Improved intermittency analysis of proton density fluctuations in NA49 ion collisions at 158 AGeV.

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Abstract. The existence of the QCD critical point is a controversial issue, which is expected to be clarified in current ion collision experiments. In the vicinity of the critical point, according to recent theoretical investigations, the chiral condensate (chiral order parameter) should exhibit critical fluctuations (self-similarity and scale-invariance). These fluctuations should also be present in the transverse momentum distributions of protons produced in ion collision experiments due to the coupling of the chiral condensate with the net-proton density.

We use intermittency analysis to calculate the 2nd scaled factorial moment (SSFM) of transverse momenta of protons produced around mid-rapidity in central A+A collisions at the NA49 experiment (SPS,CERN). Three different systems are analysed (C+C, Si+Si and Pb+Pb). We develop techniques in order to improve the traditional intermittency approach, estimating the level of non-critical background and correcting for systematic methodological errors that arise in the case of low statistics. We find evidence of power-law dependence of the SSFM (intermittency) in the case of Si+Si and Pb+Pb systems. Furthermore, we perform a suitable correlation analysis in order to test and rule out the possibility that the observed behaviour is influenced by misidentified split tracks (spurious correlations) in the analysed data sets.

1 Introduction

In order to detect the chiral critical point of QCD in nuclear collision experiments, it is necessary to employ suitable observables [1–10]. Such observables are connected to the fluctuations of the chiral condensate, $\langle \bar{q}(\mathbf{x})q(\mathbf{x})\rangle$, which is the order parameter of the chiral phase transition ($q(\mathbf{x})$ is the quark field). The quantum state which carries the critical numbers and properties of the chiral condensate is the sigma field $\sigma(\mathbf{x})$. However, the chiral condensate, though likely to be formed in heavy ion collisions, is not directly observable, as it is unstable and decays mainly into pions at time scales characteristic of the strong nuclear interaction. Its critical properties are transferred to (π^+, π^-) pairs with invariant mass slightly above their production threshold, which are experimentally observable [10]. At finite baryochemical potential, there is evidence that the sigma-field will induce critical density fluctuations also in the baryonic sector [11–16]. In particular, as indicated in [8], critical

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fluctuations are transferred to the net proton density, as well as to the proton and antiproton densities separately, which are experimentally accessible [17].

In the present work, we calculate the second factorial moment of the proton density, in transverse momentum space, from data sets of "Si"+Si ("Si"=Al,Si,P), "C"+C ("C"=C,N) and Pb+Pb collisions recorded in the NA49 experiment at maximum energy ($\sqrt{s_{NN}} \approx 17$ GeV) of the SPS (CERN). We concentrate on the development of a methodological framework allowing the consistent treatment of uncertainties related to arbitrary bin boundary placement and the possible presence of split tracks in the data.

2 The Analysed data sets

We analysed data sets on the 12.5% most central collisions of "C", "Si" and Pb nuclei on C, Si and Pb targets, respectively, collected by the NA49 experiment at the beam energy of 158A GeV. The "C" beam was a mixture of C and N ions. Likewise, the "Si" beam comprised Al, Si and P ions. In order to increase statistics as much as possible, we merged the respective beam components into one dataset per beam. Thus, in total we obtained 201,000 "C"+C ("C"=C,N), 176,000 "Si"+Si ("Si"=Al,Si,P) and 1,480,000 Pb+Pb events, with collision centrality 0-12.5%. Standard event and track selection cuts were applied, as described in [18]. Proton identification was performed with maximum purity 80% for the "C"+C and "Si"+Si systems and 90% for the Pb+Pb system. We neglected antiprotons, which we have shown to be much fewer than protons in the region of analysis. Furthermore, we selected for the analysis only protons in the mid-rapidity region (center of mass rapidity y_{CM} in the interval $[-0.75, 0.75]$), since the proton density is approximately constant in rapidity in this region [1]. Proton tracks within this interval satisfy the criterion $|y_{CM}| < y_{beam} - 0.5$ necessary for avoiding spectators in the considered datasets ($y_{beam} \approx 2.9$).

