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Mathematical modeling of physico-chemical processes in the polymerization of multicore cable products

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Abstract. In this article the question of simulation of no stationary process in the polymerization of multilayer cable product is considered. The model allows predicting time of full polymerization rubber shell.

Introduction

Processes of curing of cable products assume their transmission from specialized furnaces with a temperature of heating of $450 \div 550\,\mathrm{K}$. It is considered to be achievement on all thickness of an insulating layer of a cable of demanded [1] extents of polymerization ($\varphi \approx 0.99$).

The purpose of this work is research of differences of integrated characteristics of curing of single-core and multicore cable products.

Problem statement

Numerical modeling is executed for the typical system presented in Fig. 1. It was supposed that the cable contains some isolated conductors (copper conductor) and the general cover (rubber). Reference temperature of the product $\mathrm{T_0}$ was accepted lower than an air temperature in the heating camera $\mathrm{T_h}$.

It was taken into account air gaps near entrance sites ($z = 0$, $R_5 < r < R_6$) and output ($z = Z_1$, $R_5 < r < R_6$) cable product from vulcanizations furnace. There are considered that surrounding air flows into camera ($z = 0$, $R_5 < r < R_6$) with temperature $T_c = T_0$ and speed w_c. Mix of cold and hot air follows from outlet ($z = Z_1$, $R_2 < r < R_3$) with speed w_c. There are axisymmetric systems (Fig. 1).

The following assumptions were accepted:

1. Contact between cores and rubber is ideal.
2. Cable has a correctly cylindrical form.
3. There are cables fragment with ideal insulated ends.
4. Thermophysical characteristics core, cover and air independent from temperature.

Mathematical model

The mathematical model of heat and mass transfer can be formulated as a typical time-dependent differential equations for system "hot air – multicore cable" in cylindrical coordinates [2, 3].

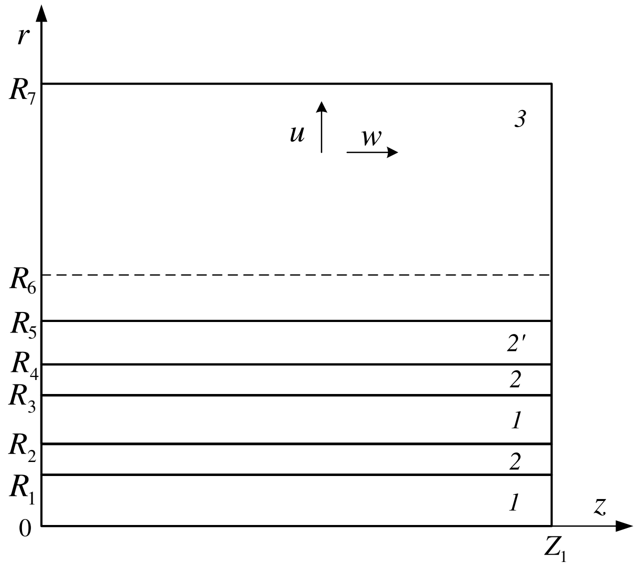

Figure 1. Scheme for solving the problem area: *1* – metal core, *2* – shell core, *2'* – shell cable, *3* – hot air in furnace.

The heat equation for cores ($0 < \mathrm{R} < \mathrm{Nr}_1$, $\mathrm{Nr}_2 < \mathrm{R} < \mathrm{Nr}_3$, $0 < \mathrm{Z} < \mathrm{Nz}$):

$$\frac{\partial T_i}{\partial t} = a_i \left[\frac{\partial^2 T_i}{\partial R^2} + \frac{1}{R}\frac{\partial T_i}{\partial R} + \frac{\partial^2 T_i}{\partial Z^2} \right], \quad \mathrm{i} = 1, 2.$$

