherent is long enough compared to the experiment's duration, hence we can assume the above phase relation to be fixed.

Next, let two Hardy atoms be placed on the two possible paths such that atom 1's Z^+_1 box lies across the photon's u path and 2's Z^-_2 box is positioned across the photon's v path. After the photon was allowed to interact with the atoms, we discard the cases in which absorption occurred (50%), getting

$$|\psi\rangle = \frac{1}{\sqrt[3]{2}}(-i|v\rangle|Z^+\rangle_1|Z^+\rangle_2 - |v\rangle|Z^-\rangle_1|Z^+\rangle_2 + i|u\rangle|Z^-\rangle_1|Z^+\rangle_2 + |u\rangle|Z^-\rangle_1|Z^-\rangle_2) \tag{13}$$

We now post-select only the cases in which a single photon reached detector D, which means that one of its paths was surely disrupted:

$$|\psi\rangle = \frac{1}{4}|d\rangle(|Z^+\rangle_1|Z^+\rangle_2 + |Z^-\rangle_1|Z^-\rangle_2) \tag{14}$$

thereby entangling the two atoms into a full-blown EPR state:

$$|EPR\rangle_{12} = \frac{1}{\sqrt{2}}(|Z^-\rangle_1|Z^+\rangle_2 + |Z^-\rangle_1|Z^-\rangle_2) \tag{15}$$

In other words, tests of Bell's inequality performed on the two atoms will show the same violations observed in the EPR case, indicating that the spin value of each atom depends on the choice of spin direction measured on the other atom, no matter how distant.

Unlike the ordinary EPR generation, where the two particles have interacted earlier, here the only common event lies in the particles' future.

One might argue that the atoms are measured only after the photon's interference, hence the entangling event still resides in the measurements' past. However, all three events, namely, the photon's interference and the two atoms' measurements, can be performed in a space-like separation, hence the entangling event may be seen as residing in the measurements' either past or future.

2.7 Nature Caught Contradicting Herself – The Quantum Liar Paradox

A closer inspection of the abovementioned inverse EPR reveals something truly remarkable. Beyond the apparent time-reversal lies a paradox that in a way is even more acute than the well-known EPR or Schrodinger's cat paradoxes. It stems not from a conflict between QM and classical physics or between relativity theory; rather, it seems to defy logic itself.

The idea underlying the experiment is very simple: In order to prove nonlocality, one has to test for Bell's inequality by repeatedly subjecting each pair of entangled particles to one out of three random measurements. Then, the overall statistics indicates that the result of each particle's measurement was determined by the choice of the measurement performed on its counterpart. A paradox inevitably ensues when one of the three measurements amounts to the question "Are you nonlocally affected by the other particle?" Let us, then, recall the gist of Bell's nonlocality proof[14] for the ordinary EPR experiment. A series of EPR particles is created, thereby having identical polarizations. Now consider three spin directions, x, y, and z. On each pair of particles, a measurement of one out of these directions should be performed, at random, on each particle.

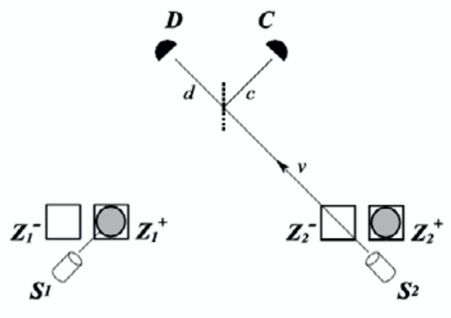

Fig. 3. Entangling two atoms.

Let many pairs be measured this way, such that all possible pairs of x, y, and z measurements are performed. Then let the incidence of correlations and anti-correlations be counted. By quantum mechanics, all same-spin pairs will yield correlations, while all different-spin pairs will yield 50%-50% correlations and anti-correlation. Indeed, this is the result obtained by numerous experiments to this day. By Bell's proof, no such result could have been pre-established in any local-realist way. Hence, the spin direction (up or down) of each particle is determined by the choice of spin angle (x, y, or z) measured on the other particle, no matter how distant.

