

Heavy Quark Symmetries: Molecular partners of the $X(3872)$ and $Z_b(10610)/Z'_b(10650)$

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Abstract. In this work, we have made use of the identification of the $X(3872)$ and $Z_b(10610)/Z'_b(10650)$ as heavy meson-heavy antimeson molecules to establish some consequences derived from the symmetries that these heavy meson-heavy antimeson systems must have. We show the most general effective lagrangian that respects these symmetries only depends on four undetermined low energy constants (LECs), which will be fitted to reproduce the experimental data about the resonances we are identifying as molecular states. Then, we obtain a whole new set of states in the spectrum that could also be thought as heavy meson-heavy antimeson molecules. Finally, using another different symmetry: Heavy Antiquark-Diquark Symmetry, we can also establish a set of pentaquark-like states taking advantage of the previous LEC calculation.

1. Introduction

The study of the QCD spectrum in the charm sector has been increasing in importance as more experimental data at high energies have become accessible. Within the years, many unexpected results appeared as much as theoretical predictions were confirmed or ruled out. One of these theoretical predictions was the existence of heavy meson-heavy antimeson molecules, predicted by Voloshin in the 70s [1]. Though this prediction seemed pretty solid taking into account the similarities between this system and the nucleon-nucleon one (which was able to form bound states), there was no clear experimental evidence of these molecular states. Till 2003, when the Belle collaboration discovered a resonance [2] very close to the $D\bar{D}^*$ threshold. This resonance is the most promising candidate to be fit in a molecular scheme. In the last ten years, many other X, Y, Z resonances that might be fitted into this scheme have also been found. Particularly important for this work is the discovery of the $Z_b(10610)/Z'_b(10650)$ [3], close to the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds, respectively.

2. EFT framework

Our analysis will be based on an effective field theory (EFT) for heavy meson-heavy antimeson molecules. This EFT turns out to be quite simple thanks to the symmetries this molecular systems show.

Table 1. Heavy meson–heavy meson combinations having the same contact terms as the $X(3872)$ and $Z_b(10610)$, and the predictions of the pole positions, which are understood to correspond to bound states except if we write “V” in parenthesis for denoting a virtual state. When we increase the strength of the potential to account for the various uncertainties, in one case (marked with † in the table) the virtual pole evolves into a bound state. The masses are given in units of MeV.

V_C	$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)	Masses ($\Lambda = 1$ GeV)
C_{0X}	$0(1^{++})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D})$	3875.87	3871.68 (input)	3871.68 (input)
	$0(2^{++})$	$D^*\bar{D}^*$	4017.3	4012_{-5}^{+4}	4012_{-12}^{+5}
	$0(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	10604.4	10580_{-8}^{+9}	10539_{-27}^{+25}
	$0(2^{++})$	$B^*\bar{B}^*$	10650.2	10626_{-9}^{+8}	10584_{-27}^{+25}
	$0(2^+)$	D^*B^*	7333.7	7322_{-7}^{+6}	7308_{-20}^{+16}
C_{0Z}	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	10604.4	10602.4 ± 2.0 (input)	10602.4 ± 2.0 (input)
	$1(1^{+-})$	$B^*\bar{B}^*$	10650.2	10648.1 ± 2.1	$10648.1_{-2.5}^{+2.1}$
	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* + D^*\bar{D})$	3875.87	3871_{-12}^{+4} (V)	3837_{-35}^{+17} (V)
	$1(1^{+-})$	$D^*\bar{D}^*$	4017.3	4013_{-11}^{+4} (V)	3983_{-32}^{+17} (V)
	$1(1^+)$	D^*B^*	7333.7	$7333.6_{-4.2}^{\dagger}$ (V)	7328_{-14}^{+5} (V)

These symmetries can be classified in two types: those due to the presence of the heavy quarks and the usual ones related to the light quarks. The most relevant heavy quark symmetries for this work are: Heavy Quark Spin Symmetry (HQSS), Heavy Flavour Symmetry (HFS) and Heavy Antiquark-Diquark Symmetry (HADS). Light quark symmetries are SU(3) light flavour symmetry and Chiral Symmetry which determines the role of pion exchanges in our theory.

At leading order, the most general lagrangian that respects HQSS we can write is [4]:

$$\begin{aligned}
 V_4 = & + \frac{C_A}{4} Tr [\bar{H}_a^{(Q)} H_a^{(Q)} \gamma_\mu] Tr [H_a^{(Q)} \bar{H}_a^{(Q)} \gamma^\mu] + \frac{C_A^\lambda}{4} Tr [\bar{H}_a^{(Q)} \lambda_{ab}^i H_b^{(Q)} \gamma_\mu] Tr [H_c^{(Q)} \lambda_{cd}^i \bar{H}_d^{(Q)} \gamma^\mu] + \\
 & + \frac{C_B}{4} Tr [\bar{H}_a^{(Q)} H_a^{(Q)} \gamma_\mu \gamma_5] Tr [H_a^{(Q)} \bar{H}_a^{(Q)} \gamma^\mu \gamma_5] \\
 & + \frac{C_B^\lambda}{4} Tr [\bar{H}_a^{(Q)} \lambda_{ab}^j H_b^{(Q)} \gamma_\mu \gamma_5] Tr [H_c^{(Q)} \lambda_{cd}^j \bar{H}_d^{(Q)} \gamma^\mu \gamma_5]
 \end{aligned} \quad (1)$$

