EPJ Web of Conferences 80, 00044 (2014)
DOI: 10.1051/epjconf/20148000044

Strong $D_2^*(2460)^0 \to D^+\pi^-$ Transition in QCD

Y. Sarac[1,a], K. Azizi[2,b], and H. Sundu[3,c]

[1] *Electrical and Electronics Engineering Department, Atilim University, 06836 Ankara, Turkey*
[2] *Physics Department, Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey*
[3] *Department of Physics, Kocaeli University, 41380 Izmit, Turkey*

Abstract. This work presents the analysis of the transition $D_2^*(2460)^0 \to D^+\pi^-$. In the calculation three point QCD sum rules method is applied and the value of the coupling constant for the considered vertex is obtained as $g_{D_2^* D\pi} = (12.72 \pm 3.56)$ GeV^{-1}.The result of the calculation is also utilized in the decay width and branching ratio calculations of the considered transition.

1 Introduction

The first observation of the orbitally excited charmed meson was made in 1986 [1] and the observation of them has continued in the following past few decades [2–11]. This period has also been accompanied by several theoretical studies on the masses, strong and electromagnetic transitions of these mesons done by using various methods (See for instance the Refs. [12–15] and the references there in). $D_2^*(2460)$ having the quantum number $I(J^P) = \frac{1}{2}(2^+)$ is among these orbitally excited mesons and was reported twenty years ago [5, 7, 8].

In the literature one may find little theoretical works on the properties of tensor mesons compared to the other types of mesons. Especially their strong transitions have not been studied much. One can attain valuable information about the internal structures and the natures of these mesons via the study of the parameters of these mesons and comparison of their results with the existing experimental findings. Through these studies one may also test the assumptions of some theoretical calculations and comprehend the experimental results which provides a better understanding of the strong interaction. This type of work may also be helpful to gain useful information related to the study of B meson since charmed tensor mesons appear as an intermediate state in B meson decays. Additionally, the possibility for a search on the transition of D_2^* meson at LHC provides motivation on the study of these mesons.

In this work we present the analysis for the transition $D_2^*(2460)^0 \to D^+\pi^-$. For this purpose we use one of a powerful nonperturbative methods, QCD sum rules, suggested firstly by Shifman [16] for the investigation of the parameters of hadrons in vacuum. Following the introduction the details of the coupling constant calculations for this transition using the QCD sum rules are presented in section 2. Section 3 gives numerical analysis of the coupling constant calculations and calculation of the decay width of the considered transition using the obtained coupling constant result.

[a] e-mail: yasemin.sarac@atilim.edu.tr
[b] e-mail: kazizi@dogus.edu.tr
[c] e-mail: hayriye.sundu@kocaeli.edu.tr

2 QCD sum rules for the coupling constants

In this section the details of the calculations of the coupling constant is presented. For the calculation we use the following three-point correlation function,

$$\Pi_{\mu\nu}(p,p',q) = i^2 \int d^4x\, d^4y\, e^{-ip\cdot x}\, e^{ip'\cdot y}\, \langle 0|\mathcal{T}\left(J^D(y)\, J^\pi(0)\, J_{\mu\nu}^{D_2^*\dagger}(x)\right)|0\rangle. \quad (1)$$

where $\mathcal{T}$ is the time ordering operator and $q = p - p'$ is transferred momentum. The J^D, J^π and $J^{D_2^*}$ are the interpolating fields and can be written in terms of the quark field operators as: $J^D(y) = i\bar{d}(y)\gamma_5 c(y)$, $J^\pi(0) = i\bar{u}(0)\gamma_5 d(0)$, and $J_{\mu\nu}^{D_2^*}(x) = (i/2)[\bar{u}(x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu (x)c(x) + \bar{u}(x)\gamma_\nu \overleftrightarrow{\mathcal{D}}_\mu (x)c(x)]$ respectively. The $\overleftrightarrow{\mathcal{D}}_\mu (x)$ is two-side covariant derivative and defined as; $\overleftrightarrow{\mathcal{D}}_\mu (x) = \frac{1}{2}\left[\overrightarrow{\mathcal{D}}_\mu (x) - \overleftarrow{\mathcal{D}}_\mu (x)\right]$, and $\overrightarrow{\mathcal{D}}_\mu (x) = \overrightarrow{\partial}_\mu (x) - i\frac{g}{2}\lambda^a A_\mu^a(x)$, $\overleftarrow{\mathcal{D}}_\mu (x) = \overleftarrow{\partial}_\mu (x) + i\frac{g}{2}\lambda^a A_\mu^a(x)$. Here the λ^a are the Gell-Mann matrices and $A_\mu^a(x)$ are the external (vacuum) gluon fields.