Let us apply this method to the abovementioned time-reversed EPR. Each Hardy atom's position, namely, whether it resides in one box or the other, constitutes a spin measurement in the z directions (as it has been split according to its spin in this direction). To perform the z measurement, then, one has to simply open the two boxes and check where the atom is. To perform x and y spin measurements, one has to re-unite the two boxes under the inverse magnetic field, and then measure the atom's spin in the desired direction. Having randomly performed all nine possible pairs of measurements on the pairs, many times, and using Bell's theorem, one can prove that the two atoms affect one another non-locally, just as in the ordinary Bell's test. A puzzling situation now emerges. In 44% of the cases (assuming random choice of measurement directions), one of the atoms will be subjected to a z measurement – namely, checking in which box it resides. Suppose, then, that the first atom was found in the intersecting box. This seems to imply that no photon has

ever crossed that path, since it is obstructed by the atom. Indeed, as the atom remains in the ground state, we know that it did not absorb any photon. But then, by Bell's proof, the other atom is still affected nonlocally by the measurement of the first atom. But then again, if no photon has interacted with the first atom, the two atoms share no causal connection, in either past or future!

The same puzzle appears when the atom is found in the non-intersecting box. In this case, we have a 100% certainty that the other atom is in the intersecting box, meaning, again, that no photon could have taken the other path. But here again, if we do not perform the which-box measurement (even though we are certain of its result) and subject the other atom to an x or y measurement, Bell-inequality violations will occur, indicating that the result was affected by the measurement performed on the first atom (Fig. 3). The situation boils down to:

1. One atom is positioned in the intersecting box.

2. It has not absorbed any photon.

3. Still, the fact that the other atom's spin is affected by this atom's position means that something has traveled the path blocked by the first atom. To prove that, let another object be placed after the first atom on the virtual photon's path. No nonlocal correlations will show up.

Thus, the very fact that one atom is positioned in a place that seems to preclude its interaction with the other atom is affected by that other atom. This is logically equivalent to the statement "this sentence has never been written". We are unaware of any other quantum mechanical experiment that demonstrates such inconsistency.

2.8 Concluding Remarks

It thus seems quite obvious that the quantum realm is unique mainly because the time-evolution it presents allows events to affect one another in both time directions. This has so far been shown with the aid of ordinary quantum measurements. A new type of measurements, more delicate and sensitive, will be employed next.

3 Weak Measurements

Superposition is quantum mechanics' most intrinsic concept, an emblem of its uniqueness. An unmeasured particle's state is not only unknown but *indeterminate*, co-sustaining mutually-exclusive states. Equally crucial (and even less understood) is "measurement" or "collapse," upon which one of these states is realized, inflicting uncertainty on conjugate variables. In view of these limitations, can there be any reason to make quantum measurement *less* precise?

It is, surprisingly, weak measurement (WM) [24-26] that overcomes these limitations as well as many others [12,13,25]. Moreover, the Two-State-Vector-Formalism (TSVF), within which WM has been conceived, predicts several peculiar phenomena occurring *between* measurements, which only WM can reveal. Consider the question "What is a particle's state between two measurements?" Obviously, measuring such a state would change it into a state *upon* measurement, rendering the question meaningless. Not so with WM: The state, almost without being disturbed, can be made known with great accuracy, moreover manifesting a host of new peculiarities.

This, however, is a non-trivial task since most projective measurements performed on the system would change its dynamics. To overcome this challenge, weak measurement was introduced [24].

Weak measurement of a quantum system enables studying it without changing its wave-function. An intuitive explanation of this feat is given in [11]. In a nutshell, strong measurement is composed of a quantum pointer and an amplification mechanism making the reading macroscopic. In order to provide an accurate result, the pointer must have a certain momentum. This way, when our particle interacts with it, the reading (in terms of momentum change) would be unambiguous. Unfortunately, the amplified interaction with this pointer results in an irreversible change of the measured system – the so-called collapse. As opposed to this ordinary "strong" measurement, weak measurement creates a loose coupling to a quantum pointer whose momentum is highly uncertain, and again the pointer reading is being amplified by the same mechanism. This combination of weak coupling and noisy reading naturally gives a very small amount of information, but also a negligible change of its dynamics. It is on the ensemble level that weak measurement gains the desired precision, overcoming its inherent inaccuracy to the extent of even surpassing the limits of ordinary quantum measurement. By the Large Numbers Law, if x_i (the different measurement outcomes) are independent and identically distributed random variables with a finite second moment, their average goes to their expectation value: $\bar{x}_n \xrightarrow{a.s} \mu$. Furthermore, since the variance (noise) is proportional to N, the relative error diminishes. We showed [11] that an ensemble, can be a horde of states of a single particle undergoing cyclic weak measurements rather than an ensemble of particles undergoing a weak measurement.

