

$\Delta I = 1/2$ Rule and \hat{B}_K : 2014

Andrzej J. Buras^{1,2,a}

¹TUM Institute for Advanced Study, Lichtenbergstr. 2a, D-85748 Garching, Germany

²Physik Department, Technische Universität München, James-Frank-Straße, D-85748 Garching, Germany

Abstract. I summarize the status of the $\Delta I = 1/2$ rule in $K \rightarrow \pi\pi$ decays within an *analytic* approach based on the dual representation of QCD as a theory of weakly interacting mesons for large N , where N is the number of colours. This approximate approach, developed in the 1980s by William Bardeen, Jean-Marc Gérard and myself, allowed us already 28 years ago to identify the dominant dynamics behind the $\Delta I = 1/2$ rule. However, the recent inclusion of lowest-lying vector meson contributions in addition to the pseudoscalar ones to hadronic matrix elements of current-current operators and the calculation of the corresponding Wilson coefficients in a momentum scheme at the NLO improved significantly the matching between quark-gluon short distance contributions and meson long distance contributions over our results in 1986. We obtain satisfactory description of the $\text{Re}A_2$ amplitude and $\text{Re}A_0/\text{Re}A_2 = 16.0 \pm 1.5$ to be compared with its experimental value of 22.3. While this difference could be the result of present theoretical uncertainties in our approach, it cannot be excluded that New Physics (NP) is here at work. The analysis by Fulvia De Fazio, Jennifer Girrbach-Noe and myself shows that indeed a tree-level Z' or G' exchanges with masses in the reach of the LHC and special couplings to quarks can significantly improve the theoretical status of the $\Delta I = 1/2$ rule while satisfying constraints from ε_K , ε'/ε , ΔM_K , LEP-II and the LHC. The ratio ε'/ε plays an important role in these considerations. I stress that our approach allows to understand the physics behind recent numerical results obtained in lattice QCD not only for the $\Delta I = 1/2$ rule but also for the parameter \hat{B}_K that enters the evaluation of ε_K . In contrast to the $\Delta I = 1/2$ rule and ε'/ε the chapter on \hat{B}_K in QCD appears to be basically closed.

1 Introduction

One of the puzzles of the 1950s was a large disparity between the measured values of the real parts of the isospin amplitudes A_0 and A_2 for a kaon to decay into two pions which on the basis of usual isospin considerations were expected to be of the same order. In 2014 we know the experimental values of the real parts of these amplitudes very precisely [1]

$$\text{Re}A_0 = 27.04(1) \times 10^{-8} \text{ GeV}, \quad \text{Re}A_2 = 1.210(2) \times 10^{-8} \text{ GeV}. \quad (1)$$

^ae-mail: aburas@ph.tum.de

As $\text{Re}A_2$ is dominated by $\Delta I = 3/2$ transitions but $\text{Re}A_0$ receives contributions also from $\Delta I = 1/2$ transitions, the latter transitions dominate $\text{Re}A_0$ which expresses the so-called $\Delta I = 1/2$ rule [2, 3]

$$R = \frac{\text{Re}A_0}{\text{Re}A_2} = 22.35. \quad (2)$$

In the 1950s QCD and Operator Product Expansion did not exist and clearly one did not know that W^\pm bosons existed in nature but using the ideas of Fermi, Gell-Mann, Feynman, Marshak and Sudarshan one could still evaluate the amplitudes $\text{Re}A_0$ and $\text{Re}A_2$ to find out that such a high value of R is a real puzzle.

In modern times we can reconstruct this puzzle by evaluating the simple W^\pm boson exchange between the relevant quarks which after integrating out W^\pm generates the current-current operator Q_2 :

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}, \quad Q_2 = (\bar{s}u)_{V-A} (\bar{u}d)_{V-A}. \quad (3)$$

We have listed here the second current-current operator, Q_1 , which we will need soon. With only Q_2 contributing we have

$$\text{Re}A_{0,2} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \langle Q_2 \rangle_{0,2}. \quad (4)$$

Calculating the matrix elements $\langle Q_2 \rangle_{0,2}$ in the strict large N limit, which corresponds to factorization of matrix elements of Q_2 into the product of matrix elements of currents, we find

$$\text{Re}A_0 = 3.59 \times 10^{-8} \text{ GeV}, \quad \text{Re}A_2 = 2.54 \times 10^{-8} \text{ GeV}, \quad R = \sqrt{2} \quad (5)$$

in plain disagreement with the data in (1) and (2). It should be emphasized that the explanation of the missing enhancement factor of 15.8 in R through some dynamics must simultaneously give the correct values for $\text{Re}A_0$ and $\text{Re}A_2$. This means that this dynamics should suppress $\text{Re}A_2$ by a factor of 2.1, not more, and enhance $\text{Re}A_0$ by a factor of 7.5.

