

# Mathematical simulation of heat transfer at coniferous tree ignition by cloud-to-ground lightning discharge

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**Abstract.** Numerical simulation results of coniferous tree heating by cloud-to-ground lightning discharge are presented. Pine tree is under consideration. The problem is solved in one-dimensional statement in cylindrical system of coordinates. Parametric investigation of volt-ampere characteristics influence on stem heating process was lead for negative and positive ground lightning discharges. Conditions of coniferous tree ignition in a typical range of influence parameters change are established.

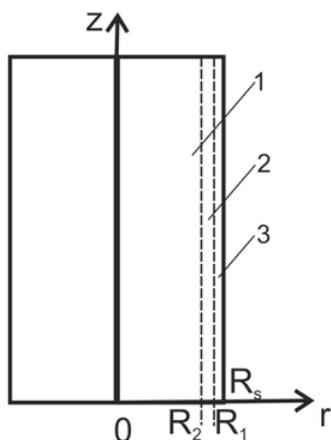
## 1. Introduction

Occurrence of forest fires is caused mainly by the anthropogenous reasons now [1]. But in taiga [2] and high-mountainous [3] regions the great value has occurrence of fires in large forests at passage of a thunder-storm as a result of cloud-to-ground lightning discharge action [4]. Various variants of deterministic-probabilistic criterion of the forest fire danger forecast [5, 6], which includes a subsystem of estimation of forest fires occurrence probability from thunder-storms, are known. But existing criteria do not consider the physical mechanism of tree ignition as a result of cloud-to-ground lightning discharge action. The basic characteristics of cloud-to-ground lightning discharges are polarity, peak current and voltage, and also action duration [7]. Therefore creation of the forest fire ignitions forecast technique on the basis of mathematical statement of a problem about tree ignition by cloud-to-ground lightning discharge is expedient. Mathematical simulation of such difficult process presumes to avoid unprofitable experimental researches. Tree ignition by an electric lightning discharge is characterized by high power.

It is not published results of mathematical simulation of trunk wood ignition processes owing to cloud-to-ground lightning discharge till now. There are no experimental data about dependences of parameters of this process on influence conditions also.

Research objective of paper is mathematical simulation of coniferous tree ignition process by cloud-to-ground lightning discharge.

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**Figure 1.** The scheme of decision area.

## 2. Physical and mathematical statement

Processes of current passage have the features in case of a lightning stroke in tree trunk of coniferous tree, for example, pines. In wood structure of angiosperms the appreciable role is played by vessels on which the moisture moves [8]. The internal part of trunk penetrated by such transport channels, is a good conductor of an electric current. One of differences of wood structure of coniferous trees is absence of vessels [8]. Therefore the central part of trunk of resinous coniferous tree has considerably greater resistance, than a bark and subcrustal layer (Fig. 1). Thereof the electric current of the discharge of lightning passes in pine mainly on the external layers sated with moisture [8].

The following physical model is considered. On the land surface separately standing tree of coniferous tree grows. Cloud-to-ground lightning discharge of certain polarity strikes in tree trunk. The electric current of cloud-to-ground lightning discharge proceeds on trunk. It is supposed, that current parametres are identical in various sections of a trunk also it proceeds in a subcrustal zone of coniferous tree. There is a warming up of wood as result of the extraction of Joule heat and at achievement of certain thermal fluxes from subcrustal zone of trunk and critical temperature is tree ignition. Influence of humidity of wood on ignition process it is neglected. Last assumption is proved enough for short-term rains with thunder-storms, and also for a rain initial stage when storm activity is usually maximum.

The problem dares for the cylinder which models tree trunk. The certain section of trunk is considered. The scheme of the decision area is presented on Fig. 1, where 1 – core, 2 – subcrustal zone, 3 – tree trunk bark;  $R_s$  – external radius of trunk,  $R_1$  – border of subcrustal zone and bark,  $R_2$  – border of core and subcrustal zone.