Weak measurements were proven to be an important tool, not only for better answering fundamental questions but also for solving practical problems such as utilization of quantum amplification, cross-correlations between quantum signals [14], and increasing the signal-to-noise ratio [27].

In the language of quantum information, weak measurements were used to construct the quantum weak channel and the weak analogies of several bounds such as Holevo's [28].

3.1 Mathematical description

Using von Neumann's arguments as in [25], a quantum measurement of the observable A is defined by the interaction:

$$H_{\text{int}}(t) = \varepsilon g(t) A P_d \tag{16}$$

where the momentum P_d is canonically conjugated to Q_d, representing the pointer's position on the measuring device. The coupling *g(t)* differs from zero only at $0 \le t \le T$ and normalized according to

$$\int_0^T g(t)dt = 1 \tag{17}$$

i.e. the measurement lasts no longer than T.

In weak measurement, the coupling Hamiltonian of Eq. 16 is small in comparison to the pointer's standard deviation, *i.e.*, the measuring device is prepared in a symmetric quantum state with standard deviation $\sigma >> \varepsilon$ and zero expectation. Without loss of generality we can refer to state $|\Psi\rangle$ in the spatial representation (which serves as our measuring base) described by a Gaussian function

$$\Psi(x) = \exp(-x^2 / 2\sigma^2) \tag{18}$$

The pointer movement in that case is connected to the weak value of the operator *A* defined by:

$$A_w = \frac{\langle\varphi|A|\psi\rangle}{\langle\varphi|\psi\rangle} \tag{19}$$

where $|\psi\rangle$ is the initial (preparation) state of the measured system, and $|\varphi\rangle$ the final state into which it is projected.

For a pre-/post-selected ensemble described by the two-state $\langle\varphi|\;|\psi\rangle$, the time evolution of the total system (measured plus measuring system) is expected to be ($\hbar = 1$) [25]:

$$\langle\varphi|\exp(-i\int H_{\text{int}}dt)|\psi\rangle|\Psi\rangle \approx \langle\varphi|\psi\rangle(1 - i\varepsilon A_W P_d)|\Psi\rangle = \langle\varphi|\psi\rangle\exp(-i\varepsilon A_W P_d)|\Psi\rangle \tag{20}$$

which results in

$$\exp(-i\varepsilon A_w p)\Psi(x) = \Psi(x - \varepsilon A_w) \tag{21}$$

For example, when weakly measuring the spin-z (described by the Pauli matrix σ^z) of an ensemble of spin-1/2 particles prepared in the X^+ direction, with coupling strength $\varepsilon = \lambda / \sqrt{N}$, the time evolution is determined by

$$W = \exp(-i\int H_{\text{int}}dt) = \exp(i\lambda\sum_{n=1}^{N}\sigma_n^z P_d / \sqrt{N}) \tag{22}$$

so for a single measurement, the evolution of the spin states becomes entangled with the pointer of Eq. 18 (when $\sigma = 1$):

$$\begin{aligned}&\frac{1}{Nf}[|\sigma_z = +1\rangle e^{-(x-\lambda/\sqrt{N})^2/2} + |\sigma_z = -1\rangle e^{-(x+\lambda/\sqrt{N})^2/2}] \approx \\ &\approx \frac{1}{Nf}[(1 - x^2 - \frac{\lambda^2}{N})|\sigma_x = +1\rangle + \frac{2x\lambda}{\sqrt{N}}|\sigma_x = -1\rangle]\end{aligned} \tag{23}$$

To complete the weak measurement, the pointer itself must be strongly measured. Then, the particle's initial state X^+ changes by only a fraction $\sim \lambda^2 / N$. Statistically, this means that only $\sim \lambda^2$ out of N particles' states have changed. When λ is small enough, this number of "flipped" spins (changed from the initial X^+ to X^-) is negligibly small compared to the ensemble's size [25].