3 Intermittency Analysis Methodology

In a pure critical system, the presence of intermittency in transverse momentum space can be revealed by calculating the dependence of the Second Scaled Factorial Moments (SSFMs) of proton tracks as a function of bin size. For that purpose, a region of transverse momentum space is partitioned into $M \times M$ equal-size bins. Consequently, the SSFMs:

$$F_2(M) = \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2} \tag{1}$$

are calculated, as an average over bins and events ($\langle \ldots \rangle$), where n_i is the number of tracks in the i-th bin and M^2 is the total number of bins. If the system exhibits critical fluctuations, $F_2(M)$ is expected to scale with M, for large values of M, as a power-law:

$$F_2(M) \sim M^{2\phi_2} \tag{2}$$

where ϕ_2 is the intermittency index. $\phi_2 = \phi_{2,cr}^B = {}^5/_6$, if the freeze-out occurs at exactly the critical point [1].

In a real experimental set of data, it is necessary to deal with the background, consisting of un-correlated and misidentified protons, in order to detect potentially intermittent behaviour. This is
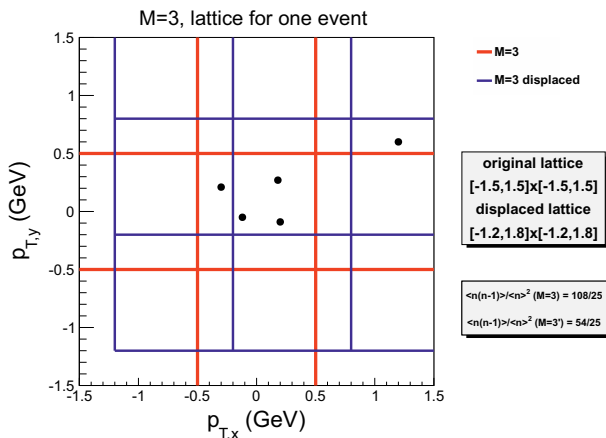
Figure 1. (Color online) Effect of displaced lattice on the SSFMs (one event). Tracks closer than the bin size can be split or grouped together by a bin boundary. Averaging over many, slightly displaced lattice positions for the whole collection of events compensates for that effect.

accomplished by constructing an artificial set of mixed events. These are created by merging tracks from different original events into new events of the same (average) multiplicities. Mixed events thus contain uncorrelated tracks, simulating background protons. Subsequently, we calculate the SSFMs of mixed events, $F_2^{(m)}(M)$ in the same manner as that of the original data, $F_2^{(d)}(M)$. The critical behaviour is then expected to be revealed in the correlator, $\Delta F_2(M) = F_2^{(d)}(M) - F_2^{(m)}(M)$, which should scale as a power law, $\Delta F_2(M) \sim M^{2\phi_2}$ with the same intermittency index as the pure critical system. Since we are dealing with a finite system, the power-law holds only in a limited range of scales. In our case, experience shows that the range $M^2 \geq 6000$ is the appropriate for a power-law fit.

We must further correct for low statistics and/or multiplicities by lattice averaging. Since bin boundaries are arbitrary, they may split pairs of neighbouring points, affecting $F_2(M)$ values (Fig.1). We compensate for that effect by repeating the calculation of SSFMs for several slightly displaced lattices and averaging $F_2(M)$ over them. The variance of $F_2(M)$ over displaced lattices provides us with an estimate of its error.

We estimate the statistical errors of $\Delta F_2(M)$ via the statistical method of bootstrapping (resampling), which consists in sampling the original set of events with replacement [19]. A new set of the same number of events is thus produced, where some of the original events are either duplicated or missing. We proceed to calculate $\Delta F_2(M)$ for each bootstrap sample in the same manner as the original. When a large number of bootstrap samples (typically 200-1000) have been analysed, we calculate their average $\overline{\Delta F_2(M)}$. We found $\overline{\Delta F_2(M)}$ to be very close to $\Delta F_2(M)$ of the original sample for all studied systems. The variance of bootstrap sample values provides an estimate of the statistical error of $\Delta F_2(M)$.