The energy equation for core insulations ($\mathrm{Nr}_1 < \mathrm{R} < \mathrm{Nr}_2$, $\mathrm{Nr}_3 < \mathrm{R} < \mathrm{Nr}_4$, $0 < \mathrm{Z} < \mathrm{Nz}$):

$$\frac{\partial T_j}{\partial t} = a_j \left[\frac{\partial^2 T_j}{\partial R^2} + \frac{1}{R}\frac{\partial T_j}{\partial R} + \frac{\partial^2 T_j}{\partial Z^2} \right], \quad j = 1, 2.$$

The energy equation for cable insulation shell ($\mathrm{Nr}_4 < \mathrm{R} < \mathrm{Nr}_5$, $0 < \mathrm{Z} < \mathrm{Nz}$):

$$\rho_3 C_3 \frac{\partial T_3}{\partial t} = \lambda_3 \left[\frac{\partial^2 T_3}{\partial R^2} + \frac{1}{R}\frac{\partial T_3}{\partial R} + \frac{\partial^2 T_3}{\partial Z^2} \right] + q_3 \rho_3 \frac{d\varphi_3}{dt},$$

$$\frac{d\varphi_3}{dt} = (1 - \varphi_3)\, k_0 \exp\left[-\frac{E}{R_t T_3} \right].$$

The energy, motion and continuity equations for air in camera ($\mathrm{Nr}_5 < \mathrm{R} < \mathrm{Nr}_7$, $0 < \mathrm{Z} < \mathrm{Nz}$):

$$\frac{\partial T_4}{\partial t} + u\frac{\partial T_4}{\partial R} + w\frac{\partial T_4}{\partial Z} = a_4 \left[\frac{\partial^2 T_4}{\partial R^2} + \frac{1}{R}\frac{\partial T_4}{\partial R} + \frac{\partial^2 T_4}{\partial Z^2} \right],$$

$$\frac{\partial \omega}{\partial \tau} + u\frac{\partial \omega}{\partial R} + w\frac{\partial \omega}{\partial Z} = \upsilon_4 \left[\frac{\partial^2 \omega}{\partial R^2} + \frac{1}{R}\frac{\partial \omega}{\partial R} + \frac{\partial^2 \omega}{\partial Z^2} \right] + \beta g \frac{\partial T_4}{\partial Z},$$

$$\frac{\partial^2 \psi}{\partial Z^2} + \frac{1}{R}\frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial R^2} = \omega.$$

Initial ($\tau = 0$) conditions: $T_i = T_0$ by $0 < R < Nr_1$, $Nr_2 < R < Nr_3$, $0 < Z < Nz$; $T_j = T_0$ by $Nr_1 < R < Nr_2$, $Nr_3 < R < Nr_4$, $0 < Z < Nz$; $T_3 = T_0$ by $Nr_4 < R < Nr_5$, $0 < Z < Nz$; $T_4 = T_v$ by $Nr_5 < R < Nr_7$, $0 < Z < Nz$; $\varphi = 0,99$ by $Nr_1 < R < Nr_2$, $Nr_3 < R < Nr_4$, $0 < Z < Nz$; $\varphi = \varphi_0$ by $Nr_4 < R < Nr_5$, $0 < Z < Nz$.

Boundary conditions ($0 < \tau < t_p$):

$$Z = 0, 0 < R < Nr_1, Nr_2 < R < Nr_3 \ \frac{\partial T_i}{\partial Z} = 0;$$

$$Z = 0, Nr_1 < R < Nr_2, Nr_3 < R < Nr_4 \quad \frac{\partial T_j}{\partial Z} = 0;$$

$$Z = 0, Nr_4 < R < Nr_5 \quad \frac{\partial T_3}{\partial Z} = 0;$$

$$Z = 0, Nr_5 < R < Nr_6 \ \ T_4 = T_v, \frac{\partial \psi}{\partial Z} = 0, \frac{\partial \psi}{\partial R} = -w_c R;$$