Moreover, if the coupling strength is $\varepsilon = \lambda / N$ rather than $\varepsilon = \lambda / \sqrt{N}$, the weak measurement process will most likely end without a single flip.

In case that all weak measurement are performed on a single particle (without constantly pre- and post-selecting it as in [11]), using a single pointer but only on one particle, it was shown [15] that the pointers undergoes a biased random walk with a log-normal distribution (see Fig. 4). When increasing the number of weak measurements, this procedure tends to a strong measurement (see Fig. 5).

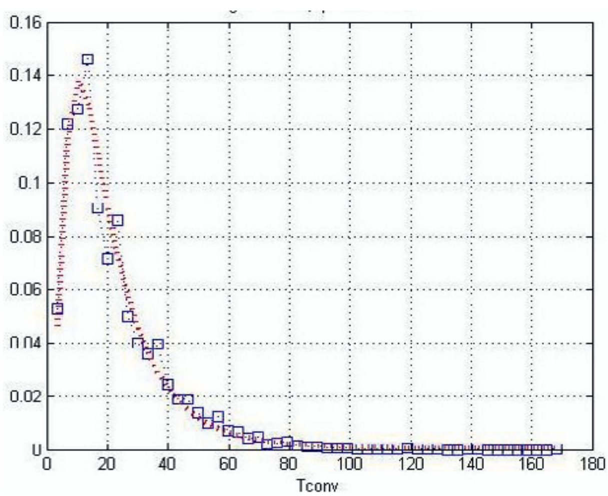

Fig. 4. Distribution of the number of measurements until the collapse given a fixed standard deviation σ.

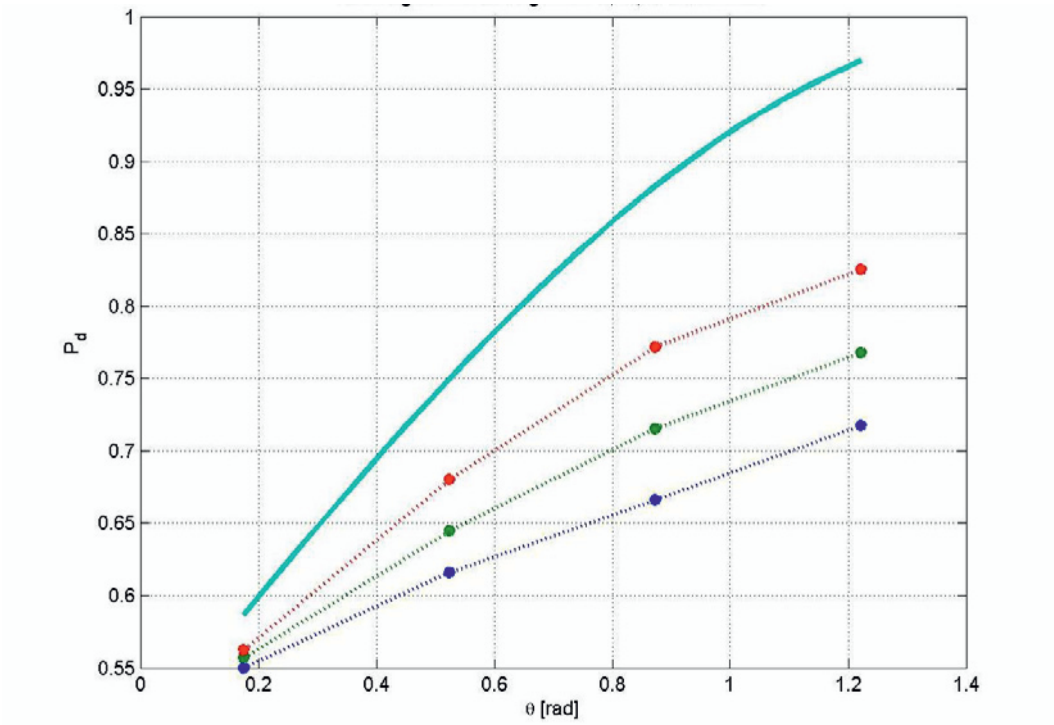

Fig. 5. Success probability for hypothesis testing with low number of weak measurements. The solid curve describes the optimal success probability for projective measurement.

3.2 Measuring Non-commuting variables of the same particle

The above technique entails an even more intriguing result: When the two (initial and final) strong measurements are made on non-commuting operators, then, for the intermediate states, these two operators can coexist with arbitrary precision.

