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Chiral-scale perturbation theory about an infrared fixed point

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Abstract. We review the failure of lowest order chiral $SU(3)_L \times SU(3)_R$ perturbation theory χPT$_3$ to account for amplitudes involving the $f_0(500)$ resonance and $O(m_K)$ extrapolations in momenta. We summarize our proposal to replace χPT$_3$ with a new effective theory χPT$_\sigma$ based on a low-energy expansion about an infrared fixed point in 3-flavour QCD. At the fixed point, the quark condensate $\langle \bar{q}q\rangle_{\text{vac}} \neq 0$ induces nine Nambu-Goldstone bosons: π, K, η and a QCD dilaton σ which we identify with the $f_0(500)$ resonance. We discuss the construction of the χPT$_\sigma$ Lagrangian and its implications for meson phenomenology at low-energies. Our main results include a simple explanation for the $\Delta I = 1/2$ rule in K-decays and an estimate for the Drell-Yan ratio in the infrared limit.

1. Three-flavor chiral expansions: Problems in the scalar-isoscalar channel

Chiral $SU(3)_L \times SU(3)_R$ perturbation theory χPT$_3$ is nowadays well established as the framework to systematically analyze the low-energy interactions of π, K, η mesons — the pseudo Nambu-Goldstone (NG) bosons of approximate chiral symmetry. The method relies on expansions about a NG-symmetry, *viz.*, low-energy scattering amplitudes and matrix elements can be described by an asymptotic series

$$\mathcal{A} = \{\mathcal{A}_{\text{LO}} + \mathcal{A}_{\text{NLO}} + \mathcal{A}_{\text{NNLO}} + \cdots\}_{\chi\text{PT}_3} \tag{1}$$

in powers and logarithms of $O(m_K)$ momentum and quark masses $m_{u,d,s} = O(m_K^2)$, with $m_{u,d}/m_s$ held fixed. The scheme works provided that contributions from the NG sector $\{\pi, K, \eta\}$ dominate those from the non-NG sector $\{\rho, \omega, \ldots\}$; an assumption known as the partial conservation of axial current (PCAC) hypothesis.

It has been observed [1], however, that the χPT$_3$ expansion (1) is afflicted with a peculiar malady: it typically *diverges* for amplitudes which involve both a 0^{++} channel and $O(m_K)$ extrapolations in

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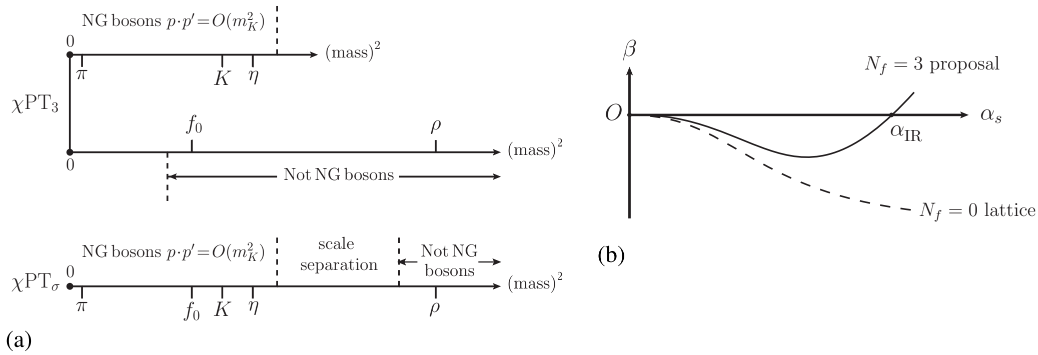

Figure 1. (a) Scale separations between Nambu-Goldstone (NG) sectors and other hadrons for each type of chiral perturbation theory χPT discussed in this proceeding. In conventional three-flavor theory χPT$_3$ (top diagram), there is *no scale separation*: the non-NG boson $f_0(500)$ sits in the middle of the NG sector $\{\pi, K, \eta\}$. Our three-flavor proposal χPT$_\sigma$ (bottom diagram) for $O(m_K)$ extrapolations in momenta implies a clear scale separation between the NG sector $\{\pi, K, \eta, \sigma = f_0\}$ and the non-NG sector $\{\rho, \omega, K^*, N, \eta', \ldots\}$. (b) Proposed β-function (solid line) for $N_f = 3$ flavor QCD with infrared fixed point α_{IR}. The dashed line shows the Yang-Mills ($N_f = 0$) lattice result [6] for continued growth in α_s with decreasing scale μ. Despite extensive literature [7] concerning the existence of α_{IR}, there is currently *no consensus* which of the above two, physically distinct, scenarios is actually realized in QCD. In particular, it is unclear how sensitive existing results are to variations in N_f. This is perhaps unsurprising, since modern calculations utilize different, nonperturbative definitions of α_s, thereby making comparisons between various analyses difficult.

momenta. The origin of this phenomenon can be traced to the $f_0(500)$ resonance, a broad 0^{++} state whose complex pole mass and residue [2]

$$m_{f_0} = 441 - i\,272 \text{ MeV} \quad \text{and} \quad |g_{f_0\pi\pi}| = 3.31 \text{ GeV} \tag{2}$$

have been determined to remarkable precision. Since χPT$_3$ classes f_0 pole terms as next-to-leading order (NLO), figure 1a shows why the low-energy expansion (1) fails: the location of f_0 and its strong coupling to π, K, η mesons invalidates the requirements of PCAC.