Finally, we must guard against the possibility of spurious correlations in the data, specifically in the form of split tracks – sections of a track that are erroneously reconstructed as a pair of tracks that are close in momentum space. Since intermittency analysis is based on the distribution of track pairs, split tracks can create false positives. Standard event and track cuts [18] remove most split tracks. We check for residual contamination via the q_{inv} distribution of track pairs:

$$q_{inv}(p_i, p_j) = \sqrt{-(p_i - p_j)^2} \tag{3}$$

where p_i is the 4-momentum of the i-th track. We calculate the ratio of distributions, $\mathcal{N}(q_{inv}^{data})/\mathcal{N}(q_{inv}^{mixed})$. A peak at low q_{inv} (below 20 MeV/c) indicates a possible split track contamination that must be removed.
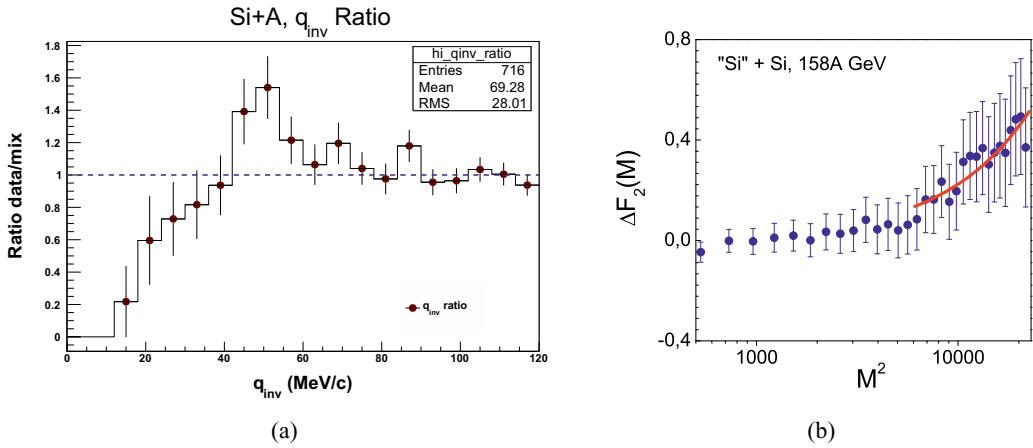
Figure 2: (Color online) (a) q_{inv} ratio distribution and (b) correlator $\Delta F_2(M)$ for the "Si"+Si system, for central collisions at 158 AGeV. Error bars in (b) are estimated by bootstrap. A semi-logarithmic scale is used to include points with negative $\Delta F_2(M)$ in the plot. Red solid line represents the results of a power-law fit to the data in the range $M^2 > 6000$.

4 Analysis Results

Intermittency analysis for the "C"+C system revealed no discernible power-law behaviour. Results of the analysis for the two remaining systems, "Si"+Si and Pb+Pb, are presented in Fig.2 and Fig.3 respectively. In Fig.2b,3b, $\Delta F_2(M)$ values were averaged over lattice positions. Error bars have two sources of origin: lattice variance and bootstrap variance. In practice, the bootstrap variance is the dominant contribution.

As we can see in Fig.2b,3b, both systems exhibit strong power-law scaling in the large M^2 region. The intermittency index resulting from a power-law fit is $\phi_2^{Si} = 1.00(17)$ for "Si"+Si and $\phi_2^{Pb} = 0.35(04)$ for Pb+Pb (fit errors). However, their q_{inv} distribution ratios differ significantly: For "Si"+Si, the ratio goes to zero at the limit of small q_{inv}, while Pb+Pb has a strong peak around 10 MeV/c. Thus we conclude that a possible contamination from split tracks cannot be excluded in the case of Pb+Pb.