3.3 Exotic mass and momentum

With the uncertainty principle thus subtly outsmarted and ordinary temporal order strained, it is perhaps not surprising that these between-measurements states revealed by WM display other physical oddities as well. Brief examination of Eq. 19 reveals that the weak values may not belong to the operator's spectrum. As a consequence, particles with odd mass or momentum, at times being even negative, are predicted by TSVF and amenable to isolation and measurement by appropriate slicing. Such effects are demonstrated elsewhere [25] leading, for instance, to violation of Bohr's correspondence principle [13].

3.4 Weak Values of Entangled States

It was shown that weak measurements can be performed on an entangled pair (or triplet, quartet etc.) without destroying the entanglement. Utilizing this possibility, we showed [12] that one can record the results of $\sigma_x, \sigma_y, \sigma_z$ of two entangled spins which will later violate

Bell's inequality. Therefore, enabling to interpret the future state vector as a hidden variable, which converts spatial nonlocality to backward affect. Weak correlations are also the basis for the Cheshire Cat Paradox [29] according to which a spin-1/2 particle can take one route of an interferometer while its spin takes the other.

A series of experiments demonstrating the theory in Secs. 3.3-3.5 and especially in [12] is expected to take place at INRiM during 2014.

3.5 Temporal Paradoxes Revisited

The method of weak measurement can validate and shed some light on the above temporal paradoxes. A recent experiment of joint weak and interaction free measurement is suggested in [30,31]. Weak measurements validating the Hardy paradox (Sec. 2.4) and negative weak values of projection operators (Sec. 3.4) are presented in [32]. Furthermore, as we concluded in [10], weak measurements (or more specifically, protective measurements [33]) can be used to track the wavefunction's changes after each partial measurement introduced in Sec. 2.3. We also suggest weak correlations measurements of the entangled pairs in Secs. 2.5-2.7 in the spirit of the ones in [12].

4 Quantum Oblivion: The Underlying Mechanism of several Quantum Feats

Let us again take a step back to grasp the emerging overall picture of the quantum realm: It is a realm describing the microscopic where time-symmetry is much more common than in the classical, macroscopic realm. Recently, we were able to pinpoint this unique quantum reversibility with a simple gedankenexperiment.

Notice first that momentum conservation is one of classical physics' most fundamental laws, which every translational symmetric system must obey. The following quantum interaction, however, seems to defy it.

Let an electron and a positron, with spin states $\left|X^{+}\right\rangle=\frac{1}{\sqrt{2}}(\left|X^{+}\right\rangle+\left|X^{-}\right\rangle)$ and momenta $P_{e^-}<P_{e^+}$, be sent along the y-direction, entering two Stern-Gerlach magnets (drawn for simplicity as beam-splitters) positioned at (t_0,x_{e^-},y_0) and (t_0,x_{e^+},y_0) respectively (Fig. 6). The magnets split the particles' paths according to their spins in the x-direction:

$$\left|\psi_{e^-}\right\rangle=\frac{1}{\sqrt{2}}(\left|1_{e^-}\right\rangle+\left|2_{e^-}\right\rangle) \text{ and } \left|\psi_{e^+}\right\rangle=\frac{1}{\sqrt{2}}(\left|3_{e^+}\right\rangle+\left|4_{e^+}\right\rangle) \tag{24}$$

We shall describe the time evolution of the process with the wave-functions above and with two-state detector: I/II.

The total wave function is:

$$|\psi\rangle = \frac{1}{2}[(|1_{e^-}\rangle + |2_{e^-}\rangle)|3_{e^+}\rangle|I\rangle + (|1_{e^-}\rangle + |2_{e^-}\rangle)|4_{e^+}\rangle|II\rangle] \tag{25}$$