Warming up process of tree by cloud-to-ground lightning discharge before ignition is described mathematically by system of the non-stationary differential equations of heat conductivity:

$$\rho_1 c_1 \frac{\partial T_1}{\partial t} = \frac{\lambda_1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_1}{\partial r} \right), \quad (1)$$

$$\rho_2 c_2 \frac{\partial T_2}{\partial t} = \frac{\lambda_2}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_2}{\partial r} \right) + JU, \quad (2)$$

$$\rho_3 c_3 \frac{\partial T_3}{\partial t} = \frac{\lambda_3}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_3}{\partial r} \right). \quad (3)$$

Boundary conditions for the Eqs. (1)–(3):

$$r = 0, \quad \lambda_1 \frac{\partial T_1}{\partial r} = 0, \quad (4)$$

$$r = R_2, \quad \lambda_1 \frac{\partial T_1}{\partial r} = \lambda_2 \frac{\partial T_2}{\partial r}, \quad T_1 = T_2, \quad (5)$$

$$r = R_1, \quad \lambda_2 \frac{\partial T_2}{\partial r} = \lambda_3 \frac{\partial T_3}{\partial r}, \quad T_2 = T_3, \quad (6)$$

$$r = R_s, \quad \lambda_3 \frac{\partial T_3}{\partial r} = \alpha(T_e - T_3). \quad (7)$$

Initial conditions for the Eqs. (1)–(3):

$$t = 0, \quad T_i(r) = T_{i0}(r), \quad i = 1, 2, 3. \quad (8)$$

Where  $T_i, \rho_i, c_i, \lambda_i$  – temperature, density, thermal capacity and heat conductivity accordingly core ( $i=1$ ), subcrustal zone ( $i=2$ ), bark ( $i=3$ ) of trunk;  $\alpha$  – heat-transfer factor;  $J$  – current of cloud-to-ground lightning discharge;  $U$  – voltage of cloud-to-ground lightning discharge;  $r$  – coordinate,  $t$  – time. Indexes “e” and “0” correspond to parameters of environment and wood parameters during the initial moment of time.

Initial data (pine wood, core) [9]:  $\rho = 500 \text{ kg/m}^3$ ;  $c = 1670 \text{ J/(kg} \cdot \text{K)}$ ;  $\lambda = 0.12 \text{ W/(m} \cdot \text{K)}$ .  
 Parametres of subcrustal layer:  $\rho = 500 \text{ kg/m}^3$ ;  $c = 2600 \text{ J/(kg} \cdot \text{K)}$ ;  $\lambda = 0.35 \text{ W/(m} \cdot \text{K)}$ .  
 Termophysical characteristics of bark:  $\rho = 500 \text{ kg/m}^3$ ;  $c = 1670 \text{ J/(kg} \cdot \text{K)}$ ;  $\lambda = 0.12 \text{ Bm/(m} \cdot \text{K)}$ .  
 Geometrical characteristics of the decision area:  $R_s = 0.25 \text{ m}$ ;  $R_1 = 0.245 \text{ m}$ ;  $R_2 = 0.235 \text{ m}$ .  
 Environment parameters:  $T_e = 300 \text{ K}$ ,  $\alpha = 80 \text{ W/(m}^2 \cdot \text{K)}$ .

