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The $X(3872)$ as a charmonium state plus an extra component due to continuum coupling effects

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Abstract. We discuss the results of an unquenched quark model (UQM) calculation of the self energy corrections to charmonium spectrum due to the coupling to the meson-meson continuum. In our approach, the effects of quark-antiquark pairs are introduced explicitly into the QM through a $^3P_0$ quark-antiquark pair-creation mechanism. Our results are fitted to the experimental data, so that the sum of the bare and self energies gives the values of the physical masses of the states of interest. Our results for the charmonium spectrum are used to discuss the nature of the $X(3872)$ resonance.

1 Introduction

The quark model (QM) [1–15] reproduces accurately enough the general trend of observables such as the spectrum and the magnetic moments. See also Refs. [16–19]. Nevertheless, it neglects continuum coupling (or pair-creation) effects, that give rise to virtual $q\bar{q} - q\bar{q}$ $(qqq - q\bar{q})$ components in the hadron wave function and determine a shift of the physical mass with respect to the bare mass [20]. These effects have already been studied by several authors in the baryon [21–25] and meson [26–39] sectors. Some examples are the resonance $\Lambda(1405)$, strongly influenced by the nearby $\bar{K}N$ channel [40], and the $f_0(980)$, that behaves remarkably as a $K\bar{K}$ molecule [41]. The pair-creation effects are also important in the case of such observables as the contribution of the orbital angular momentum to the spin of the proton [42], the flavor asymmetry of the proton [43] and the strangeness content of the nucleon [44]. See also Ref. [45]. The loop corrections can be relevant to the study of the $X(3872)$ meson [46], whose quark structure is still unknown, because there are currently two possible interpretations for the meson: a weakly-bound $D\bar{D}^*$ molecule [35, 47–50] or a $c\bar{c}$ state [31, 38, 51, 52], both with $J^{PC} = 1^{++}$.

In this contribution, we present our results for the charmonium spectrum with self energies corrections [38], computed within the UQM formalism for mesons [37–39]; the bare masses are calculated within Godfrey and Isgur's relativized QM of Ref. [4]. We use our results for the $c\bar{c}$ spectrum to discuss the nature of the $X(3872)$ resonance that, according to us, is compatible with the meson $\chi_{c1}(2^3P_1)$, with $J^{PC} = 1^{++}$. Finally, we present some results for the strong decay widths of charmonium $3S$, $2P$, $1D$ and $2D$ states [38], calculated within a modified version of the $^3P_0$ pair-creation model [37–39]. This is important to set the values of the $^3P_0$ model's parameters, that are necessary to compute the vertex function of the UQM.

2 Self energies in the UQM

We consider the UQM Hamiltonian,

$$H = H_0 + V , \tag{1}$$

where H_0 acts only in the bare meson space and V couples a meson state $|A\rangle$ to the meson-meson continuum $|BC\rangle$. The UQM approach is based on a QM, to which $q\bar{q}$ pairs with vacuum quantum numbers are added as a perturbation and where the pair-creation mechanism is inserted at the quark level [37–39, 42–44]. This is a generalization of the unitarized quark model by Törnqvist and Zenczykowski [21] (see also Ref. [53]).

The dispersive equation, resulting from a nonrelativistic Schrödinger equation, is

$$\Sigma(E_a) = \sum_{BC} \int_0^\infty q^2 dq \, \frac{\left|V_{a,bc}(q)\right|^2}{E_a - E_{bc}} \tag{2}$$

and the bare energy E_a satisfies:

$$M_a = E_a + \Sigma(E_a) . \tag{3}$$

Here, M_a is the physical mass of the meson A, with self energy $\Sigma(E_a)$; the coupling $V_{a,bc}$ between $|A\rangle$ and $|BC\rangle$ is computed within the UQM for mesons [37–39]:

$$V_{a,bc}(q) = \sum_{\ell J} \langle BC\vec{q}\,\ell J \mid T^\dagger \mid A\rangle . \tag{4}$$

The symbol $T^\dagger$ is a modified $^3P_0$ quark-antiquark pair-creation operator [37–39, 54, 55], where we substituted the $^3P_0$ model pair-creation strength γ_0 with an effective

Table 1. Parameters of the $^3P_0$ model from Ref. [38].

