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Rapidity resummation for B-meson wave functions

Yue-Long Shen[1,a] and Yu-Ming Wang[2,3,b]

[1] *College of Information Science and Engineering, Ocean University of China, Qingdao, Shandong 266100, P.R. China*
[2] *Physik Department T31, Technische Universität München, James-Franck-Straße 1, D-85748 Garching, Germany*
[3] *Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany*

Abstract. Transverse-momentum dependent (TMD) hadronic wave functions develop light-cone divergences under QCD corrections, which are commonly regularized by the rapidity ζ of gauge vector defining the non-light-like Wilson lines. The yielding rapidity logarithms from infrared enhancement need to be resummed for both hadronic wave functions and short-distance functions, to achieve scheme-independent calculations of physical quantities. We briefly review the recent progress on the rapidity resummation for B-meson wave functions which are the key ingredients of TMD factorization formulae for radiative-leptonic, semi-leptonic and non-leptonic B-meson decays. The crucial observation is that rapidity resummation induces a strong suppression of B-meson wave functions at small light-quark momentum, strengthening the applicability of TMD factorization in exclusive B-meson decays. The phenomenological consequence of rapidity-resummation improved B-meson wave functions is further discussed in the context of $B \to \pi$ transition form factors at large hadronic recoil.

1 Introduction

QCD factorization theorems are indispensable to the theoretical descriptions of high-energy processes probed at worldwide collider experiments, and the predictive power lies on the process-independence of non-perturbative hadronic functions, entering the factorization formulae. B-meson light-cone distribution amplitudes (LCDAs) and transverse-momentum dependent (TMD) wave functions are fundamental inputs for QCD calculations of exclusive B-meson decays applying collinear factorization and TMD factorization theorems. Great efforts have been devoted to the studies of renormalization properties and perturbative constraints of B-meson LCDAs in both momentum space [1–4] and dual space [5, 6], and to the explorations of QCD evolution equations of B-meson TMD wave functions [7, 8] with the standard renormalization-group (RG) method and the QCD resummation technique (for a recent review, see [9]).

Providing three-dimensional profile of a given hadron, TMD wave functions are more complicated due to the emergence of light-cone (or rapidity) singularities in the end-point region, which cancel in

[a] e-mail: shenylmeteor@ouc.edu.cn
[b] e-mail: yuming.wang@tum.de

the QCD corrections to LCDAs. Such rapidity divergences can be regularized by the rapidity (ζ) of non-light-cone Wilson lines, at the price of generating infrared enhanced double logarithm $\ln^2 \zeta^2$ [10, 11] for B-meson wave functions and single logarithm $\ln \zeta^2$ [12, 13] for pion wave functions. Resummation of these rapidity logarithms in both hadronic wave functions and hard functions are essential to make scheme-independent predictions for physical observables. In the following, we will discuss the construction of rapidity evolution equation for B-meson TMD wave functions in the Mellin and impact parameter spaces in section 2, and present the resummation improved B-meson wave functions in momentum space by performing the inverse Mellin transformation in section 3 where the resummation effects on $B \to \pi$ transition form factors are also reported.

2 Rapidity Evolution Equation

The TMD wave functions of B-meson are defined by the non-local vacuum-to-hadron matrix element in coordinate space [8]

$$\begin{aligned} &\langle 0|\bar{q}(y) W_y(n)^\dagger I_{n;y,0} W_0(n) \Gamma h(0)|\bar{B}(v)\rangle \\ &= -\frac{i f_B m_B}{4} \mathrm{Tr}\left\{ \frac{1+\not{v}}{2} \left[2\, \Phi_B^+(t, y^2) + \frac{\Phi_B^-(t, y^2) - \Phi_B^+(t, y^2)}{t} \not{y} \right] \gamma_5\, \Gamma \right\}, \end{aligned} \tag{1}$$

which depends on longitudinal ($t = v \cdot y$) and transverse (y^2) variables as well as the non-light-like gauge vector n defining the Wilson lines.

It is demonstrated in [10] that both the double rapidity logarithm $\ln^2 \zeta^2$ with $\zeta^2 = 4(v \cdot n)^2/n^2$, due to the overlap of the collinear enhancement from a loop momentum l collimated to the gauge vector n and the soft enhancement, and the mixed logarithm $\ln \mu_f \, \ln \zeta^2$ (μ_f being the renormalization scale) which has the same origin of cusp divergence in B-meson LCDAs [1] appear in the next-to-leading-order (NLO) QCD corrections to B-meson wave functions. Simultaneous resummation of two distinct types of rapidity logarithms for B-meson wavefunctions makes it subtler than that of the traditional Sudakov resummation for fast-moving light hadrons (see, however, [14] for recent progress).