It is evident that what is missing in this calculation are strong interaction effects represented these days by QCD but the question arises whether the physical picture behind the $\Delta I = 1/2$ rule as described by QCD has a simple structure. As demonstrated by Bardeen, Gérard and myself already in 1986 [4] and improved on the technical level by us recently [5] the dominant dynamics behind the $\Delta I = 1/2$ rule has in fact a simple structure.

To this end one should note that from the point of view of operator product expansion the calculation we have just performed to get (5) corresponds to

- The evaluation of the Wilson coefficient of the operator Q_2 in a free (from the point of view of strong interactions) theory of quarks, which corresponds to scales $\mu = O(M_W)$ and setting $\alpha_s(M_W) = 0$.
- The evaluation of hadronic matrix elements $\langle Q_2 \rangle_{0,2}$ in a free theory of mesons which corresponds to the factorization scale $\mu = O(m_\pi) \approx 0$ and setting N to infinity.

The second point follows from the dual representation of QCD as a theory of weakly interacting mesons for large N , advocated already in the 1970s in [6–9]. In the strict large N limit QCD becomes a free theory of mesons and in this limit the calculation of hadronic matrix elements by means of factorization method is correct within QCD [10]. But as the Wilson coefficient of Q_2 has been evaluated at $\mu = O(M_W)$ and its hadronic matrix elements at $\mu = O(m_\pi) \approx 0$ our calculation of $\text{Re}A_0$ and $\text{Re}A_2$ is incomplete. In order to complete it we have to fill the gap between these two vastly different energy scales with QCD dynamics represented by quark-gluon interactions at short distance scales and by meson interactions at long distance scales. This requires the inclusion of α_s effects at short distances and $1/N$ corrections in the meson theory at long distances.

In Section 2 I will describe the structure of our approach together with results for the $A_{0,2}$ amplitudes in three steps and will compare it with the lattice QCD approach. In this context I will also summarize the status of the parameter \hat{B}_K . In Section 3 I will summarize an analysis performed by Fulvia De Fazio, Jennifer Girrbach-Noe and myself which demonstrates that tree-level Z' or G' exchanges with masses in the reach of the LHC and special couplings to quarks can significantly improve the theoretical status of the $\Delta I = 1/2$ rule while satisfying constraints from ε_K , ε'/ε , ΔM_K , LEP-II and the LHC. Few comments in Section 4 close this brief review. I am presenting here the way I see the dynamics behind the $\Delta I = 1/2$ rule. Over the years other views have been expressed in the literature. See in particular [11] and most recent papers [12, 13] where further references can be found.

2 The Dynamics behind the $\Delta I = 1/2$ Rule

2.1 Step 1: Quark-Gluon Evolution

This step involves the calculation of the Wilson coefficients $z_{1,2}$ of the current-current operators $Q_{1,2}$ at a low energy scale $\mu = O(1 \text{ GeV})$ and fills the gap present in our simple calculation between this scale and the electroweak scale $O(M_W)$. Having them one can calculate the $K \rightarrow \pi\pi$ decay amplitudes in the Standard Model using

$$A(K \rightarrow \pi\pi) = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1,2} z_i(\mu) \langle \pi\pi | Q_i(\mu) | K \rangle, \quad (6)$$

where QCD penguin contributions have been omitted as they will be included in Step 3 below. We have indicated that the matrix elements are to be evaluated at $\mu = O(1 \text{ GeV})$ but in this step we will still keep them at $\mu \approx 0$ and use their values calculated in the strict large N limit. We will improve on this in Step 2.

The coefficients $z_i(\mu)$ have been calculated at leading order in the renormalization group improved perturbation theory in [14, 15]. This pioneering calculations of short distance QCD effects have shown that these effects indeed enhance $\text{Re}A_0$ and suppress $\text{Re}A_2$. However, the inclusion of NLO QCD corrections to $z_{1,2}$ [16, 17] made it clear, as stressed in particular in [17], that the $K \rightarrow \pi\pi$ amplitudes without the proper calculation of hadronic matrix elements of Q_i are both scale and renormalization scheme dependent. For instance setting $\mu = 0.8 \text{ GeV}$ we find

$$R_{cc}(\text{NDR} - \overline{\text{MS}}) \approx 3.0, \quad R_{cc}(\overline{\text{MOM}}) \approx 4.4, \quad (7)$$

where the subscript cc indicates that only current-current contributions have been taken into account and $\overline{\text{MOM}}$ is a momentum scheme, introduced in [5], which is particularly suited for the calculations of the amplitudes in our approach. In this scheme one finds then for $\mu = 0.8 \text{ GeV}$

$$\text{Re}A_0 = 7.1 \times 10^{-8} \text{ GeV}, \quad \text{Re}A_2 = 1.6 \times 10^{-8} \text{ GeV}. \quad (8)$$

