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Facets of Decoherence: Chiral Magnetic Effect in Heavy Ion Collisions

In memory of Professor Mikhail Polikarpov (1952 - 2013)

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Abstract. We discuss the effect of anisotropy for fluctuations of electric currents in magnetic field using the quantum measurements theory language. Possible interpretation of parity-odd phenomena like Chiral Magnetic Effect in this framework is given. It is advocated that quantum-to-classical transition caused by decoherence plays crucial role. The talk is based on papers [1–3].

1 Introduction

It has been understood for a long time that vacuum of any quantum field theory (and the Standard Model in particular) can be often insightfully viewed as a sort of medium. Needless to say that this medium is in many respects different from the conventional media we study in condensed matter physics - in particular, it usually has no rest frame and looks identically for all uniformly moving observers. However two main approaches to study properties of this (and actually of any) media stay with us since Galileo: one can either send some test particles and look how they move and interact (with each other and with the medium) or one should put some external conditions on the medium (e.g., to heat it) and study what happens. Of particular interest is a question about the fate of discrete symmetries under this or that choice of external conditions.

We know from experiment that all three most important discrete symmetries: charge conjugation **C**, time reverse **T** and parity **P** are broken both at micro- and at macro- level in our world: matter dominates over antimatter, there are several arrows of time (entropic, cosmological, psychological, electroweak), left and right chiralities act differently both in the Standard Model lagrangian and in biological systems. Let us take a closer look at the parity invariance. The fundamental result due to C.Vafa and E.Witten [4, 5] says that vacuum expectation value of any local **P**-odd observable has to vanish in vector-like theories such as QCD, e.g.

$$\langle\bar{\psi}\gamma^5\psi\rangle = 0 \ \ ; \ \ \langle G_{\mu\nu}\tilde{G}^{\mu\nu}\rangle = 0 \quad (1)$$

It does not mean of course that correlators of the corresponding operators vanish.

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In principle, one would not expect violation of (1) at finite temperature/baryon density. The situation with **P**-parity is different from **C**- and **T**-symmetries in this respect,[1] therefore it is of special interest to study possible **P**-odd effects in strong interaction context under nontrivial conditions. Despite Vafa-Witten theorem puts serious constraints on possible **P**-parity violating phenomena in the domain of strong interactions physics, such effects have been under discussion for a long time. One can mention T.D.Lee's idea of **P**-odd bubbles and A.B.Migdal's hypothesis of pion condensate in nuclei. Closely related effects of $\rho - \pi$ mixing at finite temperature [6] and sphaleron dynamics in QCD [7] were discussed.

In recent years the interest to this topic has been raised again after a series of papers by D.E.Kharzeev and coauthors [8–11] and numerous subsequent publications. The two mentioned approaches to study any vacuum nicely work together in heavy ion collision experiments with respect to the QCD vacuum. Test particles used in these experiments — heavy ions — are able to create, in the first instants after the collision, nontrivial multi-particle state, which itself plays a role of external conditions. Besides temperature and density there is also extremely strong magnetic field, of the order of $(10^3 - 10^4)$ MeV^2 in about 0.2 Fm/c after the moment of collision. The main qualitative result can be formulated as follows: if by whatever dynamical mechanism there is an excess of quarks of definite chirality inside a fireball, electric current flows along the magnetic field, whose main effect is charge asymmetry of final particles distribution between upper and lower (with respect to the interaction plane) hemispheres. On quantitative level, for free massless spinors with charge e, chemical potentials μ_L, μ_R for left-handed and right-handed ones, respectively, in constant and spatially uniform magnetic field $\mathbf{B}$ electric current is given by the following expression, known as chiral magnetic effect (CME):

$$\mathbf{J} = \frac{e^2}{2\pi^2}\mu_5\mathbf{B} \quad ; \quad \mu_5 = \frac{\mu_R - \mu_L}{2} \tag{2}$$

The expression (2) first explicitly obtained in [12] (not in heavy ion collision context) is a robust theoretical result and can be reproduced in many ways. What is special and important about the result (2) is that proportionality coefficient there is universal and fixed by the famous triangle anomaly [13, 14].