The correlation function given in Eq. (1) can be calculated following two different ways. The first way is called the phenomenological or physical side of the calculation in which it is calculated in terms of hadronic parameters. On the other hand it is calculated in terms of quark and gluon degrees of freedom by the help of the operator product expansion (OPE) in deep Euclidean region which is the theoretical or QCD side. In the sequel of the calculation of both sides matching the coefficient of same structure obtained from both sides provides the QCD sum rules for the intended physical quantity. The contribution of the higher states and continuum is suppressed by the help of double Borel transformation with respect to the variables p^2 and p'^2.

For the physical side of the calculation, Eq. (1) is saturated with the complete set of appropriate hadronic states having the same quantum numbers with the hadronic states in the equation. After performing the four-integrals over x and y the following form of the correlation function is obtained:

$$\begin{aligned}\Pi_{\mu\nu}^{Phys}(p,p',q) &= \frac{\langle 0 \mid J^\pi \mid \pi(q)\rangle\langle 0 \mid J^D \mid D(p')\rangle\langle D_2^*(p,\epsilon) \mid J_{\mu\nu}^{D_2^*} \mid 0\rangle}{(p^2 - m_{D_2^*}^2)(p'^2 - m_D^2)(q^2 - m_\pi^2)} \\ &\times \langle \pi(q)D(p') \mid D_2^*(p,\epsilon)\rangle + \cdots\,, \end{aligned} \quad (2)$$

In Eq. (2) the contributions of the higher states and continuum are represented by the $\cdots$ and the matrix elements appearing in this equation are parameterized as follows:

$$\begin{aligned}\langle 0 \mid J^\pi \mid \pi(q)\rangle &= i\frac{m_\pi^2 f_\pi}{m_d + m_u}, \\ \langle 0 \mid J^D \mid D(p')\rangle &= i\frac{m_D^2 f_D}{m_d + m_c}, \\ \langle D_2^*(p,\epsilon) \mid J_{\mu\nu}^{D_2^*} \mid 0\rangle &= m_{D_2^*}^3 f_{D_2^*} \epsilon_{\mu\nu}^{*(\lambda)}\,, \\ \langle \pi(q)D(p') \mid D_2^*(p,\epsilon)\rangle &= g_{D_2^* D\pi} \epsilon_{\eta\theta}^{(\lambda)}\, p'_\eta\, p'_\theta \end{aligned} \quad (3)$$

where f_π, f_D and $f_{D_2^*}$ are leptonic decay constants of π, D and D_2^* mesons, respectively. By the usage of the matrix elements given in Eq. (3) in Eq. (2), final form of the correlation function for the physical side becomes:

$$\Pi^{Phys}_{\mu\nu}(p,p',q) = \frac{g_{D_2^* D\pi}\, m_D^2\, m_\pi^2\, f_D\, f_\pi\, f_{D_2^*}}{(m_c+m_d)(m_u+m_d)(p^2-m_{D_2^*}^2)(p'^2-m_D^2)(q^2-m_\pi^2)} \times \Big[m_{D_2^*} p\cdot p' p'_\mu\, p_\nu - \frac{2\,(p\cdot p')^2 + m_{D_2^*}^2\, p'^2}{3\, m_{D_2^*}} p_\mu\, p_\nu - m_{D_2^*}^3 p'_\mu\, p'_\nu + m_{D_2^*}(p\cdot p')\, p_\mu\, p'_\nu + \frac{m_{D_2^*}(m_{D_2^*}^2\ \ p'^2 - (p\cdot p')^2)}{3}\, g_{\mu\nu}\Big] + \cdots , \quad (4)$$

where we apply the summation over the polarization tensor using $\sum_\lambda \varepsilon^{(\lambda)}_{\mu\nu}\varepsilon^{*(\lambda)}_{\alpha\beta} = \frac{1}{2}T_{\mu\alpha}T_{\nu\beta} + \frac{1}{2}T_{\mu\beta}T_{\nu\alpha} - \frac{1}{3}T_{\mu\nu}T_{\alpha\beta}$ with $T_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2_{D_2^*[D_{s2}^*]}}$.