Recalling that the Sudakov resummation of $\ln^2 \zeta^2$ for pion wave function can be achieved by constructing the evolution equation from varying the gauge vector n, because the collinear divergence arises from the region with a loop momentum collimated to the pion momentum and varying the vector n does not result in an additional collinear divergence. This trick however does not apply to the resummation of B-meson wave function due to the nature of rapidity logarithm explained in the above. To resum the rapidity logarithm for B-meson wave functions, we utilize the fact that the resulting collinear divergence is insensitive to the heavy-quark velocity v and the effect from varying v can then be factorized from the TMD wave functions. Trading the rapidity derivative for the differential of velocity yields

$$\zeta^2 \frac{d}{d\zeta^2} \Phi_B^{(b)}(x, k_T, \zeta^2, \mu_f) = \frac{v \cdot n}{2\epsilon_{\alpha\beta} v^\alpha n^\beta} v^+ \frac{d}{dv^+} \Phi_B^{(b)}(x, k_T, \zeta^2, \mu_f), \tag{2}$$

where $\epsilon_{\alpha\beta}$ is an anti-symmetric tensor with $\epsilon_{+-} = -\epsilon_{-+} = 1$, v^+ is the plus component of the b-quark velocity and x is the longitudinal momentum fraction of the light quark. Here, the crucial point is that the velocity derivative will be only applied to the Feynman rules involving an effective heavy b-quark

$$\frac{v \cdot n}{2\epsilon_{\alpha\beta} v^\alpha n^\beta} v^+ \frac{d}{dv^+} \frac{v^\mu}{v \cdot l} = \frac{\hat{v}^\mu}{v \cdot l}, \tag{3}$$

inducing the special vertex

$$\hat{v}^\mu \equiv \frac{v \cdot n}{2\epsilon_{\alpha\beta} v^\alpha n^\beta} \epsilon_{\rho\lambda} v^\rho \left(g^{\mu\lambda} - \frac{v^\mu l^\lambda}{v \cdot l} \right). \tag{4}$$

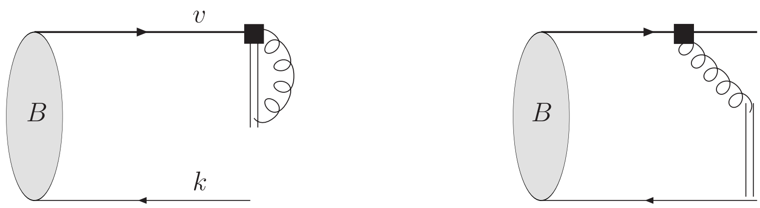

Figure 1. Leading-order soft kernel for the rapidity evolution equation, where the square denotes the special vertex of Eq. (4). Taken from [8].

The rapidity evolution equation of B-meson wave function can then be written as

$$\zeta^2 \frac{d}{d\zeta^2}\Phi_B(x,k_T,\zeta^2,\mu_f) = K^{(b,1)} \otimes \Phi_B(x,k_T,\zeta^2,\mu_f) - \frac{1}{Z_\Phi}\left(\zeta^2 \frac{d}{d\zeta^2} Z_\Phi\right)\Phi_B(x,k_T,\zeta^2,\mu_f), \tag{5}$$

where Z_Φ is the counterterm for the ultraviolet renormalization of TMD wave function. The one-loop kernel $K^{(b,1)}$ collects the soft gluon dynamics as displayed in figure 1.

Computing the two effective diagrams yields

$$K_1^{(b,1)} = -\frac{\alpha_s C_F}{4\pi}\,\Gamma(\epsilon)\left(\frac{4\pi\mu_f^2}{\lambda^2}\right)^{\epsilon}\left(\frac{v\cdot n}{\epsilon_{\alpha\beta}v^\alpha n^\beta}\right)^2, \tag{6}$$

$$\tilde{K}_2^{(1)}(N,b,\zeta^2) = \frac{\alpha_s C_F}{2\pi}\left(\frac{v\cdot n}{\epsilon_{\alpha\beta}v^\alpha n^\beta}\right)^2\left[K_0(\lambda b) - K_0\left(\sqrt{\zeta^2}\frac{m_B b}{N}\right)\right], \tag{7}$$

where the gluon mass λ is introduced to regularize the infrared divergence in each diagram, Mellin and Fourier transformations for the B-meson wave function are performed in the evaluation of irreducible contribution illustrated in the second diagram. It is then straightforward to derive