being H (\bar{H}) a meson (antimeson) hyperfield that contains the pseudoscalar and vector heavy mesons (antimesons) and λ the Gell-Mann matrices as we are assuming a SU(3) light quark flavour symmetry. This is a contact potential (zeros range) since pion exchange effects are subleading, as shown analytically and numerically [5, 6] (coupled channel effects are shown to be subleading as well).

Therefore, our dynamics depends on four low energy constants (LECs) that will be rewritten into C_{0A} , C_{0B} , C_{1A} , C_{1B} wlog. These LECs will be fitted to the experimental data. For that purpose we will identify the $X(3872)$ as a $J^{PC} = 1^{++}$ $D\bar{D}^*$ molecule and the $Z_b(10610)/Z_b'(10650)$ as isovector $J^{PC} = 1^{+-}$ $B\bar{B}^*$, $B^*\bar{B}^*$ molecules respectively. The $X(3872)$ assumption which will allow us to fix two linear combination of the LECs (for further details, see [8]) as we will also consider the ratio of the decay amplitudes $\frac{\mathcal{M}(X(3872) \rightarrow J/\psi \pi \pi)}{\mathcal{M}(X(3872) \rightarrow J/\psi \pi \pi \pi)}$ determined in [7] into our analysis, $C_{0X} = C_{0A} + C_{0B}$ and $C_{1X} = C_{1A} + C_{1B}$. The Z_b s assumptions will fix a third one: $C_{1Z} = C_{1A} - C_{1B}$.

Once we have determined our dynamics, we can obtain our results solving the Lippmann-Schwinger Equation (LSE) as resonances appear as poles in the second Riemann sheet of the T-matrix. Ultraviolet

Table 2. Predictions of the doubly-heavy baryon–heavy meson molecules. Results are given in terms of M_{th} for simplicity, and all masses are given in units of MeV. When we decrease the strength of the potential to account for the various uncertainties, in some cases (marked with † in the table) the bound state pole reaches the threshold and the state becomes virtual. The cases with a virtual state pole at the central value are marked by [V], for which †† means that the pole evolves into a bound state one and N/A means that the pole is far from the threshold with a momentum larger than 1 GeV so that it is both undetectable and beyond the EFT range.

State	$I(J^P)$	V^{LO}	Thresholds	Mass ($\Lambda = 0.5$ GeV)	Mass ($\Lambda = 1$ GeV)
$\Xi_{cc}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	5715	$(M_{\text{th}} - 10)_{-15}^{+10}$	$(M_{\text{th}} - 19)_{-44}^{\dagger}$
$\Xi_{cc}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	9031	$(M_{\text{th}} - 21)_{-19}^{+16}$	$(M_{\text{th}} - 53)_{-59}^{+45}$
$\Xi_{bb}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	12160	$(M_{\text{th}} - 15)_{-11}^{+9}$	$(M_{\text{th}} - 35)_{-31}^{+25}$
$\Xi_{bb}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	15476	$(M_{\text{th}} - 29)_{-13}^{+12}$	$(M_{\text{th}} - 83)_{-40}^{+38}$
$\Xi'_{bc} D^*$	$0(\frac{3}{2}^-)$	$C_{0a} + C_{0b}$	8967	$(M_{\text{th}} - 14)_{-13}^{+11}$	$(M_{\text{th}} - 30)_{-40}^{+27}$
$\Xi'_{bc} \bar{B}^*$	$0(\frac{3}{2}^-)$	$C_{0a} + C_{0b}$	12283	$(M_{\text{th}} - 27)_{-16}^{+15}$	$(M_{\text{th}} - 74)_{-51}^{+45}$
$\Xi_{bc}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	9005	$(M_{\text{th}} - 14)_{-13}^{+11}$	$(M_{\text{th}} - 30)_{-40}^{+27}$
$\Xi_{bc}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	12321	$(M_{\text{th}} - 27)_{-16}^{+15}$	$(M_{\text{th}} - 74)_{-51}^{+46}$
$\Xi_{bb} \bar{B}$	$1(\frac{1}{2}^-)$	C_{1a}	15406	$(M_{\text{th}} - 0.3)_{-2.5}^{\dagger}$	$(M_{\text{th}} - 12)_{-15}^{+11}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a} + \frac{2}{3} C_{1b}$	15452	$(M_{\text{th}} - 0.9)[V]_{\dagger\dagger}^{\text{N/A}}$	$(M_{\text{th}} - 16)_{-17}^{+14}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a} - \frac{1}{3} C_{1b}$	15452	$(M_{\text{th}} - 1.2)_{-2.9}^{\dagger}$	$(M_{\text{th}} - 10)_{-13}^{+9}$
$\Xi_{bb}^* \bar{B}$	$1(\frac{3}{2}^-)$	C_{1a}	15430	$(M_{\text{th}} - 0.3)_{-2.4}^{\dagger}$	$(M_{\text{th}} - 12)_{-13}^{+11}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a} - \frac{5}{3} C_{1b}$	15476	$(M_{\text{th}} - 8)_{-7}^{+8}$	$(M_{\text{th}} - 5)_{-8}^{\dagger}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a} - \frac{2}{3} C_{1b}$	15476	$(M_{\text{th}} - 2.5)_{-3.6}^{\dagger}$	$(M_{\text{th}} - 9)_{-11}^{+9}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{5}{2}^-)$	$C_{1a} + C_{1b}$	15476	$(M_{\text{th}} - 4.3)[V]_{\dagger 3.3}^{\text{N/A}}$	$(M_{\text{th}} - 18)_{-19}^{+17}$

