On the $\Delta A_{\text{CP}}$ saga

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Abstract. We discuss a model in which the large SU(3) flavor violations in singly Cabibbo suppressed decays of neutral $D$ mesons are ascribed exclusively to the final state interactions. The agreement with the experimental data on the branching ratios is obtained with large strong phase differences which are also necessary for substantial direct CP violation. While the value of the CP violating asymmetries depends on the strength of the penguin contribution, we predict an asymmetry for the decays into charged pions more than twice larger than that for charged kaons and having opposite sign.

1 Introduction

At the end of 2011 LHCb Collaboration measured the difference in the CP-violating asymmetries between the two decay channels $D \to K^+K^-$ and $D \to \pi^+\pi^-$ obtaining [3]

$$\Delta A_{\text{CP}} = A_{\text{CP}}(K^+K^-) - A_{\text{CP}}(\pi^+\pi^-) = (-0.82 \pm 0.21 \pm 0.11)\%.$$ (1)

This result was confirmed by the measurements of CDF [4] and Belle [5] Collaborations

$$\Delta A_{\text{CP}} = (-0.62 \pm 0.21 \pm 0.10)\%,$$

$$\Delta A_{\text{CP}} = (-0.87 \pm 0.41 \pm 0.06)\%,$$ (2)

respectively. These experimental results on direct CP violation in the Singly Cabibbo Suppressed (SCS) charm decays produced a large amount of interest. In the Standard Model, indeed, CP Violation (CPV) in charm decays is commonly expected to be very small [6] and so CP asymmetries of the order of one percent could be a signal of New Physics. The theoretical community have interpreted these results by considering them as a sign of New Physics [7] or compatible with the Standard Model [8, 9]. We prefer the latter hypothesis [10]. Meanwhile, new results by LHCb, using more data and different way to identify the flavour of the charmed meson, show that, at the level of $10^{-3}$, there is no evidence of CP violation in $D^0 \to K^+K^-, \pi^+\pi^-$ decays [11]

$$\Delta A_{\text{CP}} = (+0.14 \pm 0.16 \pm 0.08)\%.$$ (3)

Nevertheless, it is interesting to give an answer to the question regarding the amount of the direct CP violation in non-leptonic SCS D decays in the Standard Model. In the following we will show the main ideas and the results obtained in our paper [10].

*The phrase $\Delta A_{\text{CP}}$ saga was used for the first time by Guy Wilkinson at Beauty 2013 [1], see also [2].

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2 The model

Many years ago we presented a calculation of the decay branching ratios of $D$ and $D_s$ mesons [12] based on factorization hypothesis and a model to account for rescattering effects through nearby resonances. The results were in reasonably good agreement with the allowed experimental data and predicted CP asymmetries at least one order of magnitude smaller than what was found in [3, 4].

The experimental data however did change in the meanwhile and therefore we have carried out a new analysis by considering only the SCS decays [10]. In [12] we stressed the fact that the observed SU(3) flavor violations were essentially due to the rescattering effects. In [10], we evaluate the weak decay amplitudes assuming SU(3) symmetry so that we can consider the rescattering later. Furthermore, we approximate the hamiltonian for $D$ weak decays with its $\Delta U = 1$ part when estimating branching ratios, introducing the $\Delta U = 0$ terms only for the calculation of asymmetries. This is a reasonable approximation because of the smallness of the relevant CKM elements, $|V_{ub}V_{cb}^*| < |V_{ud(s)}V_{cd(s)}^*|$.

2.1 Decay branching ratios

The weak effective hamiltonian for SCS charmed particles is

$$
\mathcal{H}_w = \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* [C_i Q_i^1 + C_2 Q_i^2] + \frac{G_F}{\sqrt{2}} V_{us} V_{cs}^* [C_i Q_i^1 + C_2 Q_i^2] - \frac{G_F}{\sqrt{2}} V_{ub} V_{cb}^* \sum_{i=3}^{6} C_i Q_i + h.c.,
$$

(4)

where the expressions for the operators $Q_i^j$ and the numerical values used for the Wilson coefficients $C_i$ can be found in [10]. Considering the transformation properties respect to the U-spin, the effective hamiltonian can be decomposed in two parts

$$
H_{\Delta U=1} = \frac{G_F}{2 \sqrt{2}} \left( V_{us} V_{cs}^* - V_{ud} V_{cd}^* \right) \sum_{\text{u.c.}} \left[ C_1 (Q_1^1 - Q_1^0) + C_2 (Q_1^2 - Q_1^0) \right],
$$