To check the robustness of the intermittency effect in Pb+Pb, we employed a drastic q_{inv} cut, by removing from both data and mixed datasets all pairs of tracks with $q_{inv} < 20$ MeV/c (a track is randomly removed from every such pair). As can be seen in Fig.4, this cut alters significantly the measured intermittency index ϕ_2, $\phi_2^{Pb\ cut} = 0.20(02)$ for $M^2 > 6000$. However the power-law dependence of the correlator on M^2 persists. We are therefore confident that a genuine intermittency effect exists also for the Pb+Pb system.

5 Conclusions

Intermittency analysis of density fluctuations in transverse momentum space is an important tool for the detection of critical behaviour in QCD. However, the manner in which it has been traditionally handled is subject to systematic and statistical errors that can compromise the reliability of results and introduce spurious correlations. Intermittency analysis can be improved and made more robust by the application of techniques such as lattice averaging, to compensate for the arbitrary partitioning
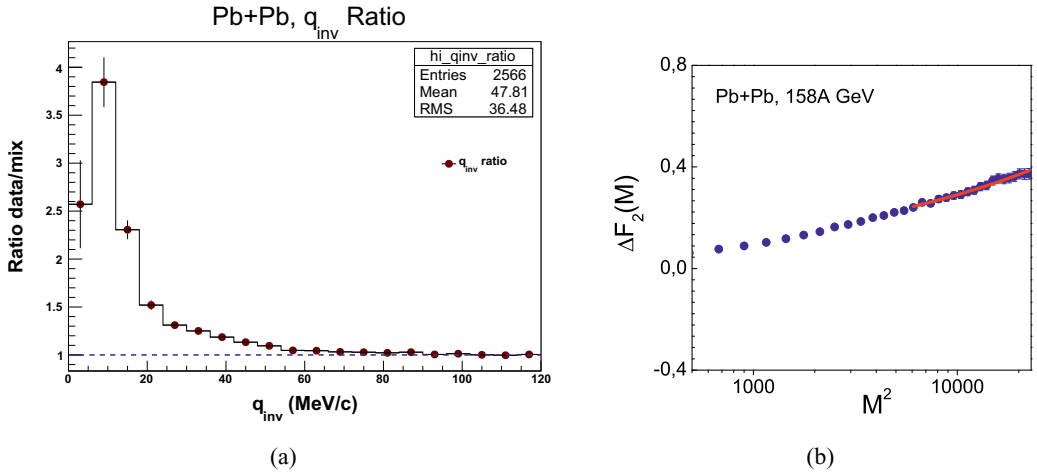
(a)

(b)

Figure 3: (Color online) (a) q_{inv} ratio distribution and (b) correlator $\Delta F_2(M)$ for the Pb+Pb system, for central collisions at 158 AGeV. Error bars in (b) are estimated by bootstrap. The same axis range as in Fig.2b is used for easy comparison. Red solid line represents the results of a power-law fit to the data in the range $M^2 > 6000$.
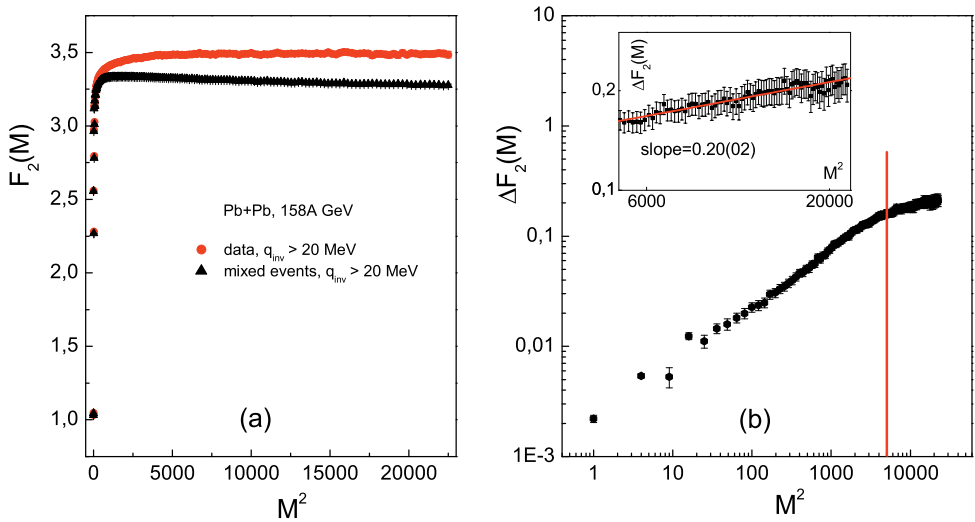

