

Studying Heavy Ion Collisions Using Methods From Cosmic Microwave Background (CMB) Analysis

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Abstract. We present and discuss a framework for studying the morphology of high-multiplicity events from relativistic heavy ion collisions using methods commonly employed in the analysis of the photons from the Cosmic Microwave Background (CMB). The analysis is based on the decomposition of the distribution of the number density of (charged) particles expressed in polar and azimuthal coordinates into a sum of spherical harmonic functions. We present an application of the method exploiting relevant symmetries to the study of azimuthal correlations arising from collective flow among charged particles produced in relativistic heavy ion collisions. We discuss perspectives for event-by-event analyses, which with increasing collision energy will eventually open entirely new dimensions in the study of ultrarelativistic heavy ion reactions.

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1 Introduction

Many advanced techniques have been developed in the last decade to analyse the single cosmic event that we have at our disposal, our Universe, through the cosmic microwave background (CMB) radiation which carries the imprint of the conditions prevailing in the Universe shortly after the Big Bang. Notably, the study of the CMB radiation and its polarization provides new insight into the fluctuations in the early Universe before inflation and can shed light on the very mechanism underlying the inflationary phase transition.

Ultra-relativistic collisions between heavy ions are often highlighted as a mechanism to produce and study ‘mini universes’ under controlled laboratory conditions. Indeed, in such collisions a tremendous amount of kinetic energy may be transferred from the colliding heavy ions (of either gold or lead) which have been accelerated in machines like RHIC and LHC to an extremely hot and energy-dense zone that contains tens of thousands of produced particles in a volume of about the one of the colliding atomic nuclei. We now know that the reaction is characterized by a high degree of transparency [1], [2], in the sense that the nucleons in the colliding nuclei pass through each other and that, after the collision, the space between them is filled with tens of thousands of quark-antiquark pairs that are created from the color field between the colliding partons.

This super-hot piece of matter, which initially has a temperature in excess of 300MeV ($\approx 4 \cdot 10^{12}\text{K}$) constitutes an isolated system that expands and cools like a tiny fragment of the early Universe. It is also established that the system initially, and before it hadronizes, is strongly coupled and hence behaves like a fluid [3], [4], [5], [6]. This fluid has been labelled the s-QGP, for ‘strongly interacting Quark Gluon Plasma’. It is the state of matter that the Universe was in up to about a microsecond after the Big Bang.

The subsequent evolution of the tiny droplet of s-QGP is complex although shortlived: the system expands, cools and hadronizes and the newly formed hadrons interact and scatter as the system continues to expand until the density has decreased so much that interactions cease (freeze-out) after about $10\text{fm}/c$ ($\approx 10^{-22}\text{sec}$). From this time on, produced hadrons stream to the detectors only affected by possible decays in flight and reinteractions with the surrounding material. In central collisions of lead ions at the LHC the energy density after about $1\text{fm}/c$ exceeds $16\text{GeV}/\text{fm}^3$ and about 17.500 charged particles are produced in a single central collision [12]. The particles that reach the detectors carry the imprint of the early stages of the collision and we may expect, dependent on the reinteractions and degree of equilibration attained during the rapid expansion that fluctuations in the final channel may reflect some of the initial conditions.

In this sense it is tempting to attempt an analysis of the number of particles detected in heavy ion reactions at the single event level using tools developed for the photons emitted from the early Universe. In the following we present such a first study focusing on large scale collective phenomena in heavy ion collisions events.

2 Spherical harmonic decomposition of heavy ion events.

Data from the CMB sky are usually presented as a temperature variation map as a function of polar angle θ and azimuthal angle ϕ . It is conventional to use the Mollweide projection depicted in Fig. 1. The azimuthal angle runs along the ‘equator’ and the polar angle runs from the ‘south pole’ to the ‘north pole’.