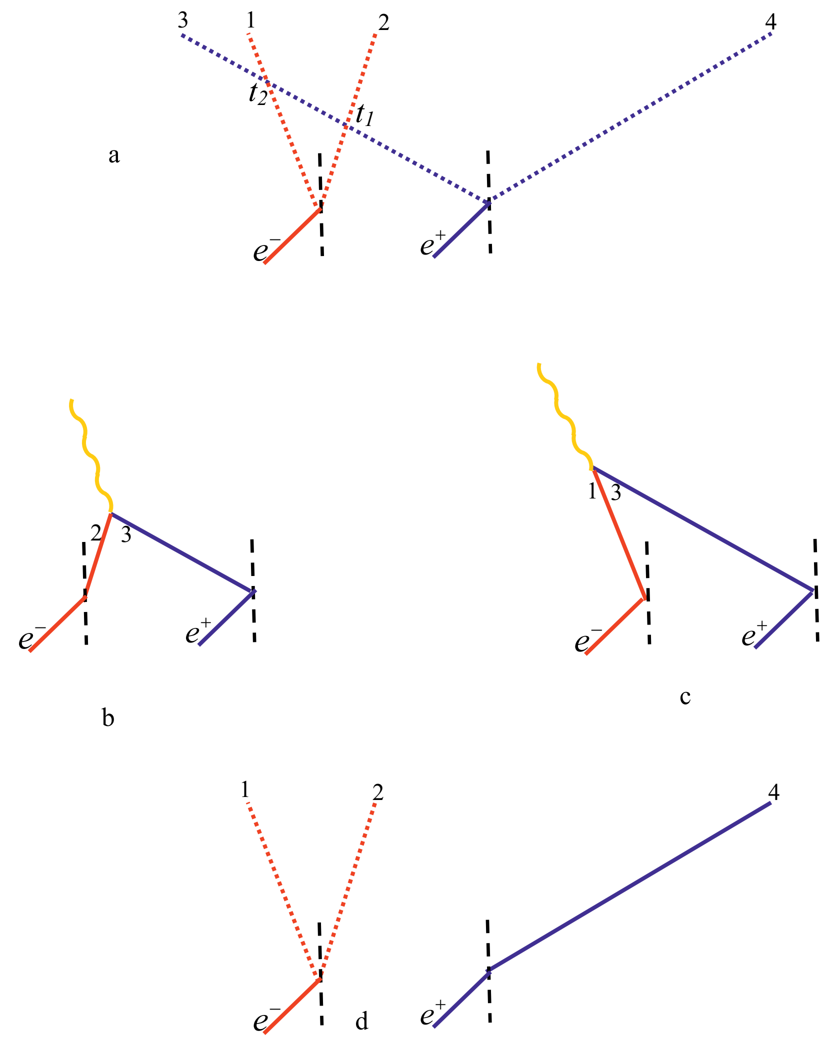

Fig. 6: Possible electron-positron interactions. (a) The setting. (b-c) Annihilations. (d) Interaction-free "collapse."

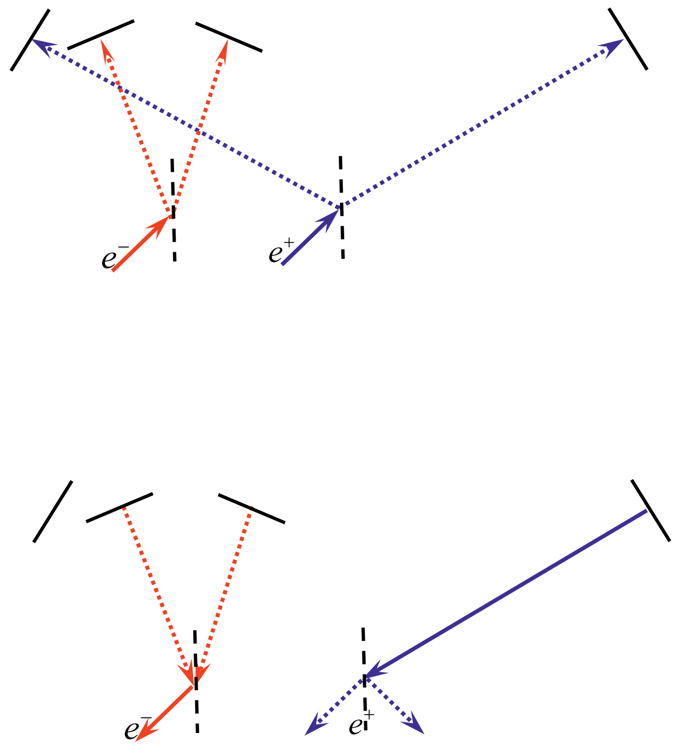

Fig. 7: The peculiar momentum exchange due to outcome (d) in Fig 6: The positron's momentum is changed while that of the electron remains intact.