The current (2) has many interesting properties. The most important one is that the corresponding transport coefficient is **P**-odd, but **T**-even. According to general arguments, this means that the current is non-dissipative and the corresponding entropy production rate equals to zero. This has close resemblance with superconductivity and superconducting current, despite the latter is by itself a temperature-dependent phenomenon, while the chiral current (2) is (at least naively) temperature-independent.

In experiments, both real and numerical, one addresses (2) indirectly, via measuring some **P**-even charge dependent correlations, which comes out, roughly speaking, as **P**-odd effect like (2) squared. From heavy ion collision experiments point of view, the charge-dependent asymmetries have been studied by STAR and PHENIX experiments at RHIC and by ALICE experiment at LHC [15] - [18]. The effect can be described as follows. For noncentral collision one can fix the reaction plane by two vectors: beam momentum and impact parameter. By convention angular momentum of the beams (and the corresponding magnetic field) is orthogonal to the reaction plane, while the azimuthal angle $\phi \in [0, 2\pi)$ is defined in the plane, orthogonal to particles momenta. With this notation, in any particular event one studies

[1] For example, **C**-invariance is intact at finite temperature, but gets broken at finite density and there is no Furry theorem at finite chemical potential.

charged particles distribution in ϕ using the following conventional parametrization

$$\frac{dN_\pm}{d\phi} \propto 1 + 2v_{1,\pm}\cos\phi + 2v_{2,\pm}\cos 2\phi + 2a_\pm \sin\phi + ... \tag{3}$$

The coefficients $v_{1,\pm}$ and $v_{2,\pm}$ account for the so called direct and elliptic flow. They are believed to be universal for positively and negatively charged particles with good accuracy. The coefficients a_+ and a_- describe charge flow along the axis, normal to the reaction plane. This **P**-parity forbidden correlation between a polar vector (electric current) and the axial one (angular momentum) is sometimes interpreted as a signature of **P**-parity violation in a given event with $a_\pm \neq 0$. On the other hand, the random nature of the process dictates $\langle a_+\rangle_e = \langle a_-\rangle_e = 0$ (there the averaging over events is taken). The averages over events and over azimuthal angle of sine and cosine functions with the weight (3) provides information about correlation coefficients. The most recent results from ALICE experiment can be found in [19].

Professor Mikhail Polikarpov was well recognized expert in lattice field theory. He was the first in lattice community who applied this powerful machinery to the physics related to (2). He was very enthusiastic about the matter. — It is the first and perhaps the only case in physics, — he motivated his students and colleagues, — when electromagnetic field has the same hundred-MeV scale, as the strong one! — We have to rethink all we know about confinement, chiral symmetry breaking, hadron spectrum etc now in the superstrong abelian magnetic field, — this was the program his group was actively realizing for the period 2008-2013. Nineteen papers about various aspects of QCD in magnetic field have been submitted to arXiv. First ever CME-related lattice results were presented in the paper "Numerical evidence of chiral magnetic effect in lattice gauge theory", co-authored by P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov [20], having more than 130 citations to date. Drastically different patterns of current fluctuations in the magnetic field above and below deconfinement phase transition were obtained. The general conclusion about existence of the desired asymmetry was positive, but obtaining precise quantitative results happened to be notoriously difficult.

2 CME and decoherence

There are a few important questions related to CME, which are left open by the expression (2). Needless to say that the list is subjective.

- How to proceed in a reliable way from nice qualitative picture of CME to quantitative predictions for charge particle correlations measured in experiments?
- What is quantum physics behind μ_5?
- How to disentangle the genuine nonabelian physics from dynamics of free massless fermions in magnetic field?
- How is quantum anomaly in microscopic current encoded in dynamics for macroscopic, effective currents (anomalous hydrodynamics)?