Parameter	Value
γ_0	0.510
α	0.500 GeV
r_q	0.335 fm
m_n	0.330 GeV
m_s	0.550 GeV
m_c	1.50 GeV

one, $\gamma_0^{\rm eff}$ [30, 37–39], and introduced a Gaussian quark form factor [37–39, 42–44, 53, 56]. A is the meson, B and C are the intermediate state mesons, with energies E_a, $E_b = \sqrt{M_b^2 + q^2}$ and $E_c = \sqrt{M_c^2 + q^2}$, $\vec{q}$ and ℓ the relative radial momentum and orbital angular momentum between B and C and $\vec{J} = \vec{J}_b + \vec{J}_c + \vec{\ell}$ is the total angular momentum. The wave functions of the mesons A, B and C are written as harmonic oscillator wave functions, depending on a single oscillator parameter $\alpha = 0.5$ GeV. The values of the pair-creation model's parameters, used to compute the strong decays of Sec. 3 and the vertices $\langle BC\vec{q}\,\ell J \right| T^\dagger \left| A \rangle$ of Eq. (4), are reported in Table 1. The value of γ_0 is fitted to the reproduction of the experimental data of Table 2.

3 Open charm strong decays in the $^3P_0$ pair-creation model

Here, we discuss our results for the open charm strong decay widths of $c\bar{c}$ mesons (see table 2) [38]. The decay widths are computed as [37–39, 54, 58, 59]

$$\Gamma_{A\to BC} = \Phi_{A\to BC}(q_0) \sum_{\ell,J} \left| \langle BC\vec{q}_0\,\ell J \right| T^\dagger \left| A \rangle \right|^2 , \qquad (5)$$

where $\Phi_{A\to BC}(q_0)$ is the standard relativistic phase space factor [37–39, 54, 58, 59],

$$\Phi_{A\to BC} = 2\pi q_0 \frac{E_b(q_0)E_c(q_0)}{M_a} , \qquad (6)$$

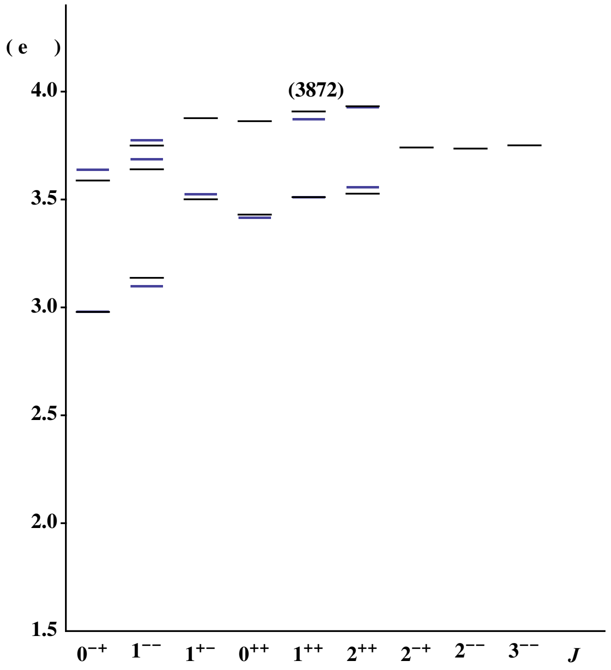
that depends on the relative momentum q_0 between B and C and on the energies of the two intermediate state mesons, $E_b = \sqrt{M_b^2 + q_0^2}$ and $E_c = \sqrt{M_c^2 + q_0^2}$.

Finally, the results of our calculation, obtained with the values of the $^3P_0$ model parameters of Table 1, are reported in Table 2.

4 Bare and self energy calculation of $c\bar{c}$ states

Here, we discuss our results of Ref. [38] for the $c\bar{c}$ spectrum with self energy corrections. The bare energies E_a's are computed within the relativized QM [4], with the values of the model parameters of Table 3, and the self energies within the UQM for mesons [37–39]. At variance with QM calculations, such as those of Ref. [4] and Table 2, second column, the quantities fitted to the experimental data [57] are the physical masses M_a's of Eq. (3), instead of the bare energies E_a's, and therefore the fitting procedure is an iterative one.

Figure 1. Comparison between the calculated masses (black lines) of $c\bar{c}$ states via Eq. (3) [38] and the experimental ones [57] (boxes). The bare masses are calculated with the values of Godfrey and Isgur's model parameters of Table 3.