(5)

$$
H_{\Delta U=0} = - \frac{G_F}{2 \sqrt{2}} V_{ub} V_{cb}^* \sum_{i=3}^{6} C_i Q_i + \frac{1}{2} \left[ C_1 (Q_1^1 + Q_1^0) + C_2 (Q_1^2 + Q_1^0) \right].
$$

(6)

Neglecting $H_{\Delta U=0}$ and observing that the neutral charmed meson $D^0$ is U-spin singlet it can be easily demonstrate that the SCS decays can be written in terms of only two independent amplitudes. In fact, there are two independent combinations of $S$-wave states having $U=1$

$$
|v_1> = \frac{1}{2} \left[ |K^+ K^- > + |K^- K^+ > - |\pi^+ \pi^- > - |\pi^- \pi^+ > \right],
$$

$$
|v_2> = \frac{\sqrt{3}}{2 \sqrt{2}} \left[ |\pi^0 \pi^0 > - |\eta_8 \eta_8 > - \frac{1}{\sqrt{3}} (|\pi^0 \eta_8 > + |\eta_8 \pi^0 >) \right],
$$

(7)

that may be rewritten in terms of the following two states with given transformation properties under SU(3)

$$
|8, U = 1 > = \frac{\sqrt{3}}{2 \sqrt{5}} \left[ |K^+ K^- > + |K^- K^+ > - |\pi^+ \pi^- > - |\pi^- \pi^+ > \\
- \left[ |\pi^0 \pi^0 > - |\eta_8 \eta_8 > - \frac{1}{\sqrt{3}} (|\pi^0 \eta_8 > + |\eta_8 \pi^0 >) \right] \right],
$$

(8)

$$
|27, U = 1 > = \frac{3}{10} \left[ |K^+ K^- > + |K^- K^+ > - |\pi^+ \pi^- > - |\pi^- \pi^+ > \\
+ \frac{3}{2} \left[ |\pi^0 \pi^0 > - |\eta_8 \eta_8 > - \frac{1}{\sqrt{3}} (|\pi^0 \eta_8 > + |\eta_8 \pi^0 >) \right] \right].
$$

(9)
providing two resonances provided by the matrix elements of the decay amplitudes with di
The angle \( \phi \) "penguin" operators. For
and isospin quantum numbers: (8 resonances in the scattering of the final particles. The possible resonances have the following SU(3)
branching ratios. In table 1 we give the values of the free parameters: for a detailed discussion see
choose to fix these parameters by comparing model predictions with experimental data on the SCS
The direct CPV occurs when the decay amplitudes for CP conjugate processes into final states
To test the model with the experimental data on the branching ratios we should consider the final state
Using these states, the two independent amplitudes can be written in terms of the diagrammatic ones, color connected (T) and color suppressed (C), in the following way
\[
\langle 8, U = 1 | H_{\Delta U = 1} | D^0 \rangle \propto T - \frac{2}{3} C, \quad \langle 27, U = 1 | H_{\Delta U = 1} | D^0 \rangle \propto T + C.
\]

Table 1. The fitted value for the free parameters.

<table>
<thead>
<tr>
<th>( C/T )</th>
<th>( \phi )</th>
<th>( \delta_0 )</th>
<th>( \delta'_0 )</th>
<th>( \delta_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.529</td>
<td>+0.389</td>
<td>±2.58</td>
<td>±0.917</td>
<td>±1.44</td>
</tr>
</tbody>
</table>

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\]

To test the model with the experimental data on the branching ratios we should consider the final state interactions. The experimental data strongly violate SU(3) symmetry. In fact, in the limit of SU(3) flavor symmetry the relation \( A(D^0 \rightarrow K^+ K^-) = -A(D^0 \rightarrow \pi^+ \pi^-) \) holds, but the experimental data on the branching ratios violate the previous relation [13]
\[
Br(D^0 \rightarrow \pi^+ \pi^-) = (1.402 \pm 0.026) \times 10^{-3},
\]
\[
Br(D^0 \rightarrow K^+ K^-) = (3.96 \pm 0.08) \times 10^{-3}.
\]