For heavy ion collisions spherical coordinates may not seem to be the natural choice, on the one hand, due to the typical cylindrical geometry of present day particle detectors and on the other hand, and more fundamentally, due to the strong peaking of the number of particles measured in the laboratory frame of reference per unit solid angle near the direction of the beam of colliding

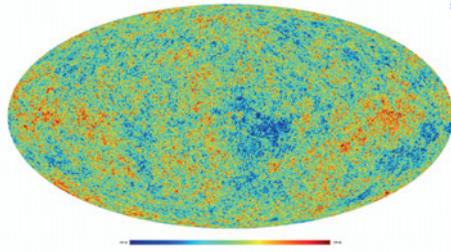


Figure 1. Mollweide projection of the CMB map of the sky. The azimuthal angle ϕ runs along the equator and the polar angle θ from the south to the north poles.

particles. Indeed, particle production is found to follow a nearly Gaussian distribution as a function of pseudorapidity $\eta = -\ln(\tan(\theta/2))$. This translates to the distribution shown in Fig. 2 as a function of the polar angle θ . The CMB map of the sky is, in contrast characterized by flatness as a function over the angular range. We shall, however, show that this is not a limitation and, that we may exploit specific symmetries to overcome such particular features in heavy ion distributions and linearize the distribution to be analyzed.

Data analysis of the CMB map of the sky is based on the expansion of the entire two-dimensional map $f(\theta, \phi)$ in terms of spherical harmonic functions $Y_{lm}(\theta, \phi)$,

$$f(\theta, \phi) = \sum_{l=1}^{l_{max}} \sum_{m=-l}^l a_{lm} Y_{lm}(\theta, \phi) \quad (1)$$

where $a_{lm} = |a_{l,m}| \exp(-im\Phi_{l,m})$ are the weight coefficients of the decomposition. Correspondingly, $|a_{l,m}|$ and $\Phi_{l,m}$ are the amplitudes and the phases of each l, m component.

The power spectrum is defined by

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^l |a_{lm}|^2. \quad (2)$$

Much of the understanding of CMB data is based on interpretation of the power spectrum up to order $l=2500$ and comparison to suitable cosmological models that try to predict the multipole spectrum.

In practice, the maximum l -value that is to be considered in the above decompositions depends on the experimentally achievable resolution in the experiment, both in terms of detector (angular) segmentation, but also in terms of detector resolution. In general, low order l -components probe large angle correlations, while high-order l values probe short scale correlations. Roughly speaking $\Delta\theta = \frac{2\pi}{l}$.

We may, for illustration purposes, use the HIJING generator [7] to generate semi-realistic heavy ion events. HIJING does not contain new physics, e.g. a QGP phase, but treats collisions starting from a Glauber description of the colliding nuclear density distributions in terms of their nucleon content and follows particle production and expansion and the hadronic and leptonic level. Fig. 2 shows the charged particle distribution for one small impact parameter (central) event as a function of pseudorapidity η (top) and polar angle θ (bottom). Fig. 3 shows the corresponding Mollweide representation of this event. Note the large density of counts around the poles in this representation. We may, however, ‘linearize’ the event by omitting the $m=0$ modes of the harmonic representation. This is due to the theta-symmetry of the spherical harmonic functions Y_{lm} for $m = 0$.

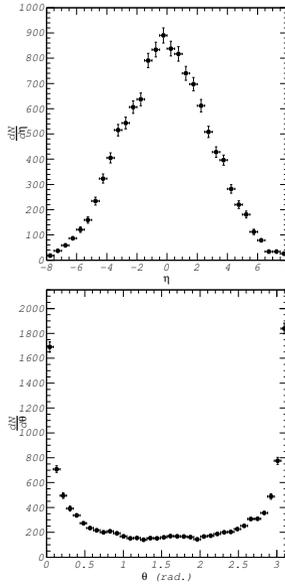


Figure 2. Top: Pseudorapidity density of charged particles as a function of pseudorapidity, as predicted by HIJING for a single semi-central collision between two Pb nuclei at current LHC energies ($\sqrt{s_{nn}} = 2.76 TeV$). Bottom: The corresponding density of charged particles as a function of their polar angle θ relative to the beam direction

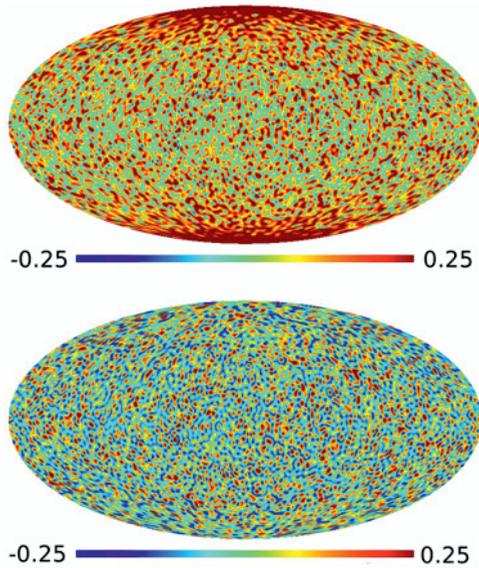


Figure 3. Top: Mollweide projection of the map of a single HIJING event (without collective flow). The color coding tracks deviations relative to the average level of counts. Bottom: same distribution but where the $m = 0$ mode has been removed from the map. The angular resolution of the maps corresponds $l_{max} = 100$ (see Eq.~1).