The self energies $\Sigma(E_a)$'s of Eq. (2) are calculated summing over a complete set of accessible $\mathrm{SU_f(4)}\otimes\mathrm{SU_{spin}(2)}$ $1S$ intermediate states. If the bare energy of the initial meson A is above the threshold BC, the self energy correction due to the channel BC is given by

$$\begin{aligned} \Sigma(E_a)(BC) &= \mathcal{P} \int_{M_b+M_c}^{\infty} \frac{dE_{bc}}{E_a - E_{bc}} \frac{qE_bE_c}{E_{bc}} \left| \langle BC\vec{q}\,\ell J \right| T^\dagger \left| A \rangle \right|^2 \\ &+ 2\pi i \left\{ \frac{qE_bE_c}{E_a} \left| \langle BC\vec{q}\,\ell J \right| T^\dagger \left| A \rangle \right|^2 \right\}_{E_{bc}=E_a} , \end{aligned} \qquad (7)$$

where the symbol $\mathcal{P}$ stands for a principal part integral and $2\pi i \left\{ \frac{qE_bE_c}{E_a} \left| \langle BC\vec{q}\,\ell J \right| T^\dagger \left| A \rangle \right|^2 \right\}_{E_{bc}=E_a}$ is the imaginary part of the self energy.

Finally, the results of our calculation, obtained with the set of parameters of Table 1, are given in Fig. 1 [38].

5 Nature of the $X(3872)$ resonance

The quark structure of the $X(3872)$ resonance, discovered by Belle [46] and later confirmed by CDF [60], D0 [61] and BABAR [62], still remains an open puzzle. Indeed, at the moment, there are two possible interpretations for the meson: a weakly bound molecule [35, 47–50] or a charmonium state [31, 38, 51, 52], both with $J^{PC} = 1^{++}$ quantum numbers. Recently, LHCb [63] has ruled out the $J^{PC} = 2^{-+}$ hypothesis.

The first possibility is to consider the $X(3872)$ as a $\chi_{c1}(2^3P_1)$ resonance [52]. Nevertheless, the relativized

Table 2. Open charm strong decay widths (in MeV) of $3S$, $2P$, $1D$ and $2D$ $c\bar{c}$ states [38]. The values of the model parameters are given in Table 1. The symbol – in the table means that a certain decay is forbidden by selection rules or that the decay cannot take place because it is below threshold. The experimental values of the meson masses (if available) are taken from the PDG [57], otherwise from the theoretical predictions of Ref. [4].

State	$D\bar{D}$	$D\bar{D}^*$ $\bar{D}D^*$	$D^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$ $\bar{D}_sD_s^*$	$D_s^*\bar{D}_s^*$	Total	Exp.
$\eta_c(3^1S_0)$	–	38.8	52.3	–	–	–	91.1	–
$\Psi(4040)(3^3S_1)$	0.2	37.2	39.6	3.3	–	–	80.3	80 ± 10
$h_c(2^1P_1)$	–	64.6	–	–	–	–	64.6	–
$\chi_{c0}(2^3P_0)$	97.7	–	–	–	–	–	97.7	–
$\chi_{c2}(2^3P_2)$	27.2	9.8	–	–	–	–	37.0	–
$\Psi(3770)(1^3D_1)$	27.7	–	–	–	–	–	27.7	27.2 ± 1.0
$\Psi_3(1^3D_3)$	1.7	–	–	–	–	–	1.7	–
$\eta_{c2}(2^1D_2)$	–	62.7	46.4	–	8.8	–	117.9	–
$\Psi(4160)(2^3D_1)$	11.2	0.4	39.4	2.1	5.6	–	58.7	103 ± 8
$\Psi_2(2^3D_2)$	–	43.5	49.3	–	11.3	–	104.1	–
$\Psi_3(2^3D_3)$	17.2	58.3	48.1	3.6	2.6	–	129.8	–

Table 3. Values of Godfrey and Isgur's model parameters, obtained by fitting the results of Eq. (3) to the experimental data [57]. Table from Ref. [38].

m_c	= 1.562 GeV	b	= 0.1477 GeV^2	α_s^{cr}	= 0.600
Λ	= 0.200 GeV	c	= 0.069 GeV	σ_0	= 1.463 GeV
s	= 2.437	ϵ_c	= −0.2500	ϵ_t	= 0.0300
$\epsilon_{so(V)}$	= −0.0314	$\epsilon_{so(S)}$	= 0.0637		

QM [4] predicts this state to be at energy of 3.95 GeV, almost 80 MeV higher than $X(3872)$'s mass. Thus, our idea is thus to see whether the introduction of loop corrections into the QM can help to improve this result for the $X(3872)$, whose unusual properties are due to its proximity to the $D\bar{D}^*$ decay threshold.