The first thing worth noting in this context is that one should carefully distinguish symmetry breaking caused by dynamics from event-by-event fluctuations. Indeed, a simple quantum-mechanical analogy can be useful to illustrate the point. In one-dimensional bound state problem with P-parity even potential $V(x) = V(-x)$ one has $\langle x \rangle = \int x dx |\psi_0(x,t)|^2 = 0$ where $\psi_0(x,t)$ is the ground state P-parity even wave function. On the other hand, performing particle position measurements on ensemble of N identical systems all in the ground state one gets sequence of positive and negative numbers $x_1, x_2, .., x_N$ (with some uncertainties determined by the measuring device properties). Quantum mechanics does not predict the result of a single measurement, but guarantees $\langle x \rangle = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} x_i = 0$. For each measurement with the outcome $x_i \neq 0$ one could say that P-invariance is broken in this particular experiment, "event-by-event". In this simple case "breaking" is clearly of statistical origin and has nothing to do with dynamics - i.e. properties of the potential $V(x)$. Therefore it is common in quantum mechanics not to use such terminology and compute instead nonzero P-parity even observables, such as, e.g. $\langle x^2 \rangle = \int x^2 dx |\psi_0(x,t)|^2 \neq 0$, characterizing the pattern of quantum fluctuations.

According to common lore a measurement is a story about interaction between two quantum systems where the first ("object") has a few degrees of freedom while the second ("device") has a lot. As is well known the role of the device is often played just by the medium an object is immersed into. Interactions with the medium lead to decoherence and transition from quantum to classical fluctuations in the process of continuous measurement. It is important in this respect not to mix quantum and classical fluctuations. In quantum case all histories (field configurations in QFT context) coexist together and simultaneously. For classical fluctuations (statistical, thermal etc) one has some concrete random value of fluctuating observable at any given moment of time (and at a point in space). There exist various theoretical frameworks to describe quantum measurements in relativistic setting. We consider two in the following: point-like detectors (Unruh-DeWitt) and filter functions (quantum corridors).

2.1 Point-like detectors

The standard Unruh–DeWitt detector coupled to the current is described by the Hamiltonian:

$$H = \int_{-\infty}^{\infty} f(\tau) d\tau \, m(\tau) \bar{\psi}(x(\tau)) \Gamma \psi(x(\tau)) \tag{4}$$

Here $x(\tau)$ parameterizes the detector's world-line, τ is the proper time along it, $m(\tau)$ - internal quantum variable (monopole momentum) of the detector whose evolution in τ is described by the standard two-level Hamiltonian with the levels E_0 and E_1, $E_1 - E_0 = \omega > 0$. The Lorentz structure of the coupling is fixed by the matrix Γ, in what follows we concentrate on the vector case, $\Gamma = n_\mu \gamma^\mu$. The dimensionless "window function" $f(\tau)$ encodes the fact that any realistic measurement takes finite time (which we denote λ in this paper), so the window function has the important property $f(\tau \lesssim \lambda) \approx 1$, $f(\tau \gg \lambda) \to 0$. By convention the measurement window is located around the point $\tau = 0$. The interested reader is refereed to [21] for detailed discussion of the finite measurement time effects.

An amplitude for the detector to "click" is given by

$$\mathcal{A} = i \int_{-\infty}^{\infty} f(\tau) d\tau \, \langle 1|m(\tau)|0\rangle \cdot \langle \Omega | j(x(\tau)) | \Omega_0 \rangle \tag{5}$$

where $j(x(\tau)) = \bar{\psi}(x(\tau))\Gamma\psi(x(\tau))$ and $|\Omega_0\rangle$ stays for initial state of the field sub-system, while $|\Omega\rangle$ represents final (after the measurement) state. The probability is proportional to $|\mathcal{A}|^2$, and the corresponding response function reads:

$$\mathcal{F}(\omega) = \int\limits_{-\infty}^{\infty} f(\tau)d\tau \int\limits_{-\infty}^{\infty} f(\tau')d\tau'\, e^{-i\omega(\tau-\tau')} \cdot G^+(\tau-\tau') \tag{6}$$

where

$$G^+(\tau-\tau') = \langle\Omega_0|j(x(\tau))j(x(\tau'))|\Omega_0\rangle \tag{7}$$