2. Three-flavor chiral-scale expansions about an infrared fixed point

In this proceeding, we summarize our proposal [3] to solve the convergence problem of χPT$_3$ expansions (1) by modifying the *leading order* (LO) of the 3-flavor theory. In short, our solution involves extending the standard NG sector $\{\pi, K, \eta\}$ to include $f_0(500)$ as a QCD dilaton σ associated with the *spontaneous* breaking of scale invariance. The scale symmetric counterpart of PCAC – partial conservation of dilatation current (PCDC) – then implies that amplitudes with σ/f_0 pole terms dominate, compared with contributions from the non-NG sector $\{\rho, \omega, K^*, N, \eta', \ldots\}$.[1]

This scenario can occur in QCD if at low energy scales $\mu \ll m_{t,b,c}$, the strong coupling α_s for the 3-flavor theory runs *nonperturbatively* to an infrared fixed point α_{IR} (Fig. 1b). At the fixed point, the gluonic term in the strong trace anomaly [9]

$$\theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu} + \big(1 + \gamma_m(\alpha_s)\big) \sum_{q=u,d,s} m_q \bar{q} q \tag{3}$$

[1] A discussion on violations of PCDC and Weinberg's power counting scheme [8] in $\gamma\gamma$ channels is contained in [3].

vanishes, which implies that in the chiral limit

$$\theta^\mu_\mu\big|_{\alpha_s=\alpha_{\rm IR}} = \big(1+\gamma_m(\alpha_{\rm IR})\big)(m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s) \to 0, \tag{4}$$

and thus $\langle \bar{q}q\rangle_{\rm vac}$ acts as a condensate for both scale and chiral $SU(3)_L \times SU(3)_R$ transformations.[2] By considering infrared expansions about the combined limit

$$m_{u,d,s} \sim 0 \quad \text{and} \quad \alpha_s \lesssim \alpha_{\rm IR}, \tag{5}$$

our proposal is to replace χPT$_3$ by chiral-scale perturbation theory χPT$_\sigma$, where the strange quark mass m_s in (4) sets the scale of $m^2_{f_0}$ as well as m^2_K and m^2_η (figure 1a, bottom diagram). As a result, the rules for counting powers of m_K are changed: f_0 pole amplitudes (NLO in χPT$_3$) are promoted to LO. That fixes the LO problem for amplitudes involving 0^{++} channels and $O(m_K)$ extrapolations in momenta. Note that we achieve this without upsetting successful LO χPT$_3$ predictions for amplitudes which do not involve the f_0; that is because the χPT$_3$ Lagrangian equals the $\sigma \to 0$ limit of the χPT$_\sigma$ Lagrangian.

In the physical region $0 < \alpha_s < \alpha_{\rm IR}$, the effective theory consists of operators constructed from the $SU(3)$ field U=$U(\pi, K, \eta)$ and chiral invariant dilaton σ, with terms classified by their scaling dimension d:

$$\mathcal{L}_{\chi{\rm PT}_\sigma} = \mathcal{L}\big[\sigma, U, U^\dagger\big] = :\mathcal{L}^{d=4}_{\rm inv} + \mathcal{L}^{d>4}_{\rm anom} + \mathcal{L}^{d<4}_{\rm mass}: . \tag{6}$$

Explicit formulas for the strong, weak, and electromagnetic interactions are obtained by scaling Lagrangian operators such as $\mathcal{K}\big[U,U^\dagger\big] = \frac{1}{4}F^2_\pi {\rm Tr}(\partial_\mu U \partial^\mu U^\dagger)$ and $\mathcal{K}_\sigma = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$ by appropriate powers of the $d=1$ field e^{σ/F_σ}. For example, the LO strong Lagrangian reads

$$\begin{aligned}
\mathcal{L}^{d=4}_{\rm inv,\,LO} &= \big\{c_1\mathcal{K} + c_2\mathcal{K}_\sigma + c_3 e^{2\sigma/F_\sigma}\big\}e^{2\sigma/F_\sigma}\,, \\
\mathcal{L}^{d>4}_{\rm anom,\,LO} &= \big\{(1-c_1)\mathcal{K} + (1-c_2)\mathcal{K}_\sigma + c_4 e^{2\sigma/F_\sigma}\big\}e^{(2+\beta')\sigma/F_\sigma}\,, \\
\mathcal{L}^{d<4}_{\rm mass,\,LO} &= {\rm Tr}(MU^\dagger + UM^\dagger)e^{(3-\gamma_m)\sigma/F_\sigma}\,,
\end{aligned} \tag{7}$$