In our calculation of Ref. [38], our result for the mass of the $\chi_{c1}(2^3P_1)$ state, i.e. 3.908 GeV, is compatible with the meson $X(3872)$, that includes an extra component due to the coupling to the meson-meson continuum, responsible for the downward energy shift. The difference between our prediction and the experimental data is within the error of a QM calculation, of the order of 30 − 40 MeV. In Table 4, our UCQM result for $\chi_{c1}(2P)$'s mass is compared to those obtained by other authors, introducing continuum coupling effects in their calculations.

Table 4. Our UCQM result for the mass of the $\chi_{c1}(2^3P_1)$ meson, that in our picture corresponds to the $X(3872)$, is compared to those of other calculations.

$\chi_{c1}(2^3P_1)$'s mass [MeV]	Reference
4007.5	[28]
3990	[30]
3896	[31]
3908	[38]
3920.5	[64]

In Refs. [28, 64], the authors calculated the charmonium spectrum with continuum coupling effects. They computed the bare masses within the Cornell potential and the continuum coupling effects with a refined version of the Cornell coupled-channel model. Their result for the mass of the $\chi_{c1}(2^3P_1)$ meson seems incompatible with this interpretation for the $X(3872)$; on the contrary, the authors suggest the assignments 1^3D_2 and 1^3D_3 for the $X(3872)$.

In Ref. [30], the author calculated the spectrum of $c\bar{c}$ mesons up to $2P$ states, considering the effects of open-charm loops on charmonium masses. The bare energies were computed within the standard non-relativistic potential model and the vertices within a $^3P_0$-type model. Their result for $\chi_{c1}(2^3P_1)$'s mass, i.e. 3990 MeV, seems incompatible with $X(3872)$'s experimental mass.

In Ref. [31], the authors computed the self energy corrections to the $c\bar{c}$ spectrum, including the effects of open and nearby closed meson-meson channels. The authors used the results of Ref. [59] for the bare masses and computed the self energy corrections within an approach related to the Dyson summation for the inverse meson propagator. Their result for the mass of the $\chi_{c1}(2^3P_1)$ seems compatible with the interpretation of the $X(3872)$ as a 2^3P_1 $c\bar{c}$ state.

The second possibility is to treat the $X(3872)$ as a $D\bar{D}^*$ molecular state, with $J^{PC} = 1^{++}$ [35, 47]. According to Refs. [65], the $D\bar{D}^*$ system with $J^{PC} = 1^{++}$ can be found by pion exchange and forms a meson-meson molecule. More recent molecular model calculations [66], that include quark exchange kernels for the transitions $D\bar{D}^* \to \rho J/\Psi$, $\omega J/\Psi$, to predict the $\omega J/\Psi$ decay mode of the $X(3872)$ [49], introduce large isospin mixing due to the mass difference between $D^0\bar{D}^{*0}$ and $D^+\bar{D}^{*-}$. Nevertheless, in Ref. [47] the authors state that the one-pion exchange binding mechanism should be taken with greater caution in the $D\bar{D}^*$ case than in the NN case (see also Refs. [51, 67, 68]).

Perhaps, the most important test for the properties of the $X(3872)$ is to calculate its strong and radiative decay rates [49, 51, 52]. Ref. [52] re-examines the re-scattering mechanism for the X(3872): its results for the ratio $R_{\rho/\omega} \approx 1$, between the $X(3872) \to J/\psi\rho$ and $X(3872) \to J/\psi\omega$ decay modes, and for the rate $X(3872) \to D^0\bar{D}^0\pi^0$, favor a charmonium interpretation for the $X(3872)$. Ref. [51] observes that the binding mechanism and the production rates are incompatible with the molecule interpretation. However, these results have been criticized in several works [67, 69–73]. In particular, Ref. [71] observes that the production rates in the molecular interpretation are compatible with Tevatron data once the charm-meson rescattering effects are considered. In Refs. [74, 75], the authors observe also prompt production at CDF and discuss whether a meson-meson molecule, with a size of a few fm and intrinsic fragility, can be promptly produced. By contrast, Refs. [35, 47–49] propose a molecular interpretation for the $X(3872)$.