For infinite measurement time, i.e. in the limit $f(\tau) \equiv 1$ one is interested in detector's excitation rate in unit time. It is determined by the power spectrum of the corresponding Wightman function:

$$\dot{\mathcal{F}}(\omega) = \int\limits_{-\infty}^{\infty} ds\, e^{-i\omega s}\, G^+(s) \tag{8}$$

In general case one has to work with the original expression (6).

First, we consider infinite measurement time case with $f(\tau) \equiv 1$. Usually in quantum measurements theory one compares response functions of a given detector in a state of inertial movement versus some non-inertial one. We are interested in another kind of asymmetry, namely between the detector oriented to measure current along the magnetic field direction and perpendicular to it. This choice is fixed by the vector $n_\mu = (0, \mathbf{n})$. With respect to its spatial movement the detector is supposed to be always at rest, so we can take $x(\tau) = (\tau, 0, 0, 0)$. Therefore it is convenient to switch to the coordinate space and as in (8) we denote $s = \tau - \tau'$. It is convenient to use the exact fermion propagator in external magnetic field given by [22]:

$$S(s) = \frac{is\gamma^0}{32\pi^2} \int\limits_0^{\infty} \frac{du}{u^3} \left(\frac{qBu}{\tan(qBu)} + \gamma^1\gamma^2 qBu \right) e^{-i\frac{s^2}{4u}} \tag{9}$$

where q stays for quark electric charge and we take $m = 0$.

The response function asymmetry given by $\delta\dot{\mathcal{F}}(\omega) = \dot{\mathcal{F}}_{33}(\omega) - (\dot{\mathcal{F}}_{11}(\omega) + \dot{\mathcal{F}}_{22}(\omega))/2$ is quadratic in B for all values of the magnetic field.[2] Explicitly, one gets:

$$G^+_{33}(s) = -\left[\frac{s}{16\pi^2} \int\limits_0^{\infty} \frac{du}{u^3} \frac{qBu}{\tan(qBu)} e^{-i\frac{s^2}{4u}} \right]^2 - \frac{(qB)^2}{16\pi^4 s^2} \tag{10}$$

$$G^+_{11}(s) = -\left[\frac{s}{16\pi^2} \int\limits_0^{\infty} \frac{du}{u^3} \frac{qBu}{\tan(qBu)} e^{-i\frac{s^2}{4u}} \right]^2 + \frac{(qB)^2}{16\pi^4 s^2} \tag{11}$$

and $G^+_{11}(s) = G^+_{22}(s)$. These results are exact for free fermions in external magnetic field in the massless limit.

To compute the response function one needs to take into account $s \to s - i\epsilon$ prescription corresponding to definition of the Wightman function (7) and switch on the temperature

[2] Notice that $\dot{\mathcal{F}}_{11}(\omega) = \dot{\mathcal{F}}_{22}(\omega)$ for our choice of the field along the third axis.

introducing sum over periodic shifts in imaginary time with $\beta = 1/kT$ and Fermi-Dirac statistics factor $(-1)^k$ for fermions:

$$\delta\dot{\mathcal{F}}(\omega) = -\frac{(qB)^2}{8\pi^4}\int\limits_{-\infty}^{+\infty} ds\, e^{-i\omega s}\left[\sum_{k=-\infty}^{+\infty}\frac{(-1)^k}{(s-i\epsilon+ik\beta)}\right]^2 \tag{12}$$

where we denoted $\delta\dot{\mathcal{F}}(\omega) = \dot{\mathcal{F}}_{33}(\omega) - \dot{\mathcal{F}}_{11}(\omega)$