This is a significant improvement over the results in (5) bringing the theory closer to the data in (1) and (2). However, this result is scale and renormalization scheme dependent. For $\text{NDR} - \overline{\text{MS}}$ scheme and $\mu \approx (2 - 3) \text{ GeV}$ as used in lattice QCD calculations this improvement would be much smaller. But, even in $\overline{\text{MOM}}$ scheme and at $\mu = 0.8 \text{ GeV}$, further enhancement of $\text{Re}A_0$ and further suppression of $\text{Re}A_2$ are needed in order to be able to understand the $\Delta I = 1/2$ rule. This brings us to Step 2 which fills the remaining gap in our original calculation.

2.2 Step 2: Meson Evolution

The renormalization group evolution down to the scales $\mathcal{O}(1 \text{ GeV})$ just performed is continued as a short but fast meson evolution down to zero momentum scales at which the factorization of hadronic matrix elements is at work. Equivalently, starting with factorizable hadronic matrix elements $\langle Q_1 \rangle_{0,2}$ and $\langle Q_2 \rangle_{0,2}$ at $\mu \approx 0$ and evolving them to $\mu = \mathcal{O}(1 \text{ GeV})$ at which $z_{1,2}$ are calculated one is able to calculate the matrix elements of these two operators at $\mu = \mathcal{O}(1 \text{ GeV})$ and properly combine them with $z_{1,2}$ calculated in the $\overline{\text{MOM}}$ scheme. Details of these calculations can be found in [4, 5] and there is no space for presenting them here. I just want to make a few comments:

- Our loop calculations in the meson theory with a cut-off $M = \mathcal{O}(1 \text{ GeV})$ include the contributions from pseudoscalars and lowest-lying vector mesons and the result can be cast in the form of evolution equations. It is remarkable that the structure of these evolution equations, in particular the anomalous dimension matrix in the meson theory, is very similar to the one in the quark-gluon picture. This allows to perform an adequate matching between the two evolutions in question thereby removing to a large extent scale and renormalization scheme dependences present in the results of Step 1.
- The inclusion of vector meson contributions in [5] in addition to pseudoscalar contributions calculated in [4] is a significant improvement over our 1986 analysis bringing the theory closer to data.
- The same comment applies to the matching between the quark-gluon and meson theory which this time has been performed at NLO in QCD. In this manner we could justify equating the physical cut-off M of the truncated meson theory (pseudoscalars and lowest-lying vector mesons) with the renormalization scale μ in the quark-gluon theory.

The resulting values

$$\text{Re}A_0 \approx (13.3 \pm 1.0) \times 10^{-8} \text{ GeV}, \quad \text{Re}A_2 \approx (1.1 \pm 0.1) \times 10^{-8} \text{ GeV}. \quad (9)$$

show a very significant improvement over the results in (8) bringing the theory closer to the data in (1) and (2). In particular within the uncertainties of our approach we can claim that the experimental value of $\text{Re}A_2$ has been reproduced. The amplitude $\text{Re}A_0$ has been enhanced in this step by almost a factor of two relative to the result in (8) but it is still by a factor of two below the data. But whereas the calculation of $\text{Re}A_2$ has been completed in this step, in order to complete the calculation of $\text{Re}A_0$ we have to include QCD penguin contribution to this amplitude. This brings us to Step 3.

2.3 Step 3: QCD Penguins

As pointed out in [18] QCD penguin operators, of which the dominant one is

$$Q_6 = -8 \sum_{q=u,d,s} (\bar{s}_L q_R)(\bar{q}_R d_L), \quad (10)$$

could play an important role in enhancing the ratio R as in the isospin limit they do not contribute to A_2 and uniquely enhance the amplitude A_0 . However, in 1975 the relevant matrix element $\langle Q_6 \rangle_0$ was unknown within QCD and its Wilson coefficient z_6 was poorly known. The first large N result for this matrix element using factorization approach has been obtained in [19] and have been subsequently confirmed in [20, 21] by using an effective Lagrangian describing the weak and strong interactions of mesons in the large N limit. It is given by

$$\langle Q_6(\mu) \rangle_0 = -4 \left[\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right]^2 (F_K - F_\pi) B_6^{(1/2)}, \quad B_6^{(1/2)} = 1, \quad (11)$$

where we have introduced the parameter $B_6^{(1/2)}$ which equals unity in the large N limit.

