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Gluonic excitations in the hadronic spectrum

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Abstract. I describe progress made by the **Hadron Spectrum Collaboration** in understanding the role that excitations of the gluonic field play in determining the spectrum of hadrons. This is achieved through calculations of the hadron spectrum using lattice QCD techniques. Clear signals for the lightest set of *hybrid* mesons and baryons are presented, and a proposal that they correspond to the same *chromomagnetic* excitation is made. The prospects for studying the decay properties of excited hadrons are briefly discussed.

1. Introduction

Hadron spectroscopy is commonly expressed in terms of an established picture known as the constituent quark model in which the systematics of mesons and baryons are described in terms of $q\bar{q}$ and qqq constructions. Such a picture describes the experimentally observed restriction of small isospin and strangeness and the gross structure of the energy spectrum of observed hadrons. On the other hand, Quantum Chromodynamics (QCD), the gauge field theory of quarks and gluons is the accepted fundamental theory, having been tested extensively at high energies where perturbation theory can be utilized. Some aspects of the relationship between non-perturbative QCD at low energy and the spectrum of hadrons are understood, while others remain obscure. For example, QCD with very light quark masses has an approximate chiral symmetry, and it is well know that this symmetry is dynamically broken giving the pion its very low mass and weak low-energy interactions. This same physics presumably also produces the heavier "effective" (constituent) mass of the quarks at low momentum scales. This aspect of strong coupling in QCD is clearly apparent in the spectrum of hadrons.

What is not clear is the role that the strong coupling within the gluon sector has on the spectrum of hadrons. One would expect there to be *glueballs*: color-singlet configurations built purely from gluonic fields (as are present in calculations in the pure-glue SU(3) Yang-Mills theory [1]), and *hybrid* mesons and baryons in which an excited gluonic field configuration couples to quarks. Experimentally we do not have a clear identification of a spectrum of glueballs and hybrid hadrons.

There is a long history of QCD-motivated model suggestions for the form that gluonic excitations might take and for the corresponding spectrum of hybrid mesons and baryons (see [2] for a review). These make different predictions for the spectra, but a common theme within them is the presence

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of *exotic* J^{PC} meson states, that is J^{PC} in the set $0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$ that cannot be constructed from solely a fermion-antifermion Fock state. Such exotic J^{PC} can occur for hybrid mesons due to the addition of quantum numbers provided by the gluonic excitation.

Exotic J^{PC} are particularly attractive targets for experiments – partial-wave analysis of multi-hadron final states projects angular dependences into amplitudes of definite J^{PC}, which can be examined separately for resonant behavior. A resonance appearing in an exotic J^{PC} amplitude is a clear signal for a state beyond the simple $q\bar{q}$ picture of mesons, and a hybrid meson is a likely explanation.

In this proceedings I will describe recent progress in determining the role of gluonic excitations in the spectrum of mesons and baryons using a first-principles numerical approach to QCD known as lattice QCD. In lattice QCD the non-perturbative field theory is considered on a finite discrete grid of points. Correlation functions can be written via the path integral formalism in terms of a sum over possible gluon field configurations. These configurations are generated using Monte-Carlo methods allowing the correlation functions to be determined numerically.

2. Obtaining a spectrum of excited hadrons from lattice QCD

The spectrum of eigenstates of the QCD Hamiltonian can be extracted from hadron two-point correlation functions in Euclidean time,

$$C_{ij}(t) = \langle 0|\, \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0)\, |0\rangle = \sum\nolimits_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t}\, \langle 0|\, \mathcal{O}_i\, |\mathfrak{n}\rangle\, \langle \mathfrak{n}|\, \mathcal{O}_j^\dagger\, |0\rangle, \tag{1}$$

where $\mathcal{O}_i$ are color-singlet operators with the quantum numbers of the desired eigenstates, constructed from of the basic quark and gluon fields.

A powerful technique to extract a spectrum of *excited* states is to work with a large basis of operators – presumably each excited state is interpolated optimally from the vacuum by some linear combination of the basis operators: $\Omega_{\mathfrak{n}}^\dagger\, |0\rangle \approx \delta_{\mathfrak{n},\mathfrak{m}}\, |\mathfrak{m}\rangle$, with $\Omega_{\mathfrak{n}}^\dagger = \sum_i v_i^{\mathfrak{n}}\, \mathcal{O}_i^\dagger$. The "best" linear combinations (in a variational sense) can be obtained by solving a linear system of equations featuring the *matrix* of correlation functions $(C_{ij}(t))$,

$$C(t)\, v^{\mathfrak{n}} = \lambda^{\mathfrak{n}}(t)\, C(t_0)\, v^{\mathfrak{n}}, \tag{2}$$

where the eigenvalues, $\lambda^{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$, whose time-dependence can be fitted to determine the spectrum, $E_{\mathfrak{n}}$.