In conclusion, we do not think that our previous arguments can, on their own, clarify the picture of the $X(3872)$ resonance completely. Analysis of other properties of the $X(3872)$, such as decay modes, is necessary to draw definitive conclusions.

6 Discussion of the results

In this contribution, we discussed the results of an unquenched quark model calculation of the self energies corrections to the spectrum of charmonia [38]. The self energies are corrections to the meson masses arising from the coupling to the meson-meson continuum.

Although the self energy corrections to the spectrum of $b\bar{b}$ [39] and $c\bar{c}$ [38] mesons are relatively small, approximately 1−2% and 2−6% of the corresponding meson mass, respectively, these continuum-coupling effects can become qualitatively important in the case of suspected non $q\bar{q}$ states, such as the $X(3872)$ [46] or $\chi_b(3P)$ mesons [76, 77], that are states close to a meson-meson decay threshold. In particular, in Ref. [38] it is shown that the continuum coupling effects of the $X(3872)$ give rise to $D\bar{D}^*$ and $D^*\bar{D}^*$ components in addition to the $c\bar{c}$ core.

In conclusion, our results of Ref. [38], that we analysed in this contribution, seem compatible with the $\chi_{c1}(2^3P_1)$ interpretation for the $X(3872)$.

References

[1] E. Eichten *et al.*, Phys. Rev. Lett. **34**, 369 (1975); E. Eichten *et al.*, Phys. Rev. D **17**, 3090 (1978); **21**, 203 (1980).

[2] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978); **19**, 2653 (1979); **20**, 1191 (1979).

[3] W. Buchmuller and S. H. H. Tye, Phys. Rev. D **24**, 132 (1981).

[4] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[5] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).

[6] F. Iachello, N. C. Mukhopadhyay and L. Zhang, Phys. Rev. D **44**, 898 (1991).

[7] W. Lucha, F. F. Schoberl and D. Gromes, Phys. Rept. **200**, 127 (1991).

[8] R. Bijker, F. Iachello and A. Leviatan, Annals Phys. **236**, 69 (1994).

[9] M. M. Giannini and E. Santopinto, Phys. Rev. C **49**, 1258 (1994); M. Ferraris, M. M. Giannini, M. Pizzo, E. Santopinto and L. Tiator, Phys. Lett. B **364**, 231 (1995); E. Santopinto, F. Iachello and M. M. Giannini, Nucl. Phys. A **623**, 100C (1997); Eur. Phys. J. A **1**, 307 (1998); M. Aiello, M. M. Giannini and E. Santopinto, J. Phys. G **24**, 753 (1998); R. Bijker, F. Iachello and E. Santopinto, J. Phys. A **31**, 9041 (1998); M. D. Sanctis, M. M. Giannini, L. Repetto and E. Santopinto, Phys. Rev. C **62**, 025208 (2000); M. M. Giannini, E. Santopinto and A. Vassallo, Eur. Phys. J. A **12**, 447 (2001); Nucl. Phys. A **699**, 308 (2002); L. Tiator, D. Drechsel, S. Kamalov, M. M. Giannini, E. Santopinto and A. Vassallo, Eur. Phys. J. A **19**, 55 (2004); M. Gorchtein, D. Drechsel, M. M. Giannini, E. Santopinto and L. Tiator, Phys. Rev. C **70**, 055202 (2004); M. De Sanctis, M. M. Giannini, E. Santopinto and A. Vassallo, Eur. Phys. J. A **19**, 81 (2004); Nucl. Phys. A **755**, 294 (2005); Phys. Rev. C **76**, 062201 (2007); E. Santopinto and M. M. Giannini, Phys. Rev. C **86**, 065202 (2012).

[10] L. Y. Glozman, D. O. Riska, Phys. Rept. **268**, 263 (1996).

[11] U. Loring, B. C. Metsch and H. R. Petry, Eur. Phys. J. A **10**, 395 (2001).

