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Classical and quantum parts in Madelung variables

Splitting the source term of the Einstein equation into classical and quantum parts

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Abstract. Postulating a particular quantum correction to the source term in the classical Einstein equation we identify the conformal content of the above action and obtain classical gravitation for massive particles, but with a cosmological term representing off-mass-shell contribution to the energy-momentum tensor.

1 Introduction

In this short paper we report an interesting observation based on a conformal treatment of the Schrödinger equation: as if the quantum mechanical problem of obtaining wave functions could be splitted to a massive and a conformal part in line with a classical – quantum partition. Identifying the quantum part as belonging to a traceless relativistic energy-momentum tensor, we suggest to modify its Bohm-Takabayashi form [1] and connect the remaining classical part to Einstein's gravity equation in form of a dust matter source of massive point particles moving on Bohm trajectories. In this scenario the quantum nature of the wave function revals itself in deviations from the classical on-mass-shell relation $P_\mu P^\mu = (mc)^2$, and our suggested natural coupling to gravity makes a simple conformal transformation of the full Einstein tensor expedient. The details of the outlined calculations are given in [2].

2 Energy-momentum tensors of the Klein-Gordon equation

In the following Greek letters denote four-indices and $g_{\mu\nu}$ is the flat Lorentzian spacetime metric. $\hbar$, c and m denotes the Planck constant, the spead of light and the mass of a particle.

The textbook form of the Klein-Gordon energy-momentum tensor reads as

$$T_{\mu\nu} = mcR^2 u_\mu u_\nu + \frac{\hbar^2}{2mc}\left(2\partial_\mu R\,\partial_\nu R - g_{\mu\nu}\left(\partial_\alpha R \partial^\alpha R + R\Box R\right)\right). \tag{1}$$

Here the wave function is represented by the real fields R and α, according to the following decomposition:

$$\psi = \frac{\hbar}{\sqrt{mc}}\, R\, e^{\frac{i}{\hbar}\alpha}, \tag{2}$$

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and the $u_\mu = \partial_\mu \alpha/(mc)$ vector field is analogous to a four-velocity with an off-mass shell dispersion relation. In a metric view this dispersion relation is:

$$g_{\mu\nu} u^\mu u^\nu = 1 + \left(\frac{\hbar}{mc}\right)^2 \frac{\Box R}{R}. \tag{3}$$

This formula, multiplied by $R^2/2$ can be rewritten as a full divergence of the *Bohm-Takabayashi* energy-momentum tensor

$$\mathcal{T}_{\mu\nu} = mcR^2 u_\mu u_\nu - \frac{\hbar^2}{2mc}\left(R\partial_\mu\partial_\nu R - \partial_\mu R\,\partial_\nu R\right). \tag{4}$$

The full energy-momentum tensor can be obtained by the linear combination of the above Klein-Gordon and Takabayashi forms, requiring, that only the classical dust contributes to the trace and the part proportional to $\hbar^2$ – the quantum contribution – is traceless. Introducing dilaton field like variable [3–6] by $\sigma = \ln(\sqrt{V}R)$, where V is constant, it can be written as:

$$\mathfrak{T}^\mu_\nu = \frac{mc}{V} e^{2\sigma} u^\mu u_\nu + \frac{\hbar^2}{2mcV} e^{2\sigma}\left(2\lambda\partial^\mu\sigma\,\partial_\nu\sigma + (\lambda-1)\partial^\mu\partial_\nu\sigma - \lambda\delta^\mu_\nu\left(2\partial_\alpha\sigma\partial^\alpha\sigma + \Box\sigma\right)\right). \tag{5}$$

3 General relativistic quantum mechanics

The previous formula is interpreted in a general relativistic framework by a conformal transformation of a slightly modified Einstein equation:

$$G^\mu_\nu + \Lambda\delta^\mu_\nu = \left[\overline{G}^\mu_\nu + (2\Box s + \partial_\alpha s\partial^\alpha s + \Lambda)\,\delta^\mu_\nu - 2\partial^\mu\partial_\nu s + 2\partial^\mu s\,\partial_\nu s\right] = \frac{8\pi G}{c^3}\, e^{-2\sigma}\,\mathfrak{T}^\mu_\nu. \tag{6}$$

Where the modification is in the source term multiplied by $e^{-2\sigma}$.

The term by term comparison of (5) and (6) reveals that $s = \sigma$ leads to a classical Einstein equation governed by the dust moving on Bohmian trajectories

$$\overline{G}^\mu_\nu = \frac{8\pi L_S}{V} u^\mu u_\nu. \tag{7}$$

We obtain also a cosmological term proportional to the off-mass-shell effect. That is, we have splitted the slightly modified Einstein equation with a cosmological term (6) in the form

$$G^{flat}_{\mu\nu} - 3\frac{\Box R}{R} g^{flat}_{\mu\nu} = \frac{8\pi G}{c^3} e^{-2\sigma}\left(T^{classical}_{\mu\nu} + T^{quantum}_{\mu\nu}\right), \tag{8}$$

and obtained a quantum interpretation for the cosmological term in flat metric. In this scenario the - on the Planck scale surprisingly small - cosmological constant stems from quantum binding with a Bohr radius a as being $\Lambda = 3/a^2$. This is the same relation as for the de Sitter cosmological horizon.

References

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