The **Hadron Spectrum Collaboration** has extracted excited meson spectra using an operator basis of the following form:

$$\mathcal{O} \sim \overline{\psi}\, \Gamma\, \overleftrightarrow{D} \ldots \overleftrightarrow{D}\, \psi, \tag{3}$$

where the gauge-covariant derivatives feature the gluon field in the usual way. The vector-like derivatives and Dirac matrix can be combined together to produce operators of definite J^{PC} – these operators are then *subduced* into irreducible representations of the cubic symmetry of the lattice. Using up to three derivatives, the basis contains operators for all J^{PC} with $J \leq 4$, with each symmetry channel typically containing a significant number of independent operators (e.g. there are 19 1^{--} operators).

In Fig. 1 I show the extracted spectrum of isovector mesons in a calculation performed on a $24^3 \times 128$ lattice (roughly $(3\,\mathrm{fm})^3$) with light quark masses set such that the pion has a mass of 391 MeV. Although technically the spectrum should be expressed in terms of irreducible representations of the cubic lattice symmetry, we find that our correlators are very close to diagonal in spin J and hence are justified in plotting the spectrum by J^{PC}, see [3] for details.

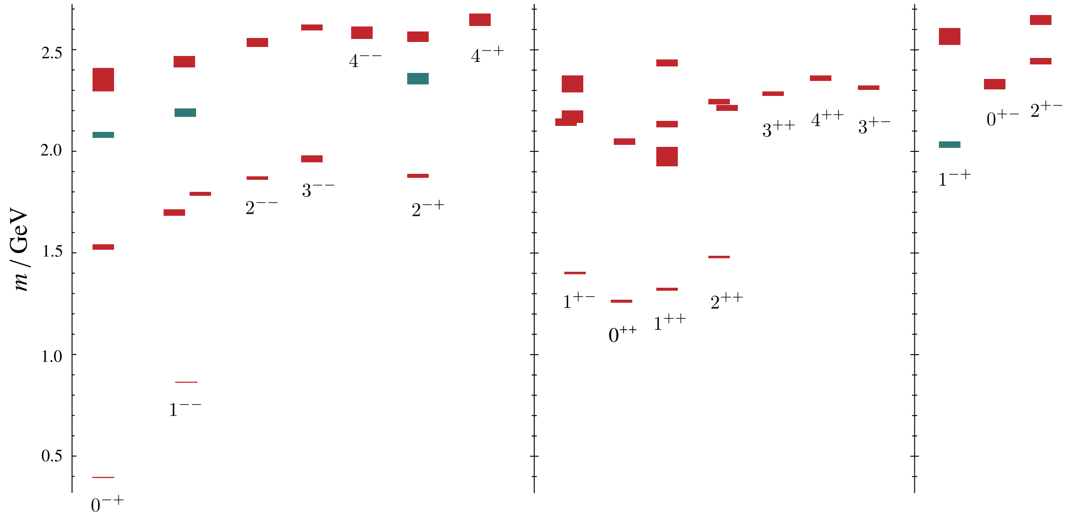

Figure 1. Spectrum of isovector meson states, plotted by J^{PC}. Calculation performed with quark masses such that $m_\pi = 391\,\text{MeV}$[4]. States highlighted in blue are discussed in the text as being candidates for the lightest hybrid meson supermultiplet.

3. Interpretation of the meson spectrum

Despite using a light quark mass significantly larger than the true physical value, the spectrum in Fig. 1 reproduces many of the features of the experimental meson spectrum, and the corresponding quark-model systematics. The "anomalously" light pion appears below a spectrum of states which can be characterized as $q\bar{q}\ (n\,^{2S+1}L_J)$ with mass scale increasing with increasing L. In the rightmost column we observe a clear spectrum of exotic J^{PC} states with a lightest 1^{-+}. It is our proposal that these exotic states are hybrid mesons.