[12] R. Bijker, M. M. Giannini and E. Santopinto, Rev. Mex. Fis. **50S2**, 88 (2004); Phys. Lett. B **595**, 260 (2004); Eur. Phys. J. A **22**, 319 (2004).

[13] E. Santopinto, Phys. Rev. C **72**, 022201 (2005); J. Ferretti, A. Vassallo and E. Santopinto, Phys. Rev. C **83**, 065204 (2011); M. De Sanctis, J. Ferretti, E. Santopinto and A. Vassallo, Phys. Rev. C **84**, 055201 (2011); AIP Conf. Proc. **1488**, 280 (2012).

[14] E. Santopinto and G. Galatá, Phys. Rev. C **75**, 045206 (2007); G. Galatá and E. Santopinto, Phys. Rev. C **86**, 045202 (2012).

[15] P. Colangelo, F. De Fazio, F. Giannuzzi and S. Nicotri, Phys. Rev. D **86**, 054024 (2012).

[16] P. Guo, A. P. Szczepaniak, G. Galata, A. Vassallo and E. Santopinto, Phys. Rev. D **77**, 056005 (2008); Phys. Rev. D **78**, 056003 (2008).

[17] S. Capstick, A. Svarc, L. Tiator, J. Gegelia, M. M. Giannini, E. Santopinto, C. Hanhart and S. Scherer *et al.*, Eur. Phys. J. A **35**, 253 (2008).

[18] A. Ostrander, E. Santopinto, A. P. Szczepaniak and A. Vassallo, Phys. Rev. D **86**, 114015 (2012).

[19] I. G. Aznauryan *et al.*, Int. J. Mod. Phys. E **22**, 1330015 (2013).

[20] C. Patrignani, T. K. Pedlar and J. L. Rosner, arXiv:1212.6552 [hep-ex].

[21] N. A. Törnqvist and P. Zenczykowski, Phys. Rev. D **29**, 2139 (1984); Z. Phys. C **30**, 83 (1986);

P. Zenczykowski, Annals Phys. **169**, 453 (1986).

[22] K. G. Horacsek, Y. Iwamura and Y. Nogami, Phys. Rev. D **32**, 3001 (1985).

[23] W. Blask, M. G. Huber and B. Metsch, Z. Phys. A **326**, 413 (1987).

[24] M. Brack and R. K. Bhaduri, Phys. Rev. D **35**, 3451 (1987).

[25] B. Silvestre-Brac and C. Gignoux, Phys. Rev. D **43**, 3699 (1991).

[26] S. Ono and N. A. Törnqvist, Z. Phys. C **23**, 59 (1984); K. Heikkila *et al.*, Phys. Rev. D **29**, 110 (1984); S. Ono *et al.*, Phys. Rev. D **34**, 186 (1986).

[27] E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003); E. van Beveren *et al.*, Phys. Rev. D **74**, 037501 (2006); G. Rupp and E. van Beveren, Eur. Phys. J. A **31**, 698 (2007).

[28] E. J. Eichten, K. Lane and C. Quigg, Phys. Rev. D **69**, 094019 (2004).

[29] D. S. Hwang and D. -W. Kim, Phys. Lett. B **601**, 137 (2004).

[30] Y. S. Kalashnikova, Phys. Rev. D **72**, 034010 (2005).

[31] M. R. Pennington and D. J. Wilson, Phys. Rev. D **76**, 077502 (2007).

[32] T. Barnes and E. S. Swanson, Phys. Rev. C **77**, 055206 (2008).

[33] B. -Q. Li and K. -T. Chao, Phys. Rev. D **79**, 094004 (2009).

[34] I. V. Danilkin and Y. .A. Simonov, Phys. Rev. D **81**, 074027 (2010).

[35] I. V. Danilkin and Yu. A. Simonov, Phys. Rev. Lett. **105**, 102002 (2010).

[36] J. -F. Liu and G. -J. Ding, Eur. Phys. J. C **72**, 1981 (2012).

[37] J. Ferretti, G. Galatá, E. Santopinto and A. Vassallo, Phys. Rev. C **86**, 015204 (2012).

[38] J. Ferretti, G. Galatá and E. Santopinto, Phys. Rev. C **88**, 015207 (2013).

[39] J. Ferretti and E. Santopinto, arXiv:1306.2874 [hep-ph].