In Fig. 4 we plot the power spectrum for this event (eq. 2). We use either the CMB GLESP package [8] for these operations or a recently developed version integrated with the ROOT package. The black histogram shows the power of the l -multipoles obtained by a Fast Fourier Transform of the Mollweide distribution of the simulated heavy-ion event. The blue histogram shows the power associated with the $m=0$ components only for each l -multipole. Finally the red histogram shows the power associated with the $m \neq 0$ components only of the respective l -multipoles. It is seen that this power spectrum is essentially flat and random as one would expect for an event not carrying any particular microscopic structure (no dominating multipoles).

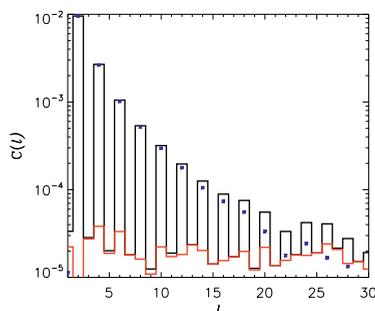


Figure 4. Left panel: power spectra for the map from Fig.3. The black histogram shows the total power spectrum, the blue dots are for the $m = 0$ mode only and the red histogram shows the power associated with $m \geq 1$ modes.

2.1 Collective flow

Two important physics signals underpin the discovery of the s-QGP [3], [4], [5], [6]. One is the quenching of jets that propagate through the dense medium, which is understood as due to energy loss of scattered partons due to stimulated gluon emission in an environment characterized by a high density of color charges. The other is the presence of significant anisotropic azimuthal flow in semi-peripheral nuclear collisions at high energy. This can be understood in terms of hydrodynamical streaming. Hydrodynamics models are able to reproduce observations using short reinteraction times and significant coupling between the partons.

Due to its importance we focus in this section on anisotropic azimuthal flow. The present exposition is a much condensed account of ref. [10], to which we refer the reader for full details. Analysis of experimental flow data typically relies on a Fourier decomposition of the number of particles that are measured as a function of azimuthal angle for a given (typically semi-peripheral) collision centrality. Often, in selected pseudorapidity regions, the analysis can be made as a function of the transverse

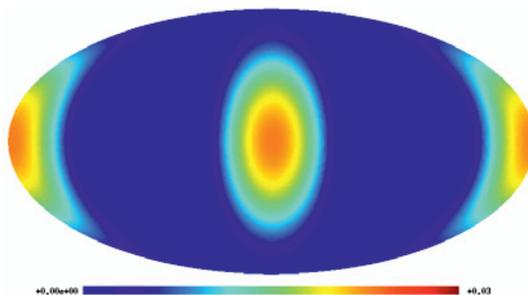


Figure 5. The Mollweide map for a structure corresponding to elliptic flow with amplitude $v_2 = 0.1$ and symmetry plane angle $\phi = 0$.

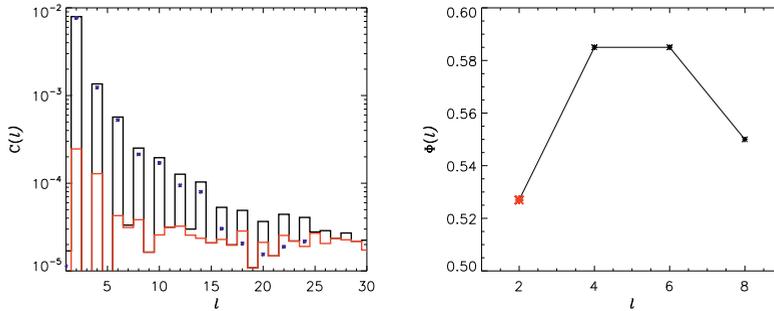


Figure 6. Left panel: the power spectrum for an event with elliptic flow $v_2 = 0.07$. The total power $C(l)$ is shown in black, the blue stars correspond to the $m = 0$ modes only, and the $m \neq 0$ power spectrum is shown in red. Right panel: the phase of the event plane Ψ_n obtained from the $l=2$ (red point) and from $l = 4, 6, 8$ (black points) modes. The input event plane angle was 30 degrees.