In our model the SU(3) breaking is given by the final state interactions, described as the effect of resonances in the scattering of the final particles. The possible resonances have the following SU(3) and isospin quantum numbers: (8, \( I = 1 \)), (8, \( I = 0 \)) and (1, \( I = 0 \)). The two states with \( I = 0 \) can mix, providing two resonances
\[
|f_0 > = \sin \phi \ |8, I = 0 > + \cos \phi \ |1, I = 0 >, \quad (10)
\]
\[
|f'_0 > = - \cos \phi \ |8, I = 0 > + \sin \phi \ |1, I = 0 >. \quad (11)
\]
The angle \( \phi \), the strong phases \( \delta_0 \), \( \delta'_0 \) and \( \delta_1 \) (corresponding to \( f_0, f'_0 \) and to the particle with isospin one, respectively) and the two weak decay amplitudes are the free parameters of our model. We choose to fix these parameters by comparing model predictions with experimental data on the SCS branching ratios. In table 1 we give the values of the free parameters: for a detailed discussion see [10].

### 2.2 CP asymmetries

The direct CPV occurs when the decay amplitudes for CP conjugate processes into final states \( f \) and \( \bar{f} \) (in our case \( f \equiv \bar{f} \)) are different in modulus. This requires the presence of at least two interfering decay amplitudes with different weak and strong phases. In our approach, the second amplitude is provided by the matrix elements of the \( \Delta U = 0 \) hamiltonian, eq. (6), that contains both \( Q_{1(2)} \) and "penguin" operators. For \( H_{\Delta U = 0} \), the independent states are the following
\[
\frac{1}{2} \left( |K^+ K^- > + |K^- K^+ > + |\pi^+ \pi^- > + |\pi^- \pi^+ > \right),
\]
\[
\frac{1}{4} \left( 3|\pi^0 \pi^0 > + |\eta_8 \eta_8 > + \sqrt{3} \left( |\pi^0 \eta_8 > + |\eta_8 \pi^0 > \right) \right),
\]
\[
\frac{1}{\sqrt{3}} \left\{ \frac{1}{4} |\pi^0 \pi^0 > + \frac{3}{4} |\eta_8 \eta_8 > - \frac{\sqrt{3}}{4} (|\pi^0 \eta_8 > + |\eta_8 \pi^0 > ) + |K^0 \bar{K}^0 > + |\bar{K}^0 K^0 > \right\}. \quad (12)
\]
As an example, we report the expressions for the second amplitude (the amplitude \(B\) [10], see also eq. (15)) for the decay modes of interest

\[
B(D^0 \to \pi^+\pi^-) = \left( P + \frac{T'}{2} \right) \left\{ \frac{1}{2} (e^{i\delta_A} + e^{i\delta_B}) + \left( -\frac{1}{6} \cos(2\phi) - \frac{7}{4\sqrt{10}} \sin(2\phi) \right) (e^{i\delta_A} - e^{i\delta_B}) \right\} \\
+ \left( T' + C' \right) \left\{ \frac{3}{20} - \frac{3}{40} (e^{i\delta_A} + e^{i\delta_B}) \right. \\
+ \left. \left[ \frac{1}{120} \cos(2\phi) + \frac{1}{4\sqrt{10}} \sin(2\phi) \right] (e^{i\delta_A} - e^{i\delta_B}) \right\} ,
\]

\[
B(D^0 \to K^+K^-) = \left( P + \frac{T'}{2} \right) \left\{ \frac{1}{4} (e^{i\delta_A} + e^{i\delta_B}) + \left( -\frac{5}{12} \cos(2\phi) + \frac{1}{4\sqrt{10}} \sin(2\phi) \right) (e^{i\delta_A} - e^{i\delta_B}) \right\} \\
+ \frac{1}{2} e^{i\delta_1},
\]

\[
+ \left( T' + C' \right) \left\{ \frac{3}{20} - \frac{1}{40} (e^{i\delta_A} + e^{i\delta_B}) + \frac{7}{120} \cos(2\phi) (e^{i\delta_A} - e^{i\delta_B}) \\
- \frac{1}{10} e^{i\delta_1} \right\} .
\]