where $F_\sigma \approx 100$ MeV is the dilaton decay constant, whose value is estimated by applying an analogue of the Goldberger-Treiman relation to analyses of NN-scattering [10]. Here the anomalous dimensions $\gamma_m = \gamma_m(\alpha_{\rm IR})$ and $\beta' = \beta(\alpha_{\rm IR})$ are evaluated at the fixed point because we expand in α_s about $\alpha_{\rm IR}$. The low-energy constants c_1 and c_2 are not fixed by symmetry arguments alone, while vacuum stability in the σ direction implies that both c_3 and c_4 are $O(M)$. From (7), one obtains formulas for the dilaton mass m_σ

$$m^2_\sigma F^2_\sigma = F^2_\pi\big(m^2_K + \tfrac{1}{2}m^2_\pi\big)(3-\gamma_m)(1+\gamma_m)\,, -\beta'(4+\beta')c_4 \tag{8}$$

and $\sigma\pi\pi$ coupling

$$\mathcal{L}_{\sigma\pi\pi} = \big\{\big(2+(1-c_1)\beta'\big)|\partial\pi|^2 - (3-\gamma_m)m^2_\pi|\pi|^2\big\}\sigma/(2F_\sigma)\,. \tag{9}$$

Note that (9) is derivative, so an on-shell dilaton is $O(m^2_\sigma)$ and consistent with σ being the broad resonance $f_0(500)$.

Our proposed replacement for χPT$_3$ possesses some desirable features, the foremost being:

1. The $\Delta I = 1/2$ rule for K-decays emerges as a *consequence* of χPT$_\sigma$, with a dilaton pole diagram (figure 2a) accounting for the large $I=0$ amplitude in $K_S \to \pi\pi$. Here, vacuum alignment [13] of the effective potential induces an interaction $\mathcal{L}_{K_S\sigma} = g_{K_S\sigma}K_S\sigma$ which mixes K_S and σ in LO. The effective coupling $g_{K_S\sigma}$ is fixed by data on $\gamma\gamma \to \pi^0\pi^0$ and $K_S \to \gamma\gamma$, with our estimate $|g_{K_S\sigma}| \approx 4.4 \times 10^3\,{\rm keV}^2$ accurate to a precision $\lesssim 30\%$ expected from a 3-flavor expansion.

[2] The former property is a simple consequence of the fact the $\bar{q}q$ is not a singlet under dilatations. The dual role of $\langle\bar{q}q\rangle_{\rm vac}$ was explored [4, 5] in some detail prior to the advent of QCD.

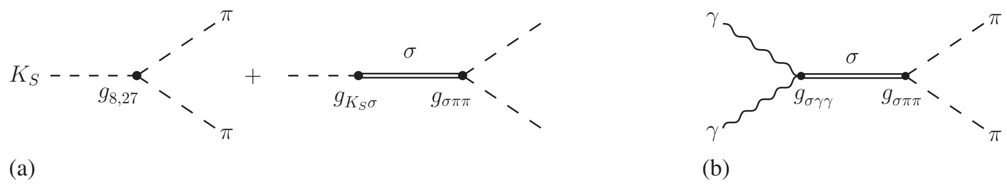

Figure 2. (a) Tree diagrams in the effective theory χPT$_\sigma$ for the decay $K_S \to \pi\pi$. The vertex amplitudes due to **8** and **27** contact couplings g_8 and g_{27} are dominated by the σ/f_0-pole amplitude. The magnitude of $g_{K_S\sigma}$ is found by applying χPT$_\sigma$ to $K_S \to \gamma\gamma$ and $\gamma\gamma \to \pi\pi$. (b) Dilaton pole in $\gamma\gamma \to \pi\pi$. Lowest order χPT$_\sigma$ includes other tree diagrams (for $\pi^+\pi^-$ production) and also $\pi^\pm$, $K^\pm$ loop diagrams (suppressed by a factor $1/N_c$) coupled to both photons.

Combined with data for the f_0 width (Eq. (2)), we find an amplitude $|A_{\sigma-\text{pole}}| \approx 0.34\,\text{keV}$ which accounts for the large magnitude $|A_0|_{\text{expt.}} = 0.33\,\text{keV}$. Consequently, the LO of χPT$_\sigma$ explains the $\Delta I = 1/2$ rule for kaon decays.

2. Our analysis of $\gamma\gamma$ channels and the electromagnetic trace anomaly [11, 12] yields a relation between the effective $\sigma\gamma\gamma$ coupling and the nonperturbative Drell-Yan ratio R_{IR} at α_{IR}:

$$g_{\sigma\gamma\gamma} = \frac{2\alpha}{3\pi F_\sigma}\left(R_{\text{IR}} - \frac{1}{2}\right). \tag{10}$$

A phenomenological value for R_{IR} is deduced by considering $\gamma\gamma \to \pi^0\pi^0$ in the large-N_c limit (Fig. 2b). Dispersive analyses [14] of this processes are able to determine the radiative width of $f_0(500)$, which in turn constrains $g_{\sigma\gamma\gamma}$ and yields the estimate $R_{\text{IR}} \approx 5$.

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