While this matrix element is much larger than the matrix elements of $Q_{1,2}$, its Wilson coefficient z_6 is strongly GIM suppressed at scales $\mathcal{O}(m_c)$ due to the fact that it results from the difference of QCD penguin diagrams with charm and up-quark exchanges. If these masses are neglected above $\mu = m_c$ then $z_6(m_c) = 0$ and its value is roughly by an order of magnitude smaller than $z_{1,2}$ at $\mu = 0.8$ GeV. In [21] an additional (with respect to previous estimates) enhancement of the QCD penguin contributions to $\text{Re}A_0$ has been identified. It comes from an incomplete GIM cancellation above the charm quark mass. But as the analyses in [4, 5] show, this enhancement is insufficient to reproduce fully the experimental value of $\text{Re}A_0$. We find that the Q_6 contribution to $\text{Re}A_0$ for $\mu \leq 1$ GeV is relevant as it is by a factor of 3 larger than $\text{Re}A_2$. Yet at $\mu = 0.8$ GeV it contributes only at the level of 15% to the experimental value of $\text{Re}A_0$.

2.4 Summary of Results

Our final results for $K \rightarrow \pi\pi$ amplitudes can be summarized as follows

$$\text{Re}A_0 \approx (17.0 \pm 1.5) \times 10^{-8} \text{ GeV}, \quad \text{Re}A_2 \approx (1.1 \pm 0.1) \times 10^{-8} \text{ GeV}, \quad R \approx 16.0 \pm 1.5. \quad (12)$$

Even if the result for $\text{Re}A_0$ is not satisfactory, it should be noted that the QCD dynamics identified by us was able to enhance the ratio R by an order of magnitude. We therefore conclude that QCD dynamics is dominantly responsible for the $\Delta I = 1/2$ rule.

In Fig. 1 we show budgets for $\text{Re}A_2$ (left) and $\text{Re}A_0$ (right) that summarize the size of different suppression mechanisms of $\text{Re}A_2$ and enhancement mechanisms of $\text{Re}A_0$. SD stands for quark-gluon evolution and LD for meson evolution. In the case of $\text{Re}A_0$ we decompose LD into contributions coming from the meson evolution involving only Q_1 and Q_2 (c_1) and the one related to the mixing of $Q_{1,2}$ and Q_6 (c_2). QCDP stands for Q_6 contribution. We set the matching scale at $\mu = 0.8$ GeV. As can be seen, we are not able to explain fully the missing $\Delta\text{Re}A_0 = 23.4 \times 10^{-8}$ GeV relative to the large N limit. Different contributions in the budget are normalized to this additive contribution required by the data. The missing piece that we presently cannot explain by QCD dynamics within our approach is represented by the white area. More details on this budget can be found in [5].

2.5 Comments on Lattice QCD Results

Lattice QCD calculations made significant progress in the last five years through the inclusion of dynamical fermions [22, 23]. Among many results the precise values for the weak decays constants F_K , F_{B_d} , F_{B_s} and F_D should be mentioned here. The values of the non-perturbative parameters B_i representing $\Delta F = 2$ operators both within the SM and in its extensions require further improvements, but it is likely that in this decade they will be known with high precision.

From my point of view, the most important lattice QCD results as far as B_i parameters are concerned are the following ones (recent FLAG update of [24] and [25]):

$$\hat{B}_K = 0.766 \pm 0.010, \quad B_8^{(3/2)}(3 \text{ GeV}) = 0.65 \pm 0.05, \quad (\text{in lattice QCD, 2014}). \quad (13)$$

The first one is relevant for the parameter ε_K and the second for the contribution of the dominant electroweak penguin operator Q_8 to the ratio ε'/ε . Unfortunately there is no reliable result on $B_6^{(1/2)}$ in (11) from lattice QCD so that ε'/ε cannot be calculated in this approach at present.

Concerning \hat{B}_K , the result in (13) confirmed with higher precision our finding in [26] that \hat{B}_K is rather close to its large N value $\hat{B}_K = 0.75$. While in 1987, including only pseudoscalar meson