Returning to the non exotic J^{PC} sector, slightly above 2 GeV, there appear three states with $J^{PC} = 0^{-+}, 1^{--}, 2^{-+}$ which do not naturally fall into any $q\bar{q}\ (n\,^{2S+1}L_J)$ supermultiplet. The nature of these states is indicated when we examine the quark-gluon composite operators onto which these states have large overlap. Focussing on the 1^{--} sector, in Fig. 2 we show the relative magnitude of overlap $\left|\langle \mathfrak{n}|\, \mathcal{O}^\dagger\, |0\rangle\right|$ for three characteristic operators for the lightest four vector states. The lightest two states are dominated by $q\bar{q}\ ^3S_1$, while the third state can be characterized as $q\bar{q}\ ^3D_1$. The fourth state, precisely the one which did not fit naturally into an $q\bar{q}$ degeneracy pattern, has large overlap onto an

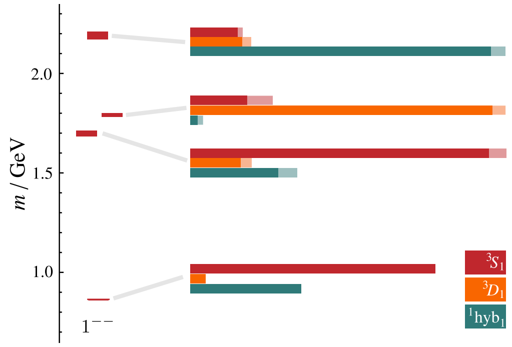

Figure 2. Normalized relative values of overlaps $|\langle 1^{--};\mathfrak{n}|\mathcal{O}^\dagger|0\rangle|$ for three characteristic operators on the four lightest vector states. Lightest two states appear to correspond dominantly to $^3S_1$ and its first radial excitation, the third to a $^3D_1$ construction and the fourth to a spin-singlet hybrid construction [5].

operator featuring the chromomagnetic component of the gluonic field-strength tensor (which we have labelled $^1\mathrm{hyb}_1$ in Fig. 2).

We conclude that the most likely explanation is that the states $(0,1,2)^{-+}, 1^{--}$ form the lightest hybrid meson supermultiplet, in which a chromomagnetic gluonic excitation couples to a color-octet $q\bar{q}$ pair. The 1^{--} state here has the unusual property that unlike conventional $q\bar{q}$ vector hadrons, its $q\bar{q}$ pair are in a spin-singlet, while the $(0,1,2)^{-+}$ have quark spins in a triplet. This interpretation is discussed in more detail in [5], where consideration is made of the quark mass dependence of the results. Similar conventional and hybrid meson systematics are present in the lattice QCD obtained charmonium spectrum [6], where we additionally observe a spectrum of positive parity hybrid mesons, which likely correspond to the same chromomagnetic gluonic excitation coupled to a color-octet $q\bar{q}$ pair in a P-wave.

4. Hybrid baryons

From the observations made above, it would appear that the lowest energy gluonic excitation is a color-octet with $J^P = 1^+$ which can form hybrid mesons by neutralizing the color of a color-octet $q\bar{q}$ pair. We might therefore wonder if the same gluonic excitation might combine with a color-octet qqq configuration to form a set of *hybrid baryons*. Using an extension of a basis of baryon operators presented in [7], the possibility of hybrid baryons was explored in [8] – the spectrum of N and Δ excitations obtained is shown in Fig. 3. Roughly 1.3 GeV heavier than the nucleon, above the first band of conventional nucleon resonances, a set of baryons that have large overlap onto operators featuring the chromomagnetic gluonic field appear. We interpret this set $N^\star\left(\frac{1}{2}^+, \frac{1}{2}^+, \frac{3}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+\right)$, $\Delta^\star\left(\frac{1}{2}^+, \frac{3}{2}^+\right)$ as the lightest hybrid baryons, and note that the energy scale of excitation, 1.3 GeV, is very close to the scale of excitation observed in the meson sector (for the lightest hybrid mesons, above the ρ in that case).

Hybrid baryons are not as attractive a prospect for experimental searches as they lack manifestly exotic quantum numbers. They would appear as supernumerary states in the excited nucleon spectrum, but even determining the presence of the full set of expected conventional states has been challenging. There is still work to do in the lattice calculation too – the volume in this calculation was rather small ($\sim (2.0\,\mathrm{fm})^3$), and clearly the correct physics of the Roper resonance as a rather light and broad resonance is not present.

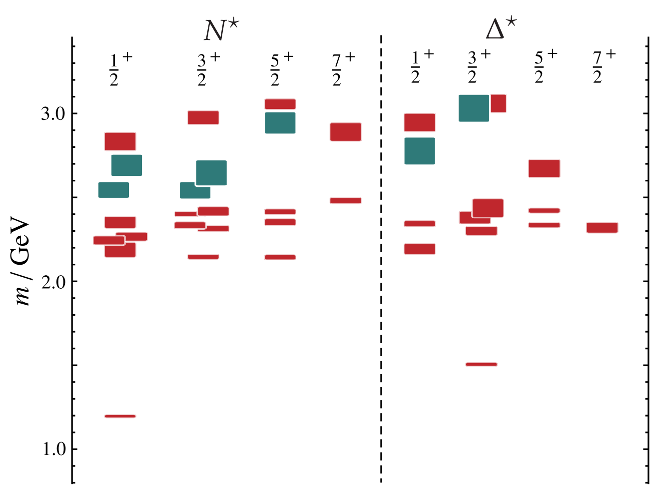

Figure 3. The baryon spectrum with $m_\pi = 391$ MeV. States in red have large overlap onto "conventional" qqq operator constructions, while those in blue overlap strongly with qqq operators also featuring a chromomagnetic gluonic excitation [8].