$$Z = 0, Nr_6 < R < Nr_7 \quad \frac{\partial T_4}{\partial Z} = 0, \psi = 0, \frac{\partial \psi}{\partial R} = 0;$$

$$Z = Nz, 0 < R < Nr_1, Nr_2 < R < Nr_3 \quad \frac{\partial T_i}{\partial Z} = 0;$$

$$Z = Nz, Nr_1 < R < Nr_2, Nr_3 < R < Nr_4 \quad \frac{\partial T_j}{\partial Z} = 0;$$

$$Z = Nz, Nr_4 < R < Nr_5 \quad \frac{\partial T_3}{\partial Z} = 0;$$

$$Z = Nz, Nr_5 < R < Nr_6 \quad T_4 = T_v, \frac{\partial \psi}{\partial Z} = 0, \frac{\partial \psi}{\partial R} = -w_c R;$$

$$Z = Nz, Nr_6 < R < Nr_7 \frac{\partial T_4}{\partial Z} = 0, \psi = 0, \frac{\partial \psi}{\partial R} = 0;$$

$$R = 0, 0 < Z < \ Nz \ \ \frac{\partial T_i}{\partial R} = 0;$$

$$R = Nr_1, 0 < Z < \ Nz - \lambda_i \frac{\partial T_i}{\partial R} = -\lambda_j \frac{\partial T_j}{\partial R}, T_i = T_j;$$

$$R = Nr_2, 0 < Z < \ Nz - \lambda_i \frac{\partial T_i}{\partial R} = -\lambda_j \frac{\partial T_j}{\partial R}, T_i = T_j;$$

$$R = Nr_3, 0 < Z < \ Nz - \lambda_i \frac{\partial T_i}{\partial R} = -\lambda_j \frac{\partial T_j}{\partial R}, T_i = T_j;$$

$$R = Nr_4, 0 < Z < \ Nz - \lambda_j \frac{\partial T_j}{\partial R} = -\lambda_3 \frac{\partial T_3}{\partial R}, T_j = T_3;$$

$$R = Nr_5, 0 < Z < \ Nz - \lambda_3 \frac{\partial T_3}{\partial R} = -\lambda_4 \frac{\partial T_4}{\partial R}, T_3 = T_4, \psi = 0, \frac{\partial \psi}{\partial Z} = w_c R;$$

$$R = Nr_7, 0 < Z < \ Nz \ \ T_4 = T_v, \psi = 0, \frac{\partial \psi}{\partial Z} = 0.$$

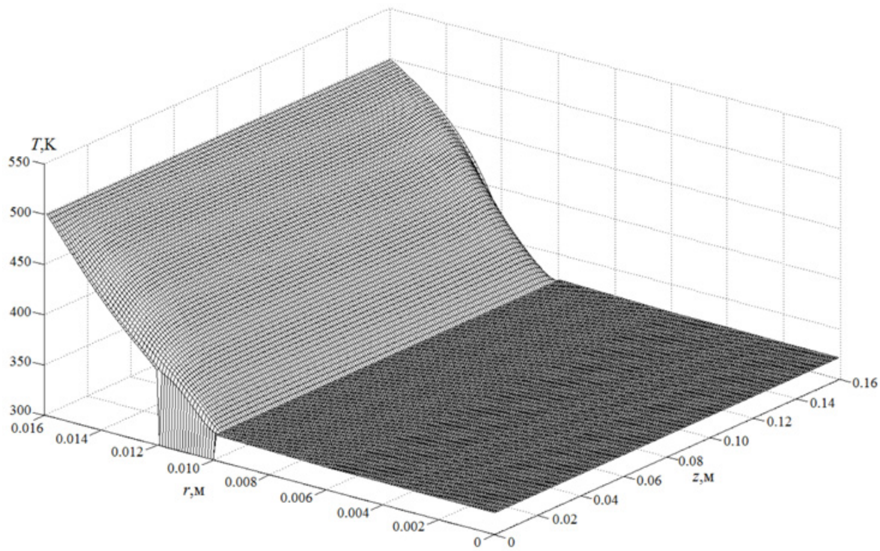

Figure 2. Temperatures field by full polymerization shell of multilayer cable ($t_p = 774.6\,s$ and $\varphi \approx 0,99$ by $Nr_4 < R < Nr_5$).