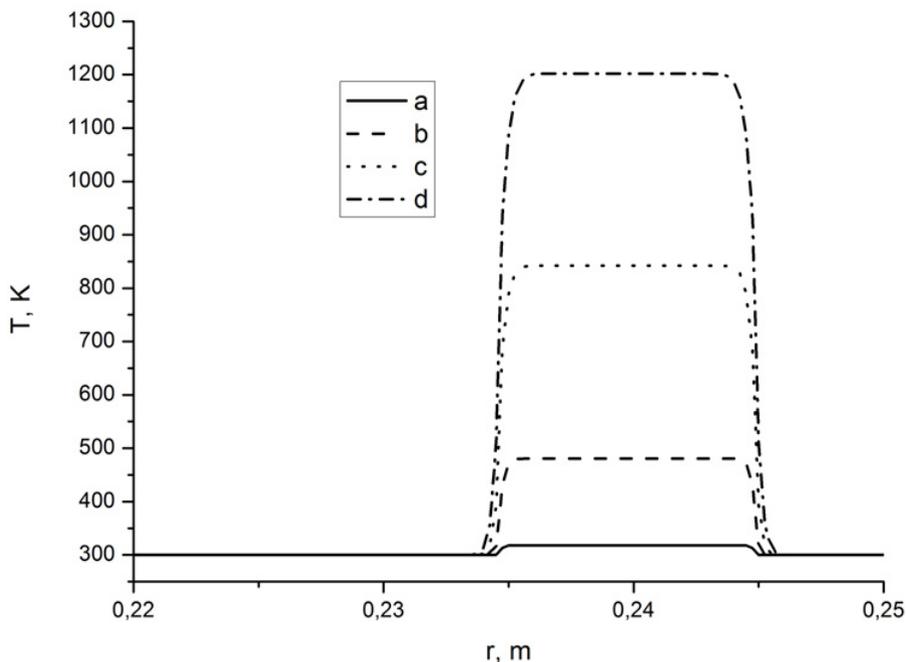
### 3. Results and discussion

Formulated mathematical model (1)–(3) with boundary and initial conditions (4)–(8) is solved by method of finite differences [10]. For the decision of difference analogues of the one-dimensional differential equations the sweep method [10] was used.

Extensive enough data on parameters of cloud-to-ground lightning discharges are known. An average peak current [11]:  $J = 23.5 \text{ kA}$  for the negative discharge and  $J=35.3 \text{ kA}$  for the positive discharge. About 16.5% of positive discharges has current less than 10 kA [12].

Parametrical research of influence of cloud-to-ground lightning discharge characteristics on process of warming up of wood trunk during lightning impulse is carried out. It is necessary to notice, that it is not published experimental data about kinetics processes of the big wood body ignition till now. Obviously, it is caused by difficulties of real experimental studying of this process. But the approach [9] in which conditions of coniferous tree (pine) ignition are described by two parameters (thermal flux and temperature of ignition surface) is known. Actually in [9] gas-phase ignition process of the condensed substance in the conditions of high thermal fluxes at surplus of an oxidizer and concerning small times of influence of heating source is modelled. Experimental data [9] on critical temperature and thermal flux to bark surface are used in the present work as criteria of ignition.

At carrying out of numerical researches was accepted, that on a pine tree influence the negative cloud-to-ground lightning discharge with duration 500 ms, peak current of 23.5 kA and voltage of 100 kV. It is typical parameters of discharge [11, 12]. Tree trunk radius temperature distribution is



**Figure 2.** Temperature distribution on tree trunk radius during the various moments of time (duration of discharge action is 500 ms): a –  $t = 0.01$  s; b – 0.1 s; c – 0.3 s; d – 0.5 s.

**Table 1.** Experimental data on conditions of pine wood ignition [9].

Ignition delay time, s	Heat flux, kW/m <sup>2</sup>	Surface temperature, K
63.5	12.5	658
45.0	21	700
11.1	42	726
2.6	84	773
0.4	210	867

presented on Fig. 2 to the various moments of time up to and at the moment of tree trunk ignition by an electric current.