Taking into account that $\sum\limits_{k=-\infty}^{\infty}\frac{(-1)^k}{x+ik} = \frac{\pi}{\sinh \pi x}$ and doing the integral with the help of residues one gets

$$\delta\dot{\mathcal{F}}(\omega) = \frac{(qB)^2}{4\pi^3}\frac{\omega}{e^{\beta\omega}-1} \tag{13}$$

Expression (13) is worth commenting. It is positive, which corresponds to the fact that the detector measuring the current along magnetic field clicks more often than measuring perpendicular currents. It is also worth noticing the change of statistics from Fermi-Dirac to Bose-Einstein - what is relevant is the statistic of operators whose fluctuations are being measured by the detector (Bose-currents in our case) and not the statistics of primary fluctuating fields.

The fact that current fluctuations are suppressed in perpendicular direction is obvious from general physics: the charged particle moving in the orthogonal plane is deflected by the magnetic field (or, using quantum mechanical language, confined to Landau levels). What is less obvious is that fluctuations along the field are enhanced (exactly by the same amount), since classically (i.e. neglecting spin effects) magnetic field has no influence on a charge moving in parallel direction. This enhancement is caused by spin interaction with the magnetic field.

At large magnetic fields

$$\frac{\dot{\mathcal{F}}_{33}(\omega) - \dot{\mathcal{F}}_{11}(\omega)}{\dot{\mathcal{F}}_{33}(\omega) + \dot{\mathcal{F}}_{11}(\omega)} \to 1 \tag{14}$$

This pattern is easy to understand in term of the corresponding energy-momentum tensor. Indeed, at zero magnetic field one has the standard thermal pressure for massless fermions

$$T_{11} = T_{22} = T_{33} = \frac{7\pi^2 T^4}{180} \tag{15}$$

which is isotropic. At large magnetic field, however, all the pressure is along the magnetic field and there is no pressure in orthogonal directions:

$$T_{11} = T_{22} \to 0 \;\; ; \;\; T_{33} = \frac{qBT^2}{12} \tag{16}$$

Now, let us study effects of finite measurement time. Again, we are to compute the main quantity of our interest: $\delta\mathcal{F}(\omega)$, the difference of the response functions corresponding to the currents parallel to the magnetic field to transverse ones. Roughly speaking, this quantity tells us which detector clicks more often (or, in other words, which one has more clicks for a given time interval): the detector measuring the currents along the field or the one oriented to measure the orthogonal currents. This difference is an integral of the following integrand:

$$\delta G^+(\tau-\tau') = \langle\Omega_0|j_3(x(\tau))j_3(x(\tau')) - j_1(x(\tau))j_1(x(\tau'))|\Omega_0\rangle =$$

$$= J^2 + \frac{(eB)^2}{8\pi^4}\frac{1}{(\tau-\tau')^2} \tag{17}$$

where J is given by (2). The first term corresponds to the usual CME current and due to its stationary nature it cannot be detected by stationary detector. The same is true for the second term, since

$$\int_{-\infty}^{\infty} ds\, e^{-i\omega s}\,\frac{1}{(s-i\xi)^2} = 0 \tag{18}$$

for $\omega > 0$. It this respect the situation at finite density is different from that at finite temperature. The thermal state has excitations of any desired energy (corresponding to poles along imaginary time axes in integral (8)), which can excite the detector. The finite density state is a state of the lowest energy which has to be excited "by hands" (i.e. by nontrivial $f(\tau)$) to get observable results.

Inserting (17) into (6) we get the following answer:

$$\delta\mathcal{F}(\omega) = \frac{(eB)^2}{4\pi^4}\left[\mu_5^2 I_0 - \frac{1}{2}I_2\right] \tag{19}$$

where

$$I_n = \int f(\tau)d\tau \int f(\tau')d\tau' e^{-i\omega(\tau-\tau')}\cdot\frac{1}{(\tau-\tau'-i\xi)^n} \tag{20}$$

Once again, the result (19) is exact in B, i.e. the asymmetry $\delta\mathcal{F}(\omega)$ gets no contributions from higher powers of magnetic field.