$$\zeta^2 \frac{d}{d\zeta^2}\tilde{\Phi}_B(N,b,\zeta^2,\mu_f) = \tilde{K}^{(1)}(N,b,\zeta^2,\mu_f)\,\tilde{\Phi}_B(N,b,\zeta^2,\mu_f), \tag{8}$$

with the renormalized evolution kernel

$$\tilde{K}^{(1)}(N,b,\zeta^2,\mu_f) = -\frac{\alpha_s C_F}{2\pi}\left[\ln\frac{\mu_f b}{2} + \gamma_E + K_0\left(\sqrt{\zeta^2}\frac{m_B b}{N}\right)\right], \tag{9}$$

where the infrared divergence cancels exactly between the two effective diagrams. The single logarithm of soft kernel $\tilde{K}^{(1)}$ can be organized by the RG resummation

$$\mathcal{K}^{(1)}(N,b,\zeta^2,\mu_f) = \tilde{K}^{(1)}(N,b,\zeta^2,\mu_c) - \int_{\mu_c}^{\mu_f}\frac{d\mu}{\mu}\lambda_K(\alpha_s(\mu))\,\theta(\mu_f-\mu_c), \tag{10}$$

where the scale $\mu_c = a\,\sqrt{\zeta^2}\,m_B/N$, a being an order-unity constant, is adjusted to diminish the logarithmic enhancement in the initial condition.

In addition, the factorization scale evolution of TMD wave function itself is computed as

$$\tilde{\Phi}_B(N,b,\zeta^2,\mu_f) = \exp\left[-\int_{\mu_0}^{\mu_f}\frac{d\mu}{\mu}\frac{\alpha_s(\mu)}{2\pi}C_F\left(\ln\zeta^2 - 2\right)\right]\tilde{\Phi}_B(N,b,\zeta^2,\mu_0)\,. \tag{11}$$

Combining the rapidity and scale evolutions and choosing $\mu_f = a\,\zeta_0\, m_B$, we obtain

$$\tilde{\Phi}_B(N,b) = \exp\left[\int_{\zeta_0^2}^{N^2\zeta_0^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2}\mathcal{K}^{(1)}(N,b,\tilde{\zeta}^2,\mu_f) - \int_{\mu_0}^{\mu_f}\frac{d\mu}{\mu}\frac{\alpha_s(\mu)}{2\pi}C_F\left(\ln\zeta_0^2 - 2\right)\right] \times\tilde{\Phi}_B(N,b,\zeta_0^2,\mu_0)\,, \quad (12)$$

with the simplified rapidity kernel

$$\mathcal{K}^{(1)}(N,b,\zeta^2,\mu_f) = -\frac{\alpha_s(\mu_c)}{2\pi}\,C_F\,\ln a - \int_{\mu_c}^{\mu_f}\frac{d\mu}{\mu}\frac{\alpha_s(\mu)}{2\pi}C_F\,\theta(\mu_f-\mu_c) \quad (13)$$

which is valid in the large N limit.

3 Resummation Improved B-meson TMD Wave Functions

We are now in a position of discussing the resummation effect on B-meson wave functions in x-space by performing the inverse Mellin transformation

$$\Phi_B^{\pm}(x,k_T) = \int_{c-i\infty}^{c+i\infty}\frac{dN}{2\pi i}(1-x)^{-N}\tilde{\Phi}_B^{\pm}(N,k_T), \quad (14)$$

for the solution given by Eq. (12). To achieve this purpose, we need to know the initial conditions of TMD wave functions which are postulated to have the factorized form

$$\Phi_B^{\pm}(x,k_T,\zeta_0^2) = \phi_B^{\pm}(x,\zeta_0^2)\,\phi(k_T)\,, \quad (15)$$

to reduce the numerical analysis. The longitudinal parts are further taken from the so-called “free-parton" model [15]

$$\phi_B^{+}(x,\zeta_0^2) = \frac{x}{2x_0^2}\,\theta(2x_0-x) \qquad \phi_B^{-}(x,\zeta_0^2) = \frac{2x_0-x}{2x_0^2}\,\theta(2x_0-x), \quad (16)$$

which in Mellin space correspond to

$$\tilde{\phi}_B^{+}(N,k_T,\zeta_0^2) = \frac{1-(1-2x_0)^N(1+2x_0N)}{2x_0^2N(N+1)}, \qquad \tilde{\phi}_B^{-}(N,k_T,\zeta_0^2) = \frac{(1-2x_0)^{N+1}+2x_0N+2x_0-1}{2x_0^2N(N+1)}.$$

The inverse Mellin transformation will be firstly performed for the resummation improved wave functions $\tilde{\Phi}_B^{\pm}(N,k_T)$ with frozen strong coupling constant α_s, to make the analytical behavior more transparent, and then with running α_s. The resummation effects on the x dependence of B-meson wave functions $\phi_B^{\pm}(x) = \Phi_B^{\pm}(x,k_T)/\phi(k_T)$ are illustrated in figure 2. The primary observations are summarized as follows:

- Faster than linear attenuation of $\phi^{\pm}(x)$ in the small x region is realized after the resummation improvement, albeit with the non-vanishing $\phi^{-}(x)$ at $x=0$ initially.
- Resummation improved B-meson wave functions become smooth and develop radiative tails.
- The normalization conditions $\int_0^1 dx\,\phi_B^{\pm}(x)(x) = 1$ are unaffected by the rapidity resummation.