momentum of the particles and/or for identified particles. Analysis techniques have progressed from measuring the distribution of particles relative to an estimated event-plane, to using advanced multi-particle correlation techniques that are less sensitive to effects that mimic collective flow (non-flow) but stem from other non-collective processes (f.ex. two body decays) [13], [14]. In all cases the analysis is carried out by averaging over the particles in each event and further averaging over a number of events. For recent reviews see, for example, refs. [15], [16].

In this study we have added flow (elliptic, e.g. of order $n=2$) to a set of simulated HIJING events. In Fig. 6 we show the resultant power spectrum. It is obvious that additional power is present in the $l=2$ and $l=4$ multipoles. In ref. [10] we explain why the $l=4$ modes also contribute (see also Fig. 7).

For each heavy ion collision the angular distribution of particles is given by the following 'stochastic equation':

$$S(\theta, \phi) = f(\theta, \phi) \left[1 + 2 \sum_n v_n \cos[n(\phi - \Psi_n)] \right] \quad (3)$$

where $S(\theta, \phi)$ is the map of particles in an event with flow v_n and, $f(\theta, \phi)$ is the map of a random distribution of particles, without flow, which is characterized by a uniform random distribution in the azimuthal direction. The flow v_n can be deterministic or even a random function with a distribution function that differs from the p.d.f of $f(\theta, \phi)$. Decomposition of Eq. (3) in spherical harmonics gives us the following equation for the corresponding coefficients:

$$b_{l,m} = a_{l,m} + \sum_n v_n (a_{l,m+n} e^{-in\Psi_n} + a_{l,m-n} e^{in\Psi_n}), \quad (4)$$

where $b_{l,m}$ are the coefficients of the spherical harmonic decomposition for $S(\theta, \phi)$.

In ref. [10] we describe a systematic analysis of simulated events with flow of various magnitudes (and orders) and varying symmetry plane angle. We also establish relationships between the b_{lm} and the v_n and ϕ_n coefficients.

Accumulating statistics for each event over a large number of simulated events yields the distributions shown in figs. 8 and 9. It is seen that it is in principle possible to determine with good precision

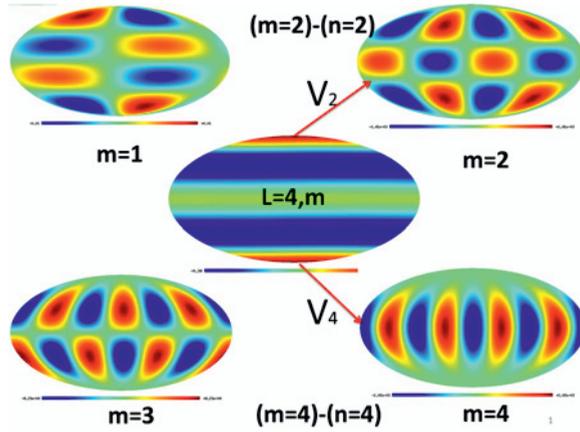


Figure 7. Schematic representation of the contribution of v_2 (elliptic) and v_4 flow to the various m components of the $b_{l=4,m}$ -coefficient. The v_2 -modulation will change the amplitude and the phase for the $b_{4,2}$ coefficient through the $m = 2 \leftrightarrow n = 2$ coupling, without contributing to the other components. The v_4 modulation will change only the $b_{4,4}$ component through the $m = 4 \leftrightarrow n = 4$ coupling. Note that the elliptic flow will provide a $b_{4,3} \leftrightarrow b_{4,1}$ coupling, but this effect is very small ($\sim v_2$), in comparison with the discussed $b_{4,2}$ and $b_{4,4}$ components.

both the flow amplitude v_n and the symmetry plane angle for every event. The precision is naturally controlled by the number of particles that can be included in the sample for every event. In the next section we elaborate on this.