The integral (20) depends on typical measurement time λ encoded in $f(\tau)$ and also on dimensionless variables $\omega\lambda$ and ξ/λ. The infinitesimal parameter ξ from Wightman's prescription has physical meaning of inverse maximal particle energy which can be measured by the detector and which is necessary a finite quantity for any realistic detector. From another point of view, it can be seen as related to the finite size of the detector. The quantum measurement theory with finite measurement windows has physical sense and can be applicable only for $\lambda \gg \xi$. However to keep $\xi \neq 0$ is important to get formally correct $\lambda \to 0$ limit, i.e. when the detector is not switched on at all and should, correspondingly, return zero count.

The concrete form of (20) depends on a chosen detector switching time profile. In the problem in question it is natural to identify it with the time profile of the magnetic field itself. The latter was computed by many authors, for example, it was suggested in [23] to use the following Ansatz:

$$B(\tau) = \frac{B_0}{(1+(\tau/\lambda)^2)^{3/2}} \tag{21}$$

where B_0 and λ are functions of impact parameter and rapidity, whose typical scale for RHIC setup is given by $B_0 \sim 10^5$ MeV^2, $\lambda \sim 0.1$ Fm$/c$. It is of course not quite correct simply replace $B \to B(\tau)$ in (17) since the exact Green's function is written for time-independent B. But since our approach anyway depends on a detector's model, we find it is a reasonable approximation to use (21) as a profile for the window function $f(\tau)$. The result for (20) with $n = 0$ follows trivially:

$$I_0 = 4\omega^2\lambda^4\, K_1^2(\kappa) \tag{22}$$

where $\kappa = \omega\lambda$. The maximum of this function in κ (i.e. optimal measurement time) is reached at $\lambda \approx 1.33/\omega$. It is easy to see that both at $\lambda \to 0$ (no measurement at all) and $\lambda \to \infty$ (stationary measurement) I_0 vanishes.

We now can rewrite (19) as

$$\delta\mathcal{F}(\omega) = \frac{(eB)^2}{4\pi^4} I_0 \left[\mu_5^2 + \frac{1}{\lambda^2} g(\kappa)\right] \tag{23}$$

with the dimensionless function $g(\kappa)$ given by

$$g(\kappa) = \frac{\kappa^2}{2} \int\limits_1^\infty dy y^2 (y-1) \left(\frac{K_1(y\kappa)}{K_1(\kappa)}\right)^2 \tag{24}$$

We have put $\xi = 0$, assuming finiteness of κ.

The expression in square brackets in (23) can be called effective axial chemical potential. The physical meaning of it is quite transparent: by energy-momentum uncertainty relation the finite observation time λ makes quarks Fermi energies uncertain, Dirac sea becomes wavy, and these fluctuations provide the vector current excess along the magnetic field even if "bare" axial chemical potential is absent. While the concrete form of the functions I_0, $g(\kappa)$ depends, of course, on the chosen window function profile, the qualitative form of the result (23) is robust.

In quantitative terms, $g(\kappa \sim 1) \approx 0.15 \div 0.25$ and the model (23) with (24) corresponds to effective axial chemical potential of order of 1 GeV, even if "bare" $\mu_5 = 0$.

2.2 Filter functions

The formalism of quantum corridors introduced in seminal paper [24] allows to take a look at the problem of interest from another prospective. Imagine one is monitoring some **P**-odd observable for a given quantum system. Then it can lead to nonzero result for measurement of correlated **P**-odd quantity. The simplest way to see it is to use a language of decoherence functionals [24], [25] and path integral formalism. Generally, for some filter function $\alpha[\Phi]$ the amplitude is given by