Depending on their positions at t_1 or t_2, the particles may (not) annihilate and consequently (not) release a pair of photons, which may in turn (not) trigger detectors positioned in the appropriate places. Let these photons, exhibiting the unique superposition emitted/not emitted, be termed "conditional photons" and let the detector's two corresponding states be denoted by I/II

At $t_0 \le t \le t_1$ the superposition does not change:

$$|\psi\rangle = \frac{1}{2}[(|1_{e^-}\rangle + |2_{e^-}\rangle)|3_{e^+}\rangle|I\rangle + (|1_{e^-}\rangle + |2_{e^-}\rangle)|4_{e^+}\rangle|II\rangle] \tag{26}$$

At $t_1 < t < t_2$, if a photon pair is emitted we know that the particles ended up in paths 2 and 3.

Otherwise,

$$|\psi\rangle = \frac{1}{\sqrt{3}}[(|1_{e^-}\rangle + |2_{e^-}\rangle)|4_{e^+}\rangle|I\rangle + |1_{e^-}\rangle|3_{e^+}\rangle|II\rangle] \quad (27)$$

which is an interesting superposition:one component of it is a definite state while the other is a superposition in itself.

At $t > t_2$, if a photon is emitted, we know that the particles ended up in paths 1 and 3. Otherwise,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1_{e^-}\rangle + |2_{e^-}\rangle)|4_{e^+}\rangle|I\rangle \quad (28)$$

which is peculiar. On the other hand the positron is physically affected by the interaction: It is not superposed anymore, hence if we time-reverse its splitting, it may fail to return to its origin. In other words, its momentum has changed. Not so with the electron: It remains superposed, hence its time-reversibility remains intact (Fig. 7). Thus only one party of the interaction "remembers" it by exhibiting change, while the other remains unaffected, apparently violating the momentum conservation law.

We have recently shown that this quantum oblivion constitutes the essential ingredient of IFM as well as all its variations presented above. Once again, quantum mechanics turns out to owe its strength to its greater temporal flexibility.

5 Zeno Going Quantum

Whereas Zeno formulated his paradox on purely logical grounds, his 20th-Century followers [4] have shown that Nature herself can make time "stop" under an appropriate form of repeated measurement. The broad philosophical implications of this effect, not yet fully explored, go beyond the scope of this paper. Here we only mention Zeilinger's application of it to enhance the IFM [17]. By an appropriate choice of cyclical measurements, he managed to raise IFM's efficiency close to 100%.

Strangely, however, this application was not explored further. This is peculiar because Hardy, in the works reviewed above (Sec. 2), has demonstrated several IFM variants even more striking than the original one. It would therefore be reasonable to expect the quantum Zeno effect to offer similar enhancements in their case too. Consider, e.g., Hardy's paradox [21] where a particle and anti-particle perform mutual IFM on one another. Augmented with the quantum Zeno effect, it can always produce the peculiar result that one member of the pair has visibly changed the other's state while it looks equally obvious that they never came into contact. This line of investigation has been recently taken up by our group, with several such intriguing effects to be reported soon.

6 Time: A geometric Parameter or the Real Source of Evolution?

What, then, is the bearing of these quantum paradoxes on the deeper issues concerning a nature of time? Following are a few speculative reflections, to which we hope later to provide firmer grounding in theory and experiment.

Classical physics, to which special and general relativity belong, treats time as a purely geometrical ingredient of the universe, alongside with the three spatial dimensions. Against the perfect logical rigor and experimental support that make relativity so powerful, many physicists find the "block universe" picture emerging from it manifestly awkward. In fact, the very notion of space-time implies that, just as all locations have the same degree of reality in space, so do all past, present, and future somehow exist along the times dimension without any moment being unique as the privileged "now."

Against this mainstream view, there are alternative accounts [34] that address a few unresolved physical issues more straightforwardly, even though still lacking empirical support. They suspect that, if we experience time so differently from space, this difference may be objective, no matter how poorly represented in present-day physics. It is well-known that even the founders of the Block Universe, including Einstein, remained highly uncomfortable with it. The minus sign assigned to the zeroth dimension in Minkowski's geometrical formulation of relativity theory is only one hint that time differs from space in a very subtle yet *objective* sense. The vast unresolved issue of the origins of time asymmetry in the universe [35] is another.