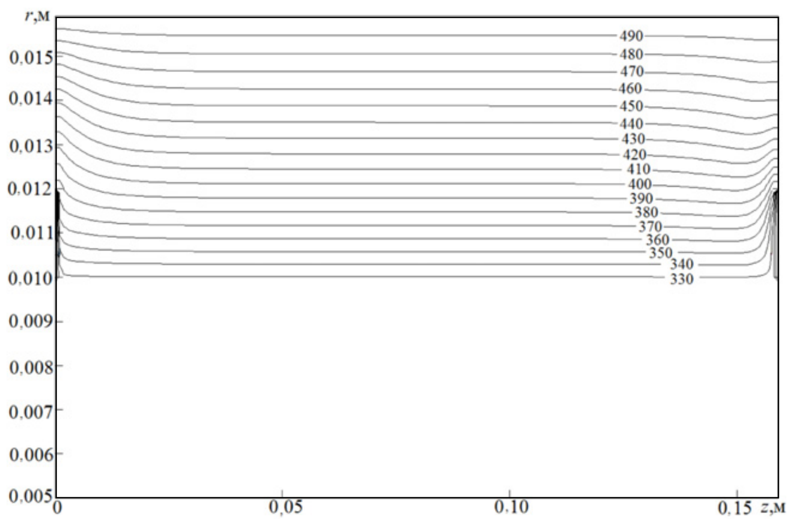

Figure 3. Isotherms by full polymerization shell of multilayer cable ($t_p = 774.6\,s$ and $\varphi \approx 0,99$ by $Nr_4 < R < Nr_5$).

The system of time-dependent differential equations solved by finite difference method [4]. The difference analogues of differential equations solved by locally one-dimensional method and alternating direction method [4]. There are applied sweep method with using a four-point implicit scheme for solving dimensional difference equations [4]. The approximation of boundary conditions for Poisson equations and vortices equation are performed analogically [5].

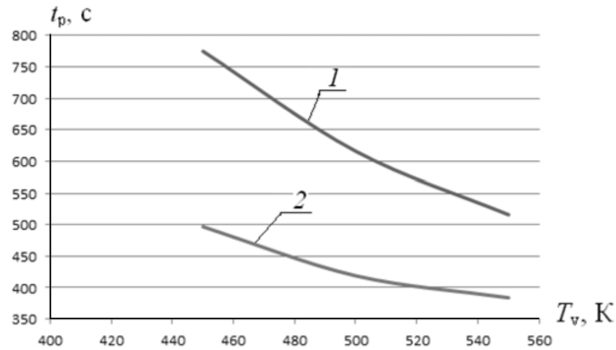

Figure 4. Dependence time of polymerization t_p from air temperature in camera T_v: *1* – multicore product, *2* – single-core product.

Results and discussion

The results (Figs. 2–4) indicate about inexpedient of excerpts normative temperature for different structure of products. Correct choice of vulcanization mode (T_h, w_c, t_p) will minimize typical times of process and provide condition of uniform heating (without overheating, melting and surface defects).

There are found, that polymerization time of multicore cable's rubber cover are exceeds analogic parameters of single-core product. Our result indicates about accounting internal structure of cable product in selecting the vulcanization mode. There is need to support more times for multicore cable (accordingly, less speed of broach) as compared with single-core.

Conclusion

This model heat and mass transfer can use as prognostic to select of typical multicore cable's vulcanization mode and corresponding parameters (camera's temperature, allowable gap dimensions on in and out of camera, time of heating, speed of broaching).

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