$$\Psi[\alpha] = \int \mathcal{D}\Phi \, \alpha[\Phi] \, e^{iS[\Phi]} \tag{25}$$

To illustrate the point on quantum-mechanical example, consider three-dimensional system given by Lagrangian $L = \dot{q}^2 - V(q)$, where $q = (x, y, z)$ and the potential $V(q)$ is invariant under **P**-parity transformation: $V(q) = V(-q)$, but not invariant under separate reflections $x \to -x$ or $y \to -y$ or $z \to -z$. Suppose that an external observer is monitoring the y-coordinate continuously in time. To describe this situation in path integral representation one has to introduce quantum corridor:

$$\int \mathcal{D}y(t) \to \int \mathcal{D}y(t) \, \exp\left(-\kappa \int\limits_0^T (y(t) - \bar{y}(t))^2 dt\right) \tag{26}$$

where the corridor width is given by $\Delta y \propto (\kappa T)^{-1/2}$. Consequently, all amplitudes and correlators computed with (26) become dependent on the function $\bar{y}(t)$ (which has the meaning of continuous observation result) and quantum corridor width Δy. For example, for x-coordinate one would have

$$\langle x(T) \rangle = X[\bar{y}(t), \kappa] \tag{27}$$

where the functional $X[\bar{y}(t),\kappa]$ depends, generally speaking, on the function $\bar{y}(t)$ in all past moments of time and vanishes at the point $\kappa = 0$, corresponding to no measurement: $X[\bar{y}(t),0] = 0$. Its exact form of course depends on the potential $V(q)$ and is of no importance for us at the moment. What is crucial is the fact that monitoring **P**-odd quantity (coordinate y in our example) can result in nonzero quantum average for some other **P**-odd quantity (coordinate x), despite the interaction is still strictly **P**-even. One can say that **P**-parity is broken "event-by-event" by measuring apparatus.

Coming back to our discussion of CME in the previous sections it is clear that crucial missing ingredient is of course the fact that in strong interaction domain the singlet axial vector current is not conserved because of triangle nonabelian anomaly:

$$\partial^\nu j^5_\nu(x) = -\eta(x) = -\frac{g^2 N_f}{16\pi^2} G^a_{\alpha\beta}(x)\tilde{G}^{a\alpha\beta}(x) \tag{28}$$

We are interested to find common distribution for the vector current and some **P**-odd quantity, which we have chosen in this section to be the field $\eta(x)$ from (28). The corresponding amplitude reads:

$$\Psi[\lambda,\kappa] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \, e^{iS_{QCD}+i\int dx\lambda(x)n_\mu j^\mu(x)+i\int dx\kappa(x)\eta(x)} \tag{29}$$

The vector current is given by the standard expression $j_\mu = \bar{\psi}Q\gamma_\mu\psi$, where Q is quark electric charges diagonal matrix in flavor space. The closed-time-path functional is given by

$$e^{iW[\lambda,\kappa;\lambda',\kappa']} = \Psi[\lambda,\kappa]\Psi^*[\lambda',\kappa'] \tag{30}$$

and the mean current is

$$\langle n_\mu j^\mu(x)\rangle[\lambda,\kappa] = -i\,\frac{\delta}{\delta\lambda(x)} e^{iW[\lambda,\kappa;\lambda',\kappa']}\Big|_{\substack{\kappa=\kappa'\\ \lambda=\lambda'}} \tag{31}$$

It is a functional of **P**-even field $\lambda(x)$ and **P**-odd field $\kappa(x)$ in the same sense as $\langle x(T)\rangle$ from (27) is a functional of $\bar{y}(t)$.

It is easy to compute $\Psi[\lambda,\kappa]$ in Gaussian approximation. It reads:

$$\Psi_{Gauss}[\lambda,\kappa] = e^{\frac{i}{2}\int dp(\lambda(p),\kappa(p))D(p)(\lambda(-p),\kappa(-p))^T} \tag{32}$$

where

$$D(p) = \begin{pmatrix} \Pi(p) & \Delta(p) \\ \Delta(p) & \Pi^5(p) \end{pmatrix} \tag{33}$$

with the components

$$\Pi(p) = i\int dx\, e^{ipx}\langle T\{j_\mu(x)j_\nu(0)\}\rangle\, n^\mu n^\nu \tag{34}$$

$$\Pi^5(p) = i\int dx\, e^{ipx}\,\langle T\{\eta(x)\eta(0)\}\rangle \tag{35}$$

$$\Delta(p) = \frac{e^2}{2\pi^2} n^\mu p^\alpha \tilde{F}_{\alpha\mu}\cdot N_c \mathrm{Tr}Q^2 \tag{36}$$

The non-diagonal terms of the matrix $D(p)$ arise due to correlation of fluctuations of the quantities of opposite **P**-parity in external abelian field.