3 Event-by-event analysis of fluctuations in heavy ion collisions.

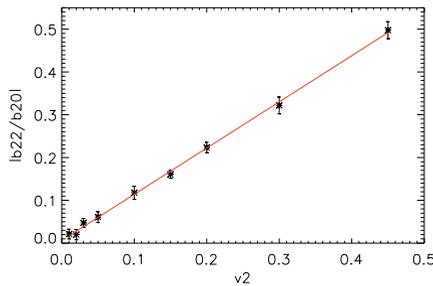


Figure 8. Estimator $|b_{2,2}|/|b_{2,0}|$ for the elliptic flow amplitude obtained for single events plotted versus the v_2 value used in the HIJING simulations. The errorbars corresponds to 68% confidence level.

A main message of this talk is, apart from drawing attention to the method outlined in ref. [10], [11] to point out that the experimental conditions at the LHC at CERN combined with powerful experiments such as ALICE [9] will, in the near future make it possible to collect very significant statistics for each event in f.ex. Pb+Pb collisions. In fact, we may estimate from the systematics presented in ref. [12] infer that at the top LHC energy for Pb+Pb collisions, which is expected to be reached in Y2015 or Y2016, the number of charged particles per single 0-5% central event will be of the order of. 23.000 particles. For semi-peripheral collisions, e.g. 40-50 % centrality, which shows

maximal elliptic flow, the particle production per event is expected to decrease by a factor of about 4, resulting in about 6000 particles per event of this class.

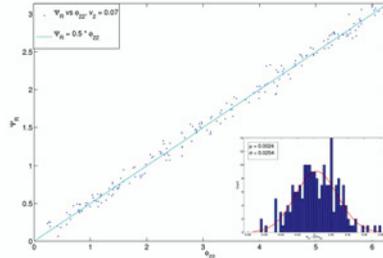


Figure 9. The correlation between the reaction plane angle Ψ_R that was an input for each of 100 HIJING events with fixed flow $v_2 = 0.07$ is plotted as a function the phase angle $\phi_{2,2}$ obtained in the harmonic decomposition analysis for each event. The insert shows the distribution of the symmetry planes found by this event-by-event analysis.

In Fig. 10 we show our estimates of the simultaneous distributions of flow amplitudes and symmetry plane angles at the event-by-event level for collections of events of different average multiplicities of charged particles.

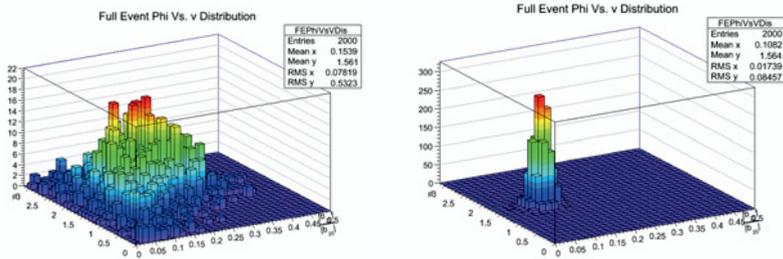


Figure 10. Simulated simultaneous resolution for the flow amplitude and the symmetry plane angle for events with 1000 particles in each event (top) and 30000 particles in each event (bottom) reconstructed using the harmonic decomposition method. The plotted distributions are for a collection of 2000 such events, each with $v_2 = 0.1$ and $\psi = 90$ degrees.

4 Conclusions and perspectives.

It is not given that the methodology outlined here will eventually prove superior to the sophisticated techniques that have been developed in recent years to study the azimuthal collective motion in selected rapidity regions (essentially a 1D analysis, even if it involves multiparticle correlation techniques). We suggest, however, that the method discussed here, being intrinsical multidimensional, (image analysis) may prove useful in studying other fluctuation phenomena at the single event level, and that the multipole amplitudes may be useful in finding and characterizing such modes. The significantly increase in multiplicity (by $\approx 30\%$) expected at the single event level at the LHC running at full energy $\sqrt{s_{nn}} = 5.5 TeV$ will open up a new doorway. A future collider operating at an order of magnitude higher energy, e.g. the VLHC which may be an option for at future hadron collider at

CERN, would lead to total charged particle production of about 60000 per central event (extrapolating from present systematics) and seriously open up for high precision event-by-event Little Big Bang physics exploration.

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