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Deformation quantization

Quantum mechanics lives and works in phase space

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Abstract. Wigner's 1932 quasi-probability Distribution Function in phase-space, his first paper in English, is a special (Weyl) representation of the density matrix. It has been useful in describing quantum flows in semiclassical limits; quantum optics; nuclear and physics; decoherence (eg, quantum computing); quantum chaos; "Welcher Weg" puzzles; molecular Talbot–Lau interferometry; atomic measurements. It is further of great importance in signal processing (time-frequency analysis).

Nevertheless, a remarkable aspect of its internal logic, pioneered by H. Groenewold and J. Moyal, has only blossomed in the last quarter-century: It furnishes a third, alternate, formulation of Quantum Mechanics, independent of the conventional Hilbert Space (the gold medal), or Path Integral (the silver medal) formulations, and perhaps more intuitive, since it shares language with classical mechanics: one need not choose sides between coordinate or momentum space variables, since it is formulated simultaneously in terms of position and momentum.

This bronze medal formulation is logically complete and self-standing, and accommodates the uncertainty principle in an unexpected manner, so that it offers unique insights into the classical limit of quantum theory. The observables in this formulation are c-number functions in phase space instead of operators, with the same interpretation as their classical counterparts, only now composed together in novel algebraic ways using *star products*.

One might then envision an imaginary world in which this formulation of quantum mechanics had preceded the conventional Hilbert-space formulation, and its own techniques and methods had arisen independently, perhaps out of generalizations of classical mechanics and statistical mechanics.

A sampling of such intriguing techniques and methods has already been published in C. K. Zachos, Int Jou Mod Phys **A17** 297-316 (2002), and T. L. Curtright, D. B. Fairlie, and C. K. Zachos, *A Concise Treatise on Quantum Mechanics in Phase Space*, (Imperial Press & World Scientific, 2014).

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