The application of the double Borel transformation with respect to the initial and final momenta squared leads us the physical side of the correlation function as:

$$\widehat{\mathbf{B}}\Pi^{Phys}_{\mu\nu}(q) = g_{D_2^* D\pi} \frac{f_D f_{D_2^*} f_\pi m_D^2 m_\pi^2 e^{-\frac{m_{D_2^*}^2}{M^2}}\, e^{-\frac{m_D^2}{M'^2}}}{(m_c+m_d)(m_u+m_d)(m_\pi^2-q^2)}\Bigg[\frac{m_{D_2^*}}{12}\Big(m_D^4 + (m_{D_2^*}^2-q^2)^2 - 2m_D^2(m_{D_2^*}^2+q^2)\Big)g_{\mu\nu} + \frac{1}{6m_{D_2^*}}\Big[m_D^4 + m_D^2(4m_{D_2^*}^2-2q^2) + (m_{D_2^*}^2-q^2)^2\Big]p_\mu p_\nu - \frac{m_{D_2^*}}{2}(m_D^2+m_{D_2^*}^2-q^2)p_\nu p'_\mu + m_{D_2^*}^3 p'_\mu p'_\nu - \frac{m_{D_2^*}}{2}(m_D^2+m_{D_2^*}^2-q^2)p_\mu p'_\nu\Bigg] + \cdots \quad (5)$$

In the next part of the calculation, using the OPE, which is done in deep Euclidean region i.e. $p^2 \to -\infty$ and $p'^2 \to -\infty$, the QCD side of the correlation function is calculated. To perform this calculation one needs to substitute the explicit form of the interpolating currents into the correlation function. Followed by the contraction of all quark pairs via Wick's theorem the expression for this side turns into

$$\Pi^{QCD}_{\mu\nu}(p,p',q) = \frac{i^5}{2}\int d^4x \int d^4y e^{-ip\cdot x} e^{ip'\cdot y} \times \Big\{Tr\Big[\gamma_5\, S_d^{ji}(-y)\gamma_5 S_c^{i\ell}(y-x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu\, (x) S_u^{\ell j}(x)\Big] + [\mu\leftrightarrow\nu]\Big\}, \quad (6)$$

where $S_c(x)$ represents the heavy quark propagator which is given in [17] and $S_u(x)$ and $S_d(x)$ are the light quark propagators.

No matter how small it is, in our calculation we also include the contribution coming from the gluon condensate. After some straightforward calculations, the correlation function in QCD side is obtained as;

$$\Pi^{QCD}_{\mu\nu}(p,p',q) = \Pi_1(q^2)p_\mu p_\nu + \Pi_2(q^2)p_\nu p'_\mu + \Pi_3(q^2)p_\mu p'_\nu + \Pi_4(q^2)p'_\mu p'_\nu + \Pi_5(q^2)g_{\mu\nu} \quad (7)$$

where $\Pi_i(q^2)$ contains the contribution coming from perturbative and nonperturbative parts of the QCD side of the calculations and it is given explicitly as

$$\Pi_i(q^2) = \int ds \int ds' \frac{\rho_i^{pert}(s,s',q^2)}{(s-p^2)(s'-p'^2)} + \Pi_i^{nonpert}(q^2), \quad (8)$$

where i=1,2,3,4,5 and the spectral density $\rho_i(s, s', q^2)$ is given by the imaginary part of the Π_i function, i.e., $\rho_i(s, s', q^2) = \frac{1}{\pi} Im[\Pi_i]$. To obtain the QCD sum rules for the coupling constant among the structure appering in Eq. 7 we consider the Dirac structure $p_\mu p'_\nu$. Following double Borel transformation, the final form of the QCD side of the correlation function for the considered Dirac structure is obtained as:

$$\widehat{\mathbf{B}}\Pi^{QCD}_{\mu\nu}(q^2) = \Big\{ \int ds \int ds' e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} \rho_3^{pert}(s, s', q^2) + \widehat{\mathbf{B}}\Pi_3^{nonpert}(q^2)\Big\} p_\mu p'_\nu + \cdots , \tag{9}$$

where

$$\begin{aligned} \rho_3^{pert}(s, s', q^2) &= \int_0^1 dx \int_0^{1-x} dy \frac{3(1 + 5y - 16y^2 - 2x - 16xy)}{16\pi^2} \\ &\times \Theta[L(s, s', q^2)], \end{aligned} \tag{10}$$

with $L(s, s', q^2) = -m_c^2 x + sx - sx^2 + q^2 y - q^2 xy - sxy + s' xy - q^2 y^2$ and the $\widehat{\mathbf{B}}\Pi_3^{nonpert}(q^2)$ is the result of the contribution coming from nonperturbative part. The contribution coming from nonperturbative part is lengthy therefore it is not presented here (for more details see Ref. [18]).

The sum rules for the coupling constant, $g_{D_2^* D\pi}$ is obtained equating the coefficients of the same Dirac structure obtained from the calculation of both sides of the correlation function. Application of double Borel transformation and the quark-hadron duality assumption provides suppression of the contribution coming from the higher states and continuum. After all, the final form of the sum rules for the coupling form factor $g_{D_2^* D\pi}$ is achieved as:

$$\begin{aligned} g_{D_2^* D\pi} &= e^{\frac{m_{D_2^*}^2}{M^2}} e^{\frac{m_D^2}{M'^2}} \frac{6(m_c + m_d)(m_d + m_{u[s]})(m_\pi^2 - q^2) m_{D_2^*}}{f_{D_2^*} f_D f_\pi m_D^2 m_\pi^2 \Big[m_D^4 + m_D^2(4m_{D_2^*}^2 - 2q^2) + (m_{D_2^*}^2 - q^2)^2 \Big]} \\ &\times \left\{ \int_{(m_c+m_u)^2}^{s_0} ds \int_{(m_c+m_d)^2}^{s'_0} ds' e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} \rho_3^{pert}(s, s', q^2) + \widehat{\mathbf{B}}\Pi_3^{non-pert}(q^2) \right\} , \end{aligned} \tag{11}$$

3 Numerical Results

This section presents the numerical analysis of the coupling constant and its fit function as a function of $Q^2 = -q^2$. The resultant fit function is also used for calculation of the the decay width and branching ratio of the considered transition. Some input parameters used in the calculation of the considered coupling constant are: $m_c = (1.275 \pm 0.025)$ GeV [19], $m_d = 4.8^{+0.5}_{-0.3}$ MeV [19], $m_u = 2.3^{+0.7}_{-0.5}$ MeV [19], $m_{D_2^*(2460)} = (2462.6 \pm 0.6)$ MeV [19], $m_D = (1869.62 \pm 0.15)$ MeV [19], $m_\pi = (139.57018 \pm 0.00035)$ MeV [19], $f_D = 206.7 \pm 8.9$ MeV [19], $f_\pi = 130.41 \pm 0.03 \pm 0.20$ MeV [19], $f_{D_2^*(2460)} = 0.0228 \pm 0.0068$ [20], $\langle \frac{\alpha_s G^2}{\pi} \rangle = (0.012 \pm 0.004)$ GeV^4.

Four auxiliary parameters, which are the Borel mass parameters M^2 and M'^2 and continuum thresholds s_0 and s'_0, are brought into the numerical analysis by the Borel transformation and quark-hadron duality assumption . These are not physical parameters therefore the results of the coupling constant calculations should have no dependency on these auxiliary parameters. As a result we need to determine the working regions of them from our analysis. Via the consideration of the relation between the continuum threshold and the energies of the excited states of initial and final mesons we obtain the interval of working region for the continuum thresholds as 7.6 GeV$^2 \leq s_0 \leq$ 8.8 GeV^2 and 4.7 GeV$^2 \leq s'_0 \leq$ 5.6 GeV^2 for the coupling constant of $D_2^* D\pi$ vertices. As for the determination of the suitable Borel mass window, Borel masses having too large value provides a good convergence of OPE however this is accompanied by that, the continuum contribution dominates the sum rules.