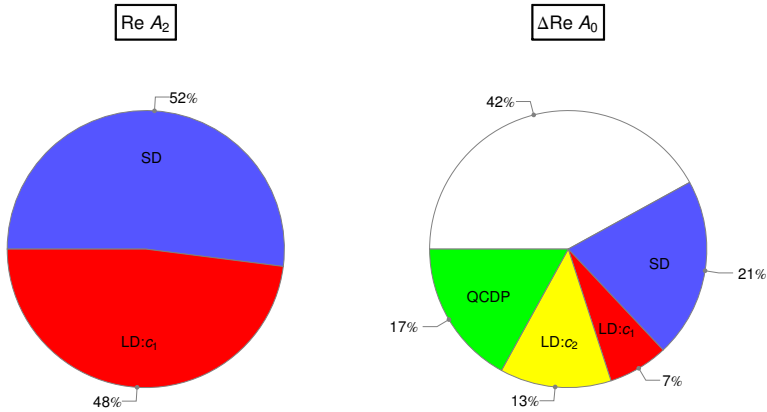


Figure 1. Budgets for $\text{Re}A_2$ (left) and $\Delta\text{Re}A_0$ (right) summarizing the size of different suppression mechanisms of $\text{Re}A_2$ and enhancement mechanisms of $\text{Re}A_0$, denoted here by $\Delta\text{Re}A_0$, for the matching scale $\mu = M = 0.8 \text{ GeV}$. SD stands for quark-gluon evolution and LD for meson evolution. In the case of $\Delta\text{Re}A_0$ we decompose LD into contributions coming from c_1 and c_2 . QCDP stands for Q_6 contribution. See the text and [5] for explanations.

contributions we found $\hat{B}_K = 0.66 \pm 0.07$, our recent calculation that takes into account also vector meson contributions and improves the matching between the meson and quark-gluon theory gives [5]

$$\hat{B}_K = 0.73 \pm 0.02, \quad (\text{in dual QCD, 2014}). \quad (14)$$

This result is in an excellent agreement with the lattice QCD value in (13) although we are aware of the fact that while lattice calculations have good control over their errors, this is not quite the case here. On the other hand, while until now lattice community did not provide, as far as I know, any explanation why after 25 years of efforts they obtained the result for \hat{B}_K within 2% from its large N value, our approach provides the explanation why $1/N$ corrections are so small. The smallness of these corrections results from an approximate cancellation between pseudoscalar and vector meson one-loop contributions. It is encouraging that such a simple analytic approach could provide some insight in the lattice results for \hat{B}_K . On the other hand there is a qualitative difference between the results in (13) and (14). While the lattice result finds $1/N$ corrections to be *positive*, Gérard has demonstrated diagrammatically in [27] that it must be *negative*. I expect therefore that future lattice results will confirm this result with higher precision than we could do it in our approach.

As far as the $\Delta I = 1/2$ rule is concerned, a detailed comparison of the results of our approach with the results from the RBC-UKQCD collaboration [25, 28–30] can be found in section 9 in [5]. The results for the amplitudes $\text{Re}A_0$ and $\text{Re}A_2$ in lattice QCD are presented in terms of the contractions (1) and (2) which are depicted in Fig. 1 of [28]. Basically, Q_2 contributes to $K^0 \rightarrow \pi^+\pi^-$ and $K^0 \rightarrow \pi^0\pi^0$ through contractions (1) and (2), respectively, while in the case of Q_1 the role of contractions is interchanged. The explicit formulae for $\text{Re}A_0$ and $\text{Re}A_2$ in terms of these contractions can be found in (122) and (123) in [5].

Now in [28] (2) ≈ -0.7 (1) has been found. This is an important result as it leads to an additional suppression of $\text{Re}A_2$ and additional enhancement of $\text{Re}A_0$ beyond the one from quark-gluon evolution, which in [28] is stopped at $\mu = 2.15 \text{ GeV}$. These suppressions and enhancements due to the different

signs of contractions in question correspond to Step 2 in our approach. Similar to the case of \hat{B}_K the authors of [28] did not provide yet the explanation for the relative sign of these two contractions while this is possible within our approach. We find [5]

$$\textcircled{1} = \frac{X_F}{\sqrt{2}}, \quad \textcircled{2} = -0.33 \frac{X_F}{\sqrt{2}}, \quad X_F = \sqrt{2} F_\pi (m_K^2 - m_\pi^2), \quad (15)$$

where the negative sign follows in our approach from the proper matching of the anomalous dimension matrices in the meson and quark-gluon pictures of QCD. It is also obtained from explicit one-loop calculation in the meson theory and can also be seen diagrammatically as discussed in [5].