In short, while Einstein and Minkowski are Parmenides' heirs in modern physics' thought, it was left to more intuitive philosophers like Bergson [36] to counter with Heraclitian dynamics. Bergson has ascribed time a genuine "flow" or "passage," characterize by "Becoming". Every event which we perceive as occurring "now" is indeed a novel phenomenon which, prior to that occurrence, *did not exist* in the most fundamental sense rather than "being already there" in the future direction of time but only inaccessible to observation.

No other than de Broglie, one of quantum mechanics' pioneers, paid the following homage to Bergson [37]:

> [I]f Bergson could have studied the quantum theories in detail, he would have noted certainly with joy that, in the image that they offer us of the evolution of the physical world, they show us nature in all its occasions hesitating between several possibilities, and he would have undoubtedly repeated, as in *La Pensée et le Mouvant*, that 'time is that very hesitation or it is nothing at all.'

Far from being mere poetical musings, these comments suggest a possible alternative to the relativistic picture of space-time. Indeed, Cramer, founder of the transactional interpretation of QM which was originally formulated within a strict Block Universe framework, discusses the "plane of the present" [38] where the advanced and retarded solutions of relativistic equation of motion perform their "handshake". He states [38]:

> [W]hile block-universe determinism is consistent with the transactional interpretation, it is not required. A part of the future is emerging into a fixed local existence with each transaction, but the future is not determining the past, and the two are not locked together in a rigid embrace.

Being well-aware of the speculative nature of our model, we briefly sketch its present stage, hopefully to mature into a physical theory.

We begin with the following assumption. Just as the speed of light is regarded as a Lorentz scalar, hence, unchanging when transforming between inertial frames, (as opposed to the space and time coordinates that change as a function of v/c), *the quantum interaction can be regarded as more fundamental than the space and time coordinates*. The space-time interval between two events following an interaction is therefore not a passive, pre-existing background for the interaction between wave-functions. Rather, it is the quantum interaction's very *outcome*. All the spatial and temporal oddities of quantum measurement, reviewed in the previous sections, would then be natural!

Consider, e.g., a quantum position measurement. A particle whose wave-function is widely spread in space is eventually found in one of all the numerous locations along this wave-function. What about all the locations where it has *not* been found? They are populated by the macroscopic bodies that have performed the interaction-free measurement that seem to indicate that the particle never went on that direction. Yet the wave-function itself, as any interference experiment can prove, did go in all these directions! The Block-Universe account for this case would allow either a "collapse," known for being incompatible with relativity, or some semi-classical hybrid a-la' "hidden variables." Can quantum mechanics offer another alternative? "No" would be the pertinent answer, but the reason for this ineffectiveness is important: It is just what QM lack to this day, namely a full quantum-mechanical description for both microscopic and macroscopic bodies. This is, in other words, the notorious measurement problem.

We submit that introducing "Becoming" into physics offers the key for the problematic "macroscopically-superposed state," a direct consequence of the quantum formalism yet never observed. *The dead-and-alive cat exists not in a well-defend space-time within the closed box, but rather as a pre-space-time interaction between numerous wave-functions, the completion of which would give either the "dead" or "alive" state with the relevant space-time configurations between all particles involved.* "Collapse," then, is the formation of the macroscopic event with its entire space-time configuration.

These admittedly speculative hypotheses rely on the insights gained from the TSVF. In contrast to the classical realm, quantum mechanics does not enable a full specification of the initial state at $t=0$ to predict the results of all measurements performed at later times. However, this fact is responsible also for our ability of defining a final state-vector describing the system (this final state is clearly redundant in classical mechanics). At first sight, this double boundary condition on the wavefunction seems to resonate with a block-universe approach. The uncertainty principle, however, provides us with freedom to define the present. As Aharonov et al. argue [39] in order to correctly simulate quantum correlations a sequence of moments should be thought of as a chain of pre- and post-selection conditions.

A preliminary model based on these assumptions is now in progress. We would feel privileged to present the following stages of its development in future meetings (whether they "already" exist somewhere in a Block Universe or awaiting genuine Becoming) of this highly inspiring forum in the very cradle of science.

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