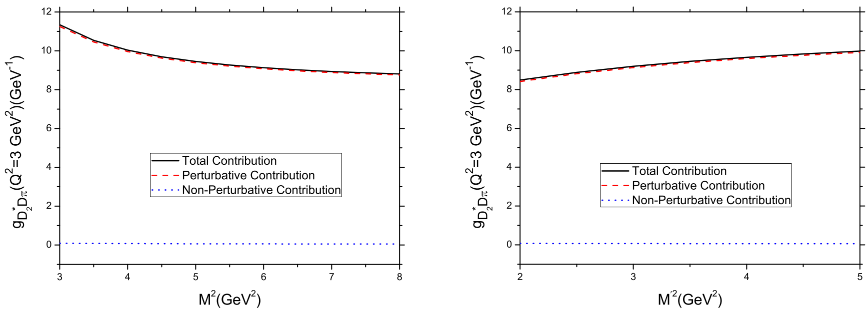

Figure 1. Left: $g_{D_2^*D\pi}(Q^2 = 3\ \text{GeV}^2)(\text{GeV}^{-1})$ as a function of the Borel mass M^2 at $M'^2 = 3.5\ \ \text{GeV}^2$. **Right:** $g_{D_2^*D\pi}(Q^2 = 3\ \text{GeV}^2)(\text{GeV}^{-1})$ as a function of the Borel mass M'^2 at $M^2 = 5\ \text{GeV}^2$.

Table 1. Parameters appearing in the fit function of the coupling constants.

$c_1(\text{GeV}^{-}1)$	$c_2(\text{GeV}^2)$	$c_3(\text{GeV}^{-1})$
11.57 ± 3.12	12.55 ± 3.51	1.13 ± 0.34

Conversely, if one chooses too small values of them, despite that the pole dominates the sum rules the OPE have a poor convergency. These considerations lead to the $3\ \text{GeV}^2 \leq M^2 \leq 8\ \text{GeV}^2$ and $2\ \text{GeV}^2 \leq M'^2 \leq 5\ \text{GeV}^2$ intervals for Borel masses in our calculation.

Determination of the working regions of the auxiliary parameters is followed by the usage of them in the analysis of the coupling constant $g_{D_2^*D\pi}$. To give an idea about our result we present the Fig. 1 which shows the dependency of the coupling constant $g_{D_2^*D\pi}$ on M^2 and M'^2.

The calculation of the coupling constant is followed by the the calculation of the decay width and branching ratio of the considered transition utilizing its value. Considering the coupling constant being the value of the form factor at $Q^2 = -m_\pi^2$ which is outside of the reliable region of our sum rules calculations, we require a suitable fit function that expands our calculation. The fit function serving that purpose is as follows: $g_{D_2^*D\pi}(Q^2) = c_1 \exp\left[-\frac{Q^2}{c_2}\right] + c_3$ where $Q^2 = -q^2$. With the value of M^2 and M'^2 from their working regions we obtain the fit parameters, c_1, c_2 and c_3, and they are presented in Table 1. Value of the $g_{D_2^*D\pi}$ coupling constant at $Q^2 = -m_\pi^2$ is obtained as $g_{D_2^*D\pi} = (12.72 \pm 3.56)\ \text{GeV}^{-1}$. The errors appearing in our result is due to the uncertainties in the input parameters as well as the auxiliary parameters. We also obtain the decay width and the branching ratio of the $D_2^*D\pi$ transition using our result of the coupling constant. The decay width for $D_2^*D\pi$ transition is attained using