divergences have been treated introducing a gaussian regulator with a cut-off $\Lambda = 0.5 - 1$ GeV, again details can be found in [8]. This induces a relation between the LECs and the cut-off Λ , but final results will be rather regulator independent.

3. Mesonic Partners of the $X(3872)$ and $Z_b(10610)/Z'_b(10650)$

After having fixed the three LECs, we now solve the LSE for different channels where the dynamics is determined from the previous conditions. The obtained results are shown in Table 1, see more details in [9]. Results displayed in this table are classified between those derived from the $X(3872)$ assumption and those derived from the $Z_b(10610)$. Among those derived from the $X(3872)$ it is important to pay attention to the $J^{PC} = 2^{++} D^* \bar{D}^*$ molecule as, having the same dynamics than the $X(3872)$ and a similar reduced mass, its existence would be a strong hint for the molecular nature of the $X(3872)$. In the $Z_b(10610)$ sector, the $Z'_b(10650)$ appears naturally and, below we have two predictions for the quantum numbers of two recently discovered resonances: the $Z_c(3900)$ [10, 11] and the $Z'_c(4025)$ [12]. According to our analysis, these resonances could be thought as virtual states with quantum numbers $J^{PC} = 1^{+-}$ of $D\bar{D}^*$ and $D^* \bar{D}^*$, respectively.

Uncertainties in the table take into account several sources of error. Uncertainties in the $X(3872)$ mass: $M_{3872} = 3871.68 \pm 0.17$ MeV, $B_{Z_b(10610)} = 2.0 \pm 2.0$, in the ratio of decay amplitudes: $K_{X(3872)} = 0.26^{+0.08}_{-0.05}$ and the errors due to the two EFT expansions used in this work: HQSS and HFS (of the order $\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_Q}\right)$) and HADS (of the order $\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_{QV}}\right)$, relevant only for Table 2). Considering $\Lambda_{QCD} \simeq 300$ MeV, $m_c \simeq 1.3$ GeV and $m_b = 4.2$ GeV, we obtain that $\Delta_{HQSS} = 20\%(7\%)$ in the charm (bottom) sector and $\Delta_{HADS} = 40\%(30\%)(20\%)$ in the charm (charm-bottom) (bottom) sector, respectively.

4. Baryonic Partners of the $X(3872)$ and $Z_b(10610)/Z'_b(10650)$

There was another heavy quark symmetry we have mentioned in the introduction: HADS. This symmetry, first discussed in 1990 [13] states that a heavy diquark behaves as a heavy antiquark up to corrections of the order $\frac{1}{m_Q v}$ (where v is the velocity of the heavy quarks), that is, a little bit worse than other heavy quark symmetries used so far in this work. This means that, as the dynamics only depends on the light degrees of freedom which are the same than in the previous case, we can still make use of the LECs obtained in the heavy meson-heavy antimeson analysis to obtain heavy meson-doubly heavy baryon systems, that is, pentaquark-like states. These states are shown in Table 2 (further details in [14]). It is worth to notice that the baryonic molecular partners of the $X(3872)$ are deeper bound than the partners of the Z'_b s. Uncertainties in the table have the same origin than those in Table 1.

5. Conclusions

In this work we have identified the $X(3872)$ and $Z_b(10610)/Z'_b(10650)$ resonances as heavy meson-heavy antimeson molecule and have made use of several symmetries to construct of a simplified EFT (contact potential with only four undetermined LECs that will be fitted to the experimental data). We have used this EFT to predict two set of molecular partners of these resonances.

The first family of resonances is based on a HQSS and HFS analysis. Here we have obtained a very robust prediction for the $2^{++} D^* \bar{D}^*$ channel and establish another predictions for the quantum numbers of two recently discovered states. The second family of resonances uses HADS and it's a very exotic contribution to the spectrum.

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