Writing the general expressions for the decay amplitudes \(D \to f\) and its CP-conjugate

\[
\mathcal{A}(f) = A e^{i\delta_A} + B e^{i\delta_B}, \\
\mathcal{A}(\bar{f}) = A^* e^{i\delta_A} + B^* e^{i\delta_B},
\]

respectively, the CP asymmetry is given by

\[
A_{CP}(f) = \frac{\left| \mathcal{A}(f) \right|^2 - \left| \mathcal{A}(\bar{f}) \right|^2}{\left| \mathcal{A}(f) \right|^2 + \left| \mathcal{A}(\bar{f}) \right|^2} = \frac{2 \mathfrak{R}(A^*B) \sin(\delta_A - \delta_B)}{|A|^2 + |B|^2 + 2 \mathfrak{R}(A^*B) \cos(\delta_A - \delta_B)},
\]

where \(\delta_{A(B)}\) are the strong phases and the amplitudes \(A\) and \(B\) contain the weak phases. In eqs. (13,14) \(P\) represents the contribution of the penguin diagram, while the terms \(T'\) and \(C'\) are related to \(T\) and \(C\) by the following relations

\[
T' = -T \frac{V_{ub} V_{cb}^*}{\sin \theta_C \cos \theta_C} \quad \text{and} \quad C' = -C \frac{V_{ub} V_{cb}^*}{\sin \theta_C \cos \theta_C} .
\]

Neglecting the \(T'\) and \(C'\) contributions, the amplitudes and thus the asymmetries get a simplified expression. If we consider, for example, the \(K^+K^-\) final state we have

\[
\mathcal{A}(K^+K^-) \simeq T \, f_T(\delta_i, \phi, C/T) + P \, f_P(\delta_i, \phi) ,
\]

and the CP-asymmetry is

\[
A_{CP}(K^+K^-) \simeq \frac{2 \mathfrak{R}(f_T f_P^*)}{|f_T|^2} \frac{\mathfrak{R}(P)}{T} ,
\]

where

\[
\frac{\mathfrak{R}(P)}{T} = \frac{|V_{ub} V_{cb}|}{\sin \theta_C \cos \theta_C} \sin \gamma \left< K^+ K^- | \sum_{i=3}^6 C_i Q_i + \frac{1}{2} (C_1 |Q_1^i + Q_1^d| + C_2 |Q_2^i + Q_2^d|) |D^0 > \right. \\
\left. - \left< K^+ K^- | C_1 (Q_1^i - Q_1^d) + C_2 (Q_2^i - Q_2^d)|D^0 > \right> \right> = 6.3 \times 10^{-4}_K .
\]
In eq. (20), \( \{Q_i\} \) indicates the penguin contraction of the operator \( Q_i \). If we choose the lowest values in each column of table 1 the asymmetries are

\[
A_{\text{CP}}(K^+K^-) = +1.469 \frac{\mathfrak{F}(P)}{T},
\]

\[
A_{\text{CP}}(\pi^+\pi^-) = -3.362 \frac{\mathfrak{F}(P)}{T}.
\]

Notice that our choice is related to the fact that the resonance \( f_0(1710) \), the one with lower mass, prefers to decay into kaons [13] and thus it should be recognized as \( f_0' \).

Putting all together we have

\[
\Delta A_{\text{CP}} = 3.03 \times 10^{-3} \kappa.
\]

Thus asymmetries of the order of percent can be obtained with a value of \( \kappa \) around three. As far as the sign of \( \Delta A_{\text{CP}} \) is concerned, \( \kappa \) and \( \Delta A_{\text{CP}} \) are negative in the factorization approximation, in agreement with the majority of experimental results. We note that if factorization is used a considerably smaller value for \( \kappa \) would be expected, due to the littleness of the Wilson coefficients of QCD penguin operators. However, large penguin contribution and penguin contraction of the current-current operators could give a value of \( \kappa \) compatible with CP asymmetries of the order of percent (as in the paper of J. Brod, A.L. Kagan and J. Zupan in [8]). We recall that the penguin diagrams were introduced many years ago in [14] as a possible explanation of the “octet enhancement”. A large contribution of these operators could successfully describe both the hyperon and the kaon non–leptonic decays. For a comprehensive discussion of the status of “\( \Delta I = 1/2 \) rule” see, for example, [15].

### 3 Conclusions

We studied the singly Cabibbo suppressed decays of the neutral \( D \) mesons in a model that ascribes the large SU(3) violations to final state interactions. We were able to reproduce the experimental data on the branching ratios and we have shown that CP violation asymmetries of the order of percent are compatible with the Standard Model if we assume an enhancement of the penguin diagrams as in the case of the non–leptonic decays of kaons.

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### References