$$\Gamma = \frac{|M(\mathbf{p}')|^2}{24\pi m_{D_2^*}}|\mathbf{p}'| \tag{12}$$

where $|M(\mathbf{p}')|^2 = g^2_{D_2^*D\pi}[\frac{2}{3m^4_{D_2^*}}(m_{D_2^*}\sqrt{\mathbf{p}'^2+m_D^2})^4 - \frac{4m_D^2}{3m^2_{D_2^*}}(m_{D_2^*}\sqrt{\mathbf{p}'^2+m_D^2})^2 + \frac{2m_D^4}{3}]$ and $|\mathbf{p}'| = \frac{1}{2m_{D_2^*}}\sqrt{m^4_{D_2^*}+m_D^4+m_\pi^4-2m^2_{D_2^*}m_\pi^2-2m_D^2m_\pi^2-2m^2_{D_2^*}m_D^2}$. Using the total decay width ($\Gamma_{D_2^*(2460)^0} = (49.0 \pm 1.3)$ MeV [19]) the branching ratio of this transition is also achieved from the outcomes of the decay width whose result with the decay width are given in Table 2.

Table 2. Numerical results of decay widths and branching ratios.

	Γ(GeV)	BR
$D_2^* D\pi$	$(1.84 \pm 0.48) \times 10^{-3}$	$(1.08 \pm 0.27) \times 10^{-1}$

To sum up, in this work we calculate the coupling constant for the transition $D_2^* D\pi$ whose value is obtained as $g_{D_2^* D\pi} = (12.72 \pm 3.56)$ GeV^{-1} using the QCD sum rules. The results of the coupling constant is also used to determine the decay width and the branching ratio of the mentioned transition.

Acknowledgment

This work has been supported in part by the Scientific and Technological Research Council of Turkey (TUBITAK) under the research project 114F018.

References

[1] Albrecht H, et al. (ARGUS Collaboration), Phys. Rev. Lett. **56**, 549 (1986).

[2] H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. B **232**, 398 (1989); H. Albrecht et al., (ARGUS Collaboration) Phys. Lett. B **231**, 208 (1989); H. Albrecht et al., (ARGUS Collaboration) Phys. Lett. B **221**, 422 (1989); H. Albrecht et al., (ARGUS Collaboration) Phys. Lett. B **230**, 162 (1989);H. Albrecht et al., (ARGUS Collaboration) Phys. Lett. B **297**, 425 (1992).

[3] J. C. Anjos et al. (E691 Collaboration), Phys. Rev. Lett. **62**, 1717 (1989).

[4] P. L. Frabetti et al. (E687 Collaboration), Phys. Rev. Lett. **72**, 324 (1994).

[5] P. Avery et al. (CLEO Collaboration), Phys. Rev. D **41**, 774 (1990); P. Avery et al. (CLEO Collaboration), Phys. Lett. B **331**, 236 (1994).

[6] J.P. Alexander et al,(CLEO Collaboration) Phys. Lett. B **303**, 377 (1993).

[7] Y. Kubota et al. (CLEO Collaboration), Phys. Rev. Lett. **72**, 1972 (1994).

[8] T. Bergfeld et al. (CLEO Collaboration), Phys.Lett. B **340**, 194 (1994).

[9] J. Link et al. (FOCUS Collaboration), Phys. Lett. B **586**, 11 (2004).

[10] K. Abe et al. (BELLE Collaboration), Phys. Rev. D **69**, 112002 (2004).

[11] V. M. Abazov et al. (D0 Collaboration), Phys. Rev. Lett. **95**, 161602 (2005).

[12] S. Godfrey, Phys. Rev. D **72**, 054029 (2005).

[13] H. Sundu, K. Azizi, Eur. Phys. J. A **48**, 81 (2012).

[14] K. Azizi, H. Sundu, J. Y. Süngü, N. Yinelek, Phys. Rev. D **88**, 036005 (2013); Phys. Rev. D **88**, 099901(E) (2013).

[15] K. Azizi, H. Sundu, A. Y. Türkan, E. Veli Veliev, J.Phys. G **41**, 035003 (2014).

[16] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979); Nucl. Phys. B **147**, 448 (1979).

[17] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127**, 1 (1985).

[18] K. Azizi, Y. Sarac, H. Sundu, arXiv:1402.6887 [hep-ph]

[19] Beringer et al. (Particle Data Group), Phys. Rev. D **86**, 010001 (2012)and 2013 partial update for the 2014 edition.

[20] H. Sundu, K. Azizi, Eur. Phys. J. A **48**, 81 (2012).