[40] R. H. Dalitz and S. F. Tuan, Phys. Rev. Lett. **2**, 425 (1959).

[41] S. M. Flatte *et al.*, Phys. Lett. B **38**, 232 (1972).

[42] E. Santopinto and R. Bijker, Few Body Syst. **44**, 95 (2008); R. Bijker and E. Santopinto, Phys. Rev. C **80**, 065210 (2009).

[43] E. Santopinto and R. Bijker, Phys. Rev. C **82**, 062202 (2010).

[44] R. Bijker, J. Ferretti and E. Santopinto, Phys. Rev. C **85**, 035204 (2012).

[45] E. Santopinto, R. Bijker and J. Ferretti, Few Body Syst. **50**, 199 (2011); E. Santopinto and R. Bijker, Few Body Syst. **54**, 761 (2013).

[46] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003); D. Acosta *et al.* [CDF II Collaboration], Phys. Rev. Lett. **93**, 072001 (2004).

[47] C. Hanhart *et al.*, Phys. Rev. D **76**, 034007 (2007).

[48] V. Baru *et al.*, Phys. Rev. D **84**, 074029 (2011).

[49] E. S. Swanson, Phys. Lett. B **588**, 189 (2004); **598**, 197 (2004).

[50] F. Aceti, W. H. Liang, E. Oset, J. J. Wu and B. S. Zou, Phys. Rev. D **86**, 114007 (2012).

[51] M. Suzuki, Phys. Rev. D **72**, 114013 (2005).

[52] C. Meng and K.-T. Chao, Phys. Rev. D **75**, 114002 (2007).

[53] P. Geiger and N. Isgur, Phys. Rev. D **55**, 299 (1997).

[54] L. Micu, Nucl. Phys. B **10**, 521 (1969); A. Le Yaouanc *et al.*, Phys. Rev. D **8**, 2223 (1973); **9**, 1415 (1974); Phys. Lett. B **71**, 397 (1977); **72**, 57 (1977).

[55] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).

[56] P. Geiger and N. Isgur, Phys. Rev. Lett. **67**, 1066 (1991); Phys. Rev. D **44**, 799 (1991); **47**, 5050 (1993).

[57] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012).

[58] E. S. Ackleh, T. Barnes and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996).

[59] T. Barnes, S. Godfrey and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).

[60] D. Acosta *et al.* [CDF Collaboration], Phys. Rev. Lett. **93**, 072001 (2004).

[61] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **93**, 162002 (2004).

[62] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **71**, 071103 (2005).

[63] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **110**, 222001 (2013).

[64] E. J. Eichten, K. Lane and C. Quigg, Phys. Rev. D **73**, 014014 (2006).

[65] M. B. Voloshin and L. B. Okun, JETP Lett. **23**, 333 (1976) [Pisma Zh. Eksp. Teor. Fiz. **23**, 369 (1976)]; A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. Lett. **38**, 317 (1977); N. A. Tornqvist, Phys. Rev. Lett. **67**, 556 (1991).

[66] N. A. Tornqvist, Phys. Lett. B **590**, 209 (2004).

[67] E. Braaten, M. Lu and J. Lee, Phys. Rev. D **76**, 054010 (2007).

[68] S. Fleming, M. Kusunoki, T. Mehen and U. van Kolck, Phys. Rev. D **76**, 034006 (2007).

[69] C. M. Zanetti, M. Nielsen and R. D. Matheus, Phys. Lett. B **702**, 359 (2011).

[70] M. Nielsen and C. M. Zanetti, Phys. Rev. D **82**, 116002 (2010).

[71] P. Artoisenet and E. Braaten, Phys. Rev. D **81**, 114018 (2010) [arXiv:0911.2016 [hep-ph]].

[72] C. E. Thomas and F. E. Close, Phys. Rev. D **78**, 034007 (2008).

[73] E. S. Swanson, Phys. Rept. **429**, 243 (2006).

[74] C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa and C. Sabelli, Phys. Rev. Lett. **103**, 162001 (2009).

[75] G. Bauer [CDF Collaboration], Int. J. Mod. Phys. A **20**, 3765 (2005).

[76] G. Aad *et al.* [ATLAS Collaboration], Phys. Rev. Lett. **108**, 152001 (2012).

[77] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **86**, 031103 (2012).