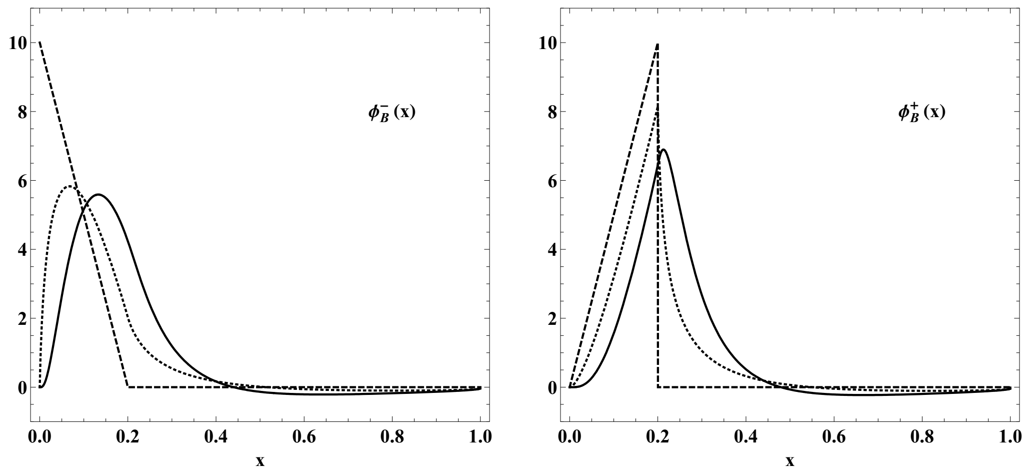

Figure 2. The dashed, dotted and solid curves correspond to the x dependence of initial condition $\phi_B^{\pm}(x, \zeta_0^2)$, and the resummation improved $\phi_B^{\pm}(x)$ for fixed $\alpha_s = 0.3$ and for running α_s with $\zeta_0 = e/10$ and $a = 1$. Taken from [8].

It will be interesting to inspect the rapidity resummation effect for phenomenological observables which are relevant for the precision test of CKM mechanism (for recent reviews, see [16, 17]) and for the hunting of beyond standard model physics. In this respect, we consider the $B \to \pi$ transition form factors $f_{B\pi}^{\pm}(q^2)$ at large hadronic recoil as illuminative examples, which are essential to the golden-channel determination of matrix element $|V_{ub}|$ exclusively. Numerically the resummation effects are found to decrease the above two form factors by approximately 25% at $q^2 = 0$, due to the strong suppression of B-meson wave functions at the end-point. Notice that such improvement is sizeable and must be taken into account in the future calculations of $B \to \pi$ form factors applying TMD factorization theorem to catch up with the precision achieved in the calculations from light-cone QCD sum rules [18, 19].

4 Conclusion

Applying the QCD resummation technique with non-light-like Wilson lines, we construct rapidity evolution equation for TMD wave functions of B-meson collecting the double logarithm $\ln^2 \zeta^2$ and mixed logarithm $\ln \mu_f \, \ln \zeta^2$. Technically, the rapidity resummation of B-meson wave functions is novel due to the different nature of collinear divergence in QCD correction from that of energetic light mesons. The resummation improved B-meson wave functions induce a strong suppression in the small x region and hence enhance the applicability of TMD factorization in exclusive B-meson decays. Sizeable corrections to the $B \to \pi$ transition form factors are also observed due to the resummation improvement. Many open questions concerning the TMD wave functions of B-meson remain to be answered, and these include (a) What are the relations between TMD wave functions and LCDAs at large transverse momentum? (b) What are the operator-product-expansion constraints of TMD wave functions? (c) Can we gain some insights of TMD wave functions at low energy scale from non-perturbative approaches? As a final remark, the rapidity resummation technique presented here can be generalized immediately to the TMD wave functions of Λ_b- baryon entering the factorization formulae of many exclusive decays [20–22] which are of increasing interest at the LHC and Tevatron.

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