Even if with $X_F = 0.0298 \text{ GeV}^3$ the values of the contractions in (15) appear at first sight to be much smaller than the ones presented in [28], it should be noted that lattice groups work with other renormalization schemes and different scales. In fact one can demonstrate, as seen in (15), that in our case the factor relating $\textcircled{2}$ and $\textcircled{1}$ must be smaller in magnitude. Therefore the numerical comparison of the results of [28] with ours must also involve the Wilson coefficients z_i . The fact that our approach and lattice approach predict similar values for $\text{Re}A_2$ implies the compatibility of both approaches as far as $\Delta I = 3/2$ transitions are concerned. Indeed the lattice result for $\text{Re}A_2$ in [25] reads:

$$\text{Re}A_2 = (1.13 \pm 0.21) \times 10^{-8} \text{ GeV}, \quad (16)$$

where the error is dominated by systematics. This result is in agreement with the data and, within uncertainties, with our result. We find it remarkable that the central value in (16) differs from our central value in (12) by only a few percent. This is still another support for the dual picture of QCD. There is no reliable result for $\text{Re}A_0$ from lattice QCD yet but on the basis of present calculations $R \approx 11$, still by a factor of two below the data. As QCD penguin contributions at $\mu = (2 - 3) \text{ GeV}$ are found to be small, we expect that future lattice calculations of hadronic matrix elements of $Q_{1,2}$ will imply significantly larger values of R .

I would like to end this comparison with lattice QCD with a few personal comments:

- I find the study of $K \rightarrow \pi\pi$ decays in lattice QCD very important but as long as lattice calculations of hadronic matrix elements are performed at $\mu = (2 - 3) \text{ GeV}$ I do not expect that we will gain a satisfactory physical understanding of the dynamics behind the $\Delta I = 1/2$ rule from this approach. Obtaining just two numbers for $\text{Re}A_0$ and $\text{Re}A_2$ from very demanding computer simulations without the understanding of the dynamics behind them would be rather disappointing after almost 60 years of efforts to understand the $\Delta I = 1/2$ rule. I believe that combining the physical insight on the dynamics behind the $\Delta I = 1/2$ rule gained through dual QCD approach presented above with lattice QCD calculations could eventually completely uncover the puzzles of the 1950s on $K \rightarrow \pi\pi$ decays.
- On the other hand, from the present perspective only lattice simulations with dynamical fermions can provide precise values of $\text{Re}A_{0,2}$ one day, but this may still take several years of intensive efforts by the lattice community [22, 23, 31]. Having precise SM values for $\text{Re}A_{0,2}$ would determine precisely the room for NP contribution left not only in $\text{Re}A_0$ but also $\text{Re}A_2$. In turn this would give us two observables which could be used to constrain NP.
- While the issue of the $\Delta I = 1/2$ rule is important, in my opinion more pressing is the calculation of $B_6^{(1/2)}$ as this would allow one to constrain a number of NP scenarios with the help of ε'/ε .

Other applications of large N ideas to $K \rightarrow \pi\pi$ and \hat{B}_K , but sometimes in a different spirit than our original approach, are reviewed in [32]. I refer in particular to [33–43]. A recent review of $SU(N)$ gauge theories at large N can be found in [44].

Finally I hope that the community of lattice experts will *eventually* acknowledge the physical relevance of our simple analytical approach and give us credit for a number of findings, listed in [5], that they confirmed 28 years later. Afterall, our approach provided an insight into the dynamics behind the $\Delta I = 1/2$ rule and offered the explanation why \hat{B}_K is so close to 0.75. At least three colleagues in Rome [45] gave us credit for the signs of $1/N$ corrections in QCD to $K \rightarrow \pi\pi$ matrix elements and B_K that are opposite to the ones obtained using vacuum insertion approximation.

3 Z', G' Effects in $K \rightarrow \pi\pi$

3.1 $\Delta I = 1/2$ Rule

As we have seen, presently the value of $\text{Re}A_0$ within dual QCD approach is by 30% below the data and even more in the case of lattice QCD. While this deficit could be the result of theoretical uncertainties in both approaches, it cannot be excluded that the missing piece in $\text{Re}A_0$ comes from NP. This question has been addressed in [46] and I will briefly report on the results of this work.

In this paper we have first demonstrated that a significant part of the missing piece in $\text{Re}A_0$ can be explained by tree-level FCNC transitions mediated by a heavy colourless Z' gauge boson with flavour violating *left-handed* coupling $\Delta_L^{sd}(Z')$ and approximately universal flavour diagonal *right-handed* coupling $\Delta_R^{qq}(Z')$ to quarks. The approximate flavour universality of the latter coupling assures negligible NP contributions to $\text{Re}A_2$. This property together with the breakdown of GIM mechanisms at tree-level allows to enhance significantly the contribution of the leading QCD penguin operator Q_6 to $\text{Re}A_0$. A large fraction of the missing piece in the $\Delta I = 1/2$ rule can be explained in this manner for $M_{Z'}$ in the reach of the LHC, while satisfying constraints from ε_K , ε'/ε , ΔM_K , LEP-II and the LHC. The presence of a small right-handed flavour violating coupling $\Delta_R^{sd}(Z') \ll \Delta_L^{sd}(Z')$ and of enhanced matrix elements of $\Delta S = 2$ left-right operators allows to satisfy simultaneously the constraints from $\text{Re}A_0$ and ΔM_K , although this requires some fine-tuning. The result of this analysis is summarized by the left plot in Fig. 2.