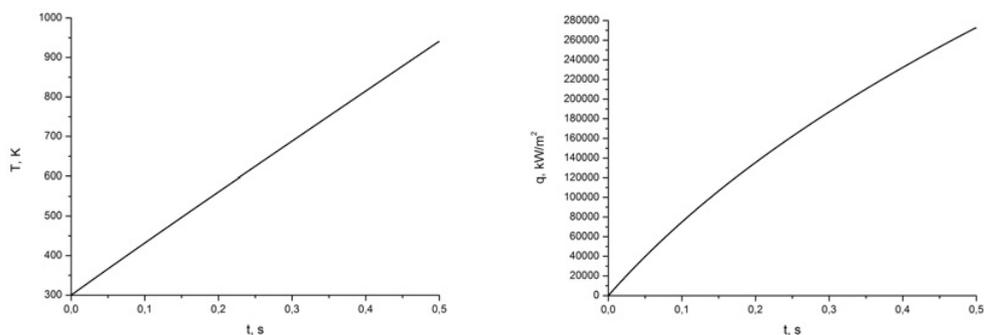
Dependence of thermal flux from subcrustal zone to ignition surface of tree trunk from time (a), and temperature of this border (b) during the various moments of time is shown on Fig. 3. Ignition conditions of wood trunk were defined on experimental data [9] (Table 1).

Results of numerical calculations of ignition conditions shown in Table 2 (depending on voltage of cloud-to-ground lightning discharge are resulted at duration of 500 ms impulse).

Ignition conditions depending on current of cloud-to-ground lightning discharge are shown in Table 3 at voltage  $U = 100$  kV.

The analysis of the results presented on Fig. 2, shows, that tree trunk in subcrustal zone is warmed up as a result of cloud-to-ground lightning discharge action to temperatures at which natural combustible materials burn (more than 1200 K). Analyzed results allow to drawn conclusion, that tree trunk ignites and, at least, chars. Besides, at such temperatures occurs ablation of wood material to formation of furrows on trunk outer side. This conclusion corresponds to data of observations of thunder-storms [13] where it is specified, that tree charred as a result of cloud-to-ground lightning discharge action.

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**Figure 3.** Temperature of subcrustal zone border of coniferous tree trunk (a) and thermal flux to ignition surface of tree trunk from subcrustal zone (b) during the various moments of time.

**Table 2.** Ignition condition of tree depending on voltage of the discharge at current  $J = 23.5$  kA.

Voltage, U, kV	Accordence to conditions [9]	Surface temperature, K	Heat flux from subcrustal zone to surface, kW/m <sup>2</sup>
1–85	No	<867	<210
90	Yes	867	242
95	Yes	867	246
100	Yes	867	249
105	Yes	867	252
110	Yes	867	255

**Table 3.** Ignition condition of tree trunk depending on current at voltage  $U = 100$  kV.

Current, J, kA	Accordence to conditions [9]	Surface temperature, K	Heat flux from subcrustal zone to surface, kW/m <sup>2</sup>
1–20	No	<867	<210
23.5	Yes	867	249
30	Yes	867	264
35	Yes	867	274

The analysis of dependences of thermal flux value and temperature of subcrustal zone border (Fig. 3) shows following results. Ignition conditions (temperature 867 K and thermal flux 249 kW/m<sup>2</sup>) for the considered discharge are reached for enough typical parameters of cloud-to-ground lightning discharge. Obtained results also show, that the estimation of forest fire occurrence conditions as result of cloud-to-ground lightning discharges can be spent at use enough simple mathematical model (1)–(8). This model can be easily realized as a part of forest fire danger forecast systems [14]. Parameters of cloud-to-ground lightning discharge and wood characteristic serve as initial data for operation of such model. Voltage, current and duration of the concrete discharge can be estimated or even fixed by means of modern registration systems of storm activity [7, 11, 12]. Thermophysical characteristics of any types of wood can be defined in simple enough ways [9] for each level of moisture content.

## 4. Conclusion

Result of the present research is numerical realization of mathematical model of coniferous tree warming up as result of lightning stroke electric current. Parametrical research conditions of realization of the

considered phenomenon are lead. This phenomenon is characteristic for typical range of parameters of cloud-to-ground lightning discharge. Possibility of coniferous tree ignition by the cloud-to-ground lightning discharge in the conditions of thunder-storm is shown. Presented physical and mathematical models can be included in subsystem for estimation of forest fire occurrence probability[14]. Besides, obtained results supplement theoretical base for the further development of models for fire-dangerous materials ignitions.

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