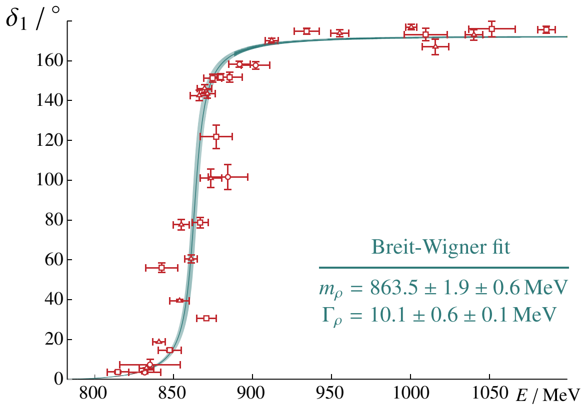

Figure 4. The elastic scattering phase-shift for $\pi\pi$ scattering with isospin=1 and $\ell = 1$. Calculation has quark masses such that $m_\pi = 391$ MeV. Data points correspond to discrete energy states in finite volumes $(16^3, 20^3, 24^3)$ in a number of moving frames. There is a clear resonant behavior which can be fitted with a Breit-Wigner form yielding the mass and width values shown. The small width is a result of the reduced phase-space for decay into our relatively heavy pions. [12]

5. Excited hadrons as resonances

Within a finite-volume having periodic boundary conditions, there can be no continuum of hadron-hadron scattering states as we have in experiments. The boundary conditions quantize the allowed momentum and we have only a discrete spectrum. It can be shown though, that the energy distribution of these states in a finite-volume can be related to the scattering amplitudes in infinite volume (see Lüscher [9] and papers citing this).

The **Hadron Spectrum Collaboration** explored this formalism initially in non-resonant $\pi\pi$ scattering in isospin $= 2$ [10, 11] where the spectrum was obtained using operators constructed to resemble a pair of pions with definite relative and total momentum. The expected weak repulsive scattering was extracted in both $\ell = 0$ and $\ell = 2$.

Subsequently the $\pi\pi$ elastic $I = 1$, $\ell = 1$ scattering channel was explored using both local $q\bar{q}$-like operators of the type described in Sect. 2 and $\pi\pi$-like operators. By considering a range of lattice volumes, calculating the spectrum in a number of moving frames, the phase-shift shown in Fig. 4 was obtained [12]. A clear ρ resonance signal is visible and could be described by a Breit-Wigner parameterization. The small decay width extracted is in line with the expectation of smaller phase-space for decay into the 391 MeV pions used in this calculation.

6. Summary

I have reported on the progress of the **Hadron Spectrum Collaboration** in understanding the role of gluonic excitations, and hadron spectroscopy more broadly using lattice QCD calculations [3, 4, 6–8, 10–18]. We have found that there is a lightest exotic hybrid meson with $J^{PC} = 1^{-+}$ partnered with non-exotic $(0, 2)^{-+}$ and 1^{--} hybrid mesons, with the internal structure being a color-octet $q\bar{q}$ pair in S-wave coupled to a chromomagnetic gluonic excitation (color-octet with $J^{PC} = 1^{+-}$). Positive parity hybrid mesons likely exist slightly heavier in mass as indicated by calculations in the charmonium sector. Hybrid baryons have also been predicted, likely appearing roughly 1.3 GeV heavier than the nucleon.

The current frontier in lattice QCD calculations of hadron spectroscopy is the resolution of excited states as resonances in hadronic scattering amplitudes. These amplitudes can be extracted from calculations of the volume-dependence of the spectrum, and a successful application of the method to the example of elastic $\pi\pi$ scattering was presented, where the ρ appears as a narrow resonance. Ongoing calculations are studying the possibility of resonances in coupled-channels [19], moving toward understanding decay systematics of excited hadrons. Other important directions include computing the coupling of excited meson and baryon states to photons, which is of particular relevance for photo- and electro- production experiments like those to be performed using Jefferson Lab's GlueX and CLAS12 detectors.

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