We have also investigated whether a colour octet of heavy neutral gauge bosons (G') could also help in fully explaining the $\Delta I = 1/2$ rule. It turns that due to various colour factors and different LHC constraints on its mass, G' is even more effective than Z' : it provides, within theoretical uncertainties, the missing piece in $\text{Re}A_0$ for $M_{G'} = (3.5 - 4.0)$ TeV. Indeed we find

$$R = \frac{\text{Re}A_0}{\text{Re}A_2} \approx 18 (Z'), \quad R = \frac{\text{Re}A_0}{\text{Re}A_2} \approx 21 (G') \quad (17)$$

with the second result summarized by the right chart in Fig. 2.

The results presented in [46] and summarized above used the preliminary LHC bounds on the relevant quark couplings provided by Maikel de Vries. In [47] an update on these results has been presented. In particular de Vries points out that the upper bounds on the couplings in the full theory, relevant for our analysis, are slightly softer than the ones following directly from four-quark effective operators at hadron colliders which he provided for our analysis in [46]. This result not only puts our bounds on the size of NP effects on firm footing but also allows for slightly larger NP contributions to $\text{Re}A_0$. Specifically, the bounds on the relevant couplings in (105), (107), (139) and (140) in [46] receive additional corrections represented by the additional terms between the last square brackets in the formulae below:

$$|\Delta_R^{qq}(Z')| \leq 1.0 \left[\frac{M_{Z'}}{3 \text{ TeV}} \right] \left[1 + \left(\frac{1.3 \text{ TeV}}{M_{Z'}} \right)^2 \right], \quad (18)$$

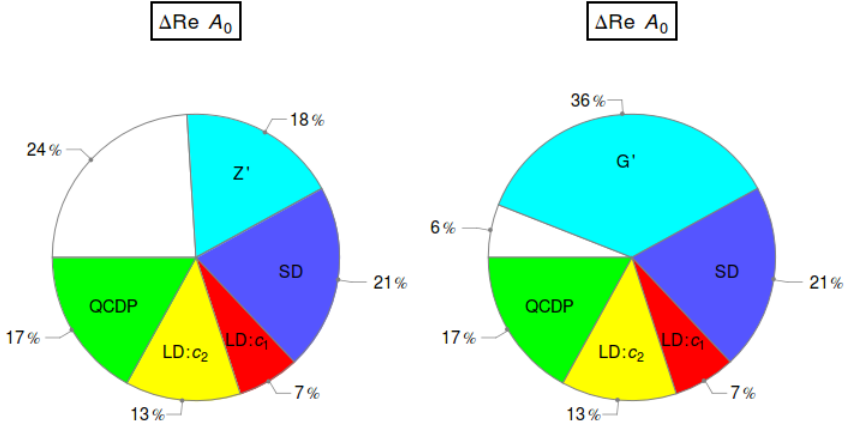


Figure 2. Budgets of different enhancements of $\text{Re}A_0$, denoted here by $\Delta\text{Re}A_0$. Z' and G' denote the contributions calculated in [46]. The remaining coloured contributions come from the SM dynamics as calculated in [5] and shown in Fig. 1. The white region stands for the missing piece.

$$|\Delta_L^{sd}(Z')| \leq 2.3 \left[\frac{M_{Z'}}{3 \text{ TeV}} \right] \left[1 + \left(\frac{1.3 \text{ TeV}}{M_{Z'}} \right)^2 \right], \quad (19)$$

$$|\Delta_R^{qq}(G')| \leq 2.0 \left[\frac{M_{G'}}{3.5 \text{ TeV}} \right] \left[1 + \left(\frac{1.4 \text{ TeV}}{M_{G'}} \right)^2 \right], \quad (20)$$

$$|\Delta_L^{sd}(G')| \leq 2.6 \left[\frac{M_{G'}}{3.5 \text{ TeV}} \right] \left[1 + \left(\frac{1.4 \text{ TeV}}{M_{G'}} \right)^2 \right]. \quad (21)$$

These bounds correspond to the excluded blue regions in Fig. 3 and should be compared with the ones in Figs. 3 and 9 in [46].

The important feature of these results is that all corrections are above unity. In this manner the region representing Z' in Fig. 2 can easily be 20% and in the case of G' the white region can be practically removed. Of course all these changes are within the uncertainties of the analysis in [46] but it is gratifying that the results in [47] put our analysis on firmer footing.