To make the above picture suitable for concrete computations let us take a model profile for the κ-field, corresponding to the measurement taking place inside 3-dimensional "decoherence volume" $V \sim R^3$ for the time period τ starting from the moment $t = 0$. We chose Gaussian Ansatz

$$\kappa(t, \mathbf{x}) = \kappa \cdot f(t, \mathbf{x}) = \kappa \cdot \exp\left(-\mathbf{x}^2/2R^2\right)\, \exp\left(-t^2/2\tau^2\right) \tag{37}$$

It leads to the following expression for the current parallel to magnetic field:

$$\langle j_3(t, \mathbf{x})\rangle[0, \kappa] = -\kappa B \cdot \left(\frac{N_c \mathrm{Tr} Q^2}{2\pi^2}\right) \cdot \left(\frac{t \cdot f(t, \mathbf{x})}{\tau^2}\right) \cdot e^{-\int dp\, \kappa(p) \Im\{\Pi^5(p)\} \kappa(-p)} \tag{38}$$

where we switched off the **P**-even filter ($\lambda = 0$), but has kept the **P**-odd one. The above expression is a generalization of (2). The current is linear both in κ and in B and vanishes being integrated over κ in symmetric limits. Notice that due to the form-factor the current flows only inside the volume (where the measurement has been done).

Expression (38) remarkably demonstrates two different faces of decoherence. On one hand, the mere existence of this quantum current is due to classical nature of the field κ. On the other hand, the last exponent in the right hand side of (38) is responsible for current damping due to decoherence. This is analogous to measuring electric current by applying external electromagnetic field. If the field is weak and/or momentum of its dominant Fourier modes k is small, linear response theory works well and one gets conductivity as proportionality coefficient between applied field and induced current via Green-Kubo formula. If however the quanta of the field have $k^2 > 4m^2$, they decay into real charged particles with the mass m instead of being absorbed by charged current carriers and the linear response picture is not valid anymore. Quantitatively this process is controlled by the discussed factor with the imaginary part of the corresponding polarization operator in the exponent.

3 Instead of conclusion

Besides many other gifts, Mikhail Igorevich Polikarpov had outstanding sense of humor. In particular, he was fond of photos by René Maltête (1930 - 2000) and often used them in his scientific or pedagogical talks to illustrate various physical ideas. One of his favorite pictures was the photo known as "La majorité". Another was the photo "Traces". He did the following trick with the latter photo. He covered by a sheet of paper the narrow strip on the left hand side of the picture in order to hide the two guys on the motorbikes. Of course, a person looking at the picture for the first time (often it was a new student, who had just come seeking for joining the group) was puzzled. Then the paper was removed, the one saw the bicyclists, got the point of the photo in a couple of seconds, and started smiling. — Look, — commented Mikhail Igorevich, — imagine you meet some situation in your personal or professional life, in business or in science, some situation which you cannot get. You just do not understand what happens, how it can be like that. You are really puzzled. Then it is useful to remember, that each situation in life has its boundary, and perhaps a key, a small piece of reality which puts all things at their right places and allows you to catch what is going on is just one inch beyond this boundary. Do not be afraid to look beyond boundaries!

Warm memories about this great man - the Scientist and the Teacher - will stay with people who knew him for ever, together with his unique lessons - the above "practical holography" one and many others.

Requiescat in pacem

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Figure 1. La majorité

Figure 2. Traces