Figure 4: (Color online) (a) SSFMs $F_2(M)$ for data and mixed events in Pb+Pb, after the application of a q_{inv} cut. (b) Correlator $\Delta F_2(M)$ and power-law fit ($M^2 > 6000$, inlaid plot) for the same system (log-log scale). Intermittent behaviour persists, albeit with a lower ϕ_2 value in the fit range.

of transverse momentum space in bins, and bootstrap (resampling) to estimate the statistical error of SSFMs where statistics is insufficient.

Furthermore, it is possible to guard against spurious correlations by examining the q_{inv} distributions of the analysed systems. If the statistics is large, the offending tracks can be removed by a q_{inv} cut. However, the distortion of SSFMs in this case is large enough to affect the intermittency index, and only the presence of power-law behaviour can be inferred, at best.

With the aforementioned precautions, we have found intermittency analysis of proton density fluctuations to be valuable in the search for the QCD critical point. In particular, the theoretically expected value of the intermittency index, $\phi_{2,cr}^B = {}^5/_6$, lies in the range of the results obtained from intermittency analysis of the "Si"+Si and Pb+Pb systems. This is a strong indication that the freeze-out of these systems is in the vicinity of the critical point.

Further analysis of collision data from the ongoing NA61 and STAR experiments can benefit from intermittency analysis techniques in the search for the QCD critical point.

References

[1] N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis and K. S. Kousouris, Phys. Rev. Lett. **97**, (2006) 032002.

[2] P. Seyboth, Nucl. Phys. **A 862**, 54 (2011).

[3] M. Gazdzicki, J. Phys. **G 38**, 124024 (2011).

[4] M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998).

[5] H. Fujii, Phys. Rev. **D 67**, 094018 (2003).

[6] M. A. Stephanov, Phys. Rev. Lett. **102**, 032301 (2009).

[7] B. Mohanty, Nucl. Phys. **A 830**, 899c (2009).

[8] Y. Hatta and M. A. Stephanov, Phys. Rev. Lett. **91**, 102003 (2003).

[9] N. G. Antoniou, Y. F. Contoyiannis, F. K. Diakonos, A. I. Karanikas and C. N. Ktorides, Nucl. Phys. **A 693**, 799 (2001).

[10] N. G. Antoniou, Y. F. Contoyiannis, F. K. Diakonos and G. Mavromanolakis, Nucl. Phys. **A 761**, 149 (2005).

[11] K. Fukushima and T. Hatsuda, Rep. Prog. Phys. **74**, 014001 (2011).

[12] D. Bower and S. Gavin, Phys. Rev. **C 64**, 051902(R) (2001).

[13] N. G. Antoniou, Nucl. Phys. B-Proc. Suppl. **92**, 26 (2001).

[14] N. G. Antoniou, Acta Phys. Pol. **B 33**, 1521 (2002).

[15] Y. Hatta and T. Ikeda, Phys. Rev. **D 67**, 014028 (2003).

[16] M. A. Stephanov, Int. J. Mod. Phys. **A 20**, 4387 (2005).

[17] M. M. Aggarwal *et al*, Phys. Rev. Lett. **105**, 022302 (2010).

[18] T. Anticic *et al*, Phys. Rev. **C 83**, 054906 (2011).

[19] W. J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).