Finally, it should be stressed that the allowed ranges for NP contributions in Fig. 2 are independent of $M_{Z'}$ and $M_{G'}$ as with increased values of these masses the propagator suppression in $\text{Re}A_0$ is compensated by the increase of the allowed ranges for the couplings. The additional corrections in the formulae above introduce weak mass dependence for masses above 3 TeV which should be used in any case to be on the safe side. Of course one has to stay within the perturbative bounds for the couplings involved. This feature tells us that even if Z' and G' would not be found at the LHC, they could still play a role in the $\Delta I = 1/2$ rule if their masses were below 10 TeV. But to find it out would require the study of other observables as discussed in [46]. Moreover, new bounds from the upgraded LHC could further restrict NP contributions to this rule.

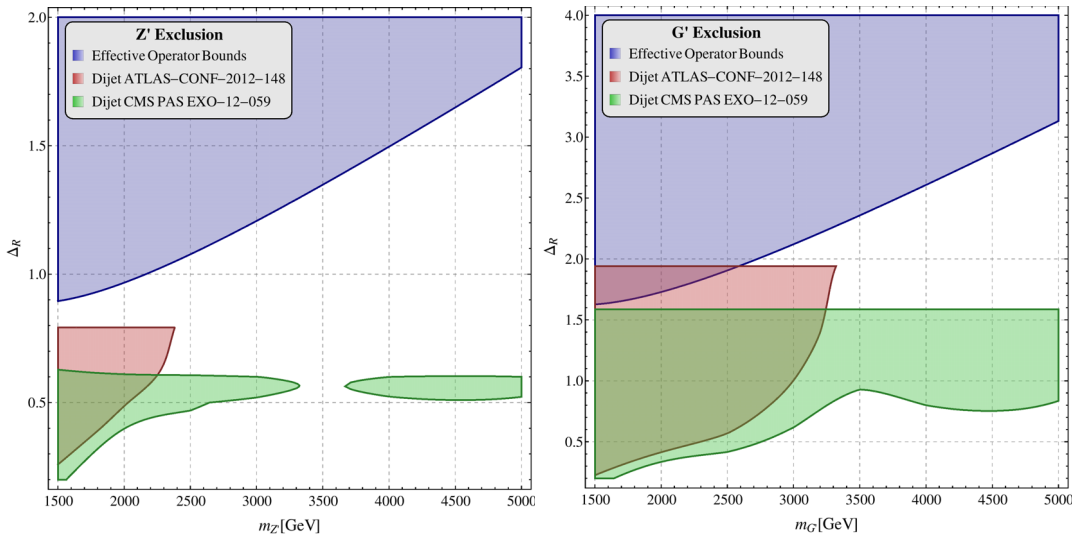


Figure 3. Exclusion limits for the Z' and G' in the mass-coupling plane, from various searches at the LHC based on [47]. The blue region is excluded by effective operator limits studied by ATLAS [48] and CMS[49]. The red and green contours are excluded by dijet resonance searches by ATLAS and CMS, respectively.

3.2 ε'/ε

In view of the improved value for $B_8^{(3/2)}$ from [25] in (13) we have updated in [46] the value of ε'/ε in the SM stressing various uncertainties, originating in the values of $|V_{ub}|$ and $|V_{cb}|$ and also in the parameter $B_6^{(1/2)}$. In particular we have found that the best agreement of the SM with the data is obtained for $B_6^{(1/2)} \approx 1.0$, that is close to the large N limit of QCD. In this paper one can also find the impact of Z' , G' and Z with flavour violating couplings on ε'/ε . There is no doubt that in the 2020s the ratio ε'/ε could become a star of flavour physics as it was in the 1990s.

4 Conclusions

I have reviewed the present understanding of the $\Delta I = 1/2$ rule that emerged within the dual approach to QCD as a theory of weakly interacting mesons for large N already 28 years ago in [4] and has been put on a firmer footing recently in [5]. While lattice QCD will eventually provide much more accurate values for $\text{Re}A_0$ and $\text{Re}A_2$ than it is possible in our approach, our approach provided in my opinion better insight into the dynamics behind this rule than it was possible with lattice QCD until now. But the story is not over as we presently do not know whether at a level of (10 – 30)% NP could be responsible for the measured value of R . Lattice QCD could make an important contribution in answering this question in the coming years.

I am looking forward to improved results on $\text{Re}A_0$ and $\text{Re}A_2$ from lattice QCD and to possible discoveries of Z' and G' at LHC2 in order to see whether these heavy gauge bosons have anything to say in the context of the $\Delta I = 1/2$ rule. But the most pressing now is an accurate evaluation of $B_6^{(1/2)}$ by lattice QCD as ε'/ε is much more sensitive to NP and very short distance scales than the amplitudes $\text{Re}A_0$ and $\text{Re}A_2$.

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