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Puzzling out the proton radius puzzle

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Abstract. The discrepancy between the proton charge radius extracted from the muonic hydrogen Lamb shift measurement and the best present value obtained from the elastic scattering experiments, remains unexplained and represents a burning problem of today's nuclear physics: after more than 50 years of research the radius of a basic constituent of matter is still not understood. This paper presents a summary of the best existing proton radius measurements, followed by an overview of the possible explanations for the observed inconsistency between the hydrogen and the muonic-hydrogen data. In the last part the upcoming experiments, dedicated to remeasuring the proton radius, are described.

1 Introduction

"How big is the proton?" is one of the fundamental physics questions. The proton has been studied since the early days of experimental hadronic physics [1]. Through the years many different measurements of its properties have been performed, ranging from the pioneering experiments done in the 1960s to the high precision measurements done in the last few years. In particular, its radius has been determined by various electron scattering experiments and many atomic Lamb shift measurements (see Figure 1). Both approaches gave consistent results. Unfortunately their average does not agree with the findings of recent very precise Lamb shift measurements in muonic-hydrogen [2, 3], which report a new value for the proton charge radius, 8σ away from the previously accepted value. This discrepancy, known as the proton radius puzzle, represents an important open question of today's nuclear physics.

2 Nuclear scattering experiments

In a typical scattering experiment the radius of a proton is determined indirectly by measuring the cross-section for elastic scattering of electrons on hydrogen [4]. The measured cross-section depends on electric and magnetic form-factors G_E^p and G_M^p, which carry information about the charge and magnetization distribution in the proton and are extracted from the measured data via Rosenbluth separation. The charge radius is extracted from the slope of the electric form-factor at $Q^2 = 0$:

$$r_E^2 \equiv -6\hbar^2 \, \frac{d}{dQ^2} G_E(Q^2)\Big|_{Q^2=0} ,$$

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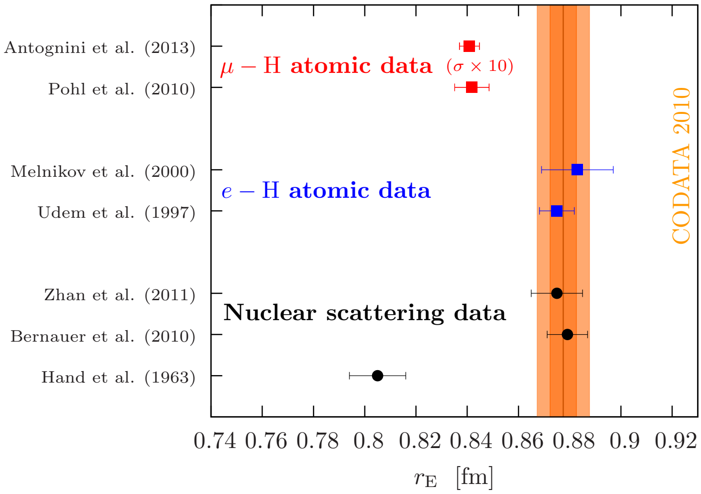

Figure 1. An overview of the best proton charge radius measurements. Full circles show findings of the scattering experiments. Full squares represent values obtained from the Lamb shift spectroscopy. The values determined from the muonic hydrogen measurements are colored red. The uncertainties of muonic hydrogen data are multiplied by factor 10 for clarity [5].

where Q^2 represents the square of the momentum transfer four-vector. Unfortunately, the data for $Q^2 < 0.005\,(\mathrm{GeV}/c)^2$ that would allow for a reliable and precise determination of this slope do not yet exist [6]. Therefore, an extrapolation of available G_E^p points to $Q^2 \to 0$ is used to estimate r_E^2. The extracted value of r_E^2 is extremely sensitive to the details of this extrapolation, which in turn strongly depends on the precision and accuracy of the values of G_E^p themselves.

Currently the most precise existing nuclear scattering data are those of Bernauer [7]. In this experiment elastic cross-section was measured, by using three high-resolution spectrometers of the A1-Collaboration at MAMI (Mainz, Germany) in combination with a liquid hydrogen target and an electron beam with different energies and currents ranging from 1 nA to 10 μA. The cross-section was measured for 1400 different kinematic points, covering the Q^2 from 1 $(\mathrm{GeV}/c)^2$ down to as low as $4 \cdot 10^{-3}\,(\mathrm{GeV}/c)^2$ with the statistical uncertainty smaller than 0.2 %. In the analysis the measured points were not sorted in terms of Q^2 and used in the Rosenbluth separation. Instead, an alternative approach was considered, where models for the elastic form-factors were fitted directly to all measured points [8]. Various models were tested which all gave similar results with $\chi^2/1400 \approx 1.14$. The obtained fit for $G_E^p(Q^2)$ could then be directly used to estimate the proton charge radius. The best estimate for the proton charge radius was determined to be $r_E^{\mathrm{Scat.}} = (0.879 \pm 0.008)$ fm.

3 Hydrogen spectroscopy

The spectroscopy of the hydrogen atom plays an important role in the development of the modern physics, because it can be used to precisely test the predictions of the theory of Quantum Electrodynamics (QED). The theory has two free parameters, the Rydberg constant (R_y) and the mean charge radius of the proton which need to be determined elsewhere (for instance in the scattering experiments) before being able to confront it to precise atomic data. Alternatively, one could decide to trust QED and combine it with the measured spectroscopic spectra to extract these two parameters, thus offering a complementary way of determining the proton charge radius.

An important effect on the atomic levels is the Lamb shift [9], a small energy level splitting in hydrogen, which arises due to the vacuum fluctuations. In the first approximation the effect is proportional to the probability of finding an electron with the principal and orbital quantum numbers

(n,l) at the center of the atom:

$$\Delta E^{nl}_{\text{Lamb}} \propto |\Psi_{nm}(0)|^2 \ .$$

Figure 2 shows the electron probability distributions $|\Psi_{nm}(\vec{r})|^2$ in the hydrogen atom for first few quantum numbers. In the S-states electron spends most of its time at the center, resulting in a non-zero Lamb correction. Hence, the energy levels for the S-states will shift due to this effect. On the other hand, for the P- and D-states, the probability for finding electron at the center is almost zero, which leads to a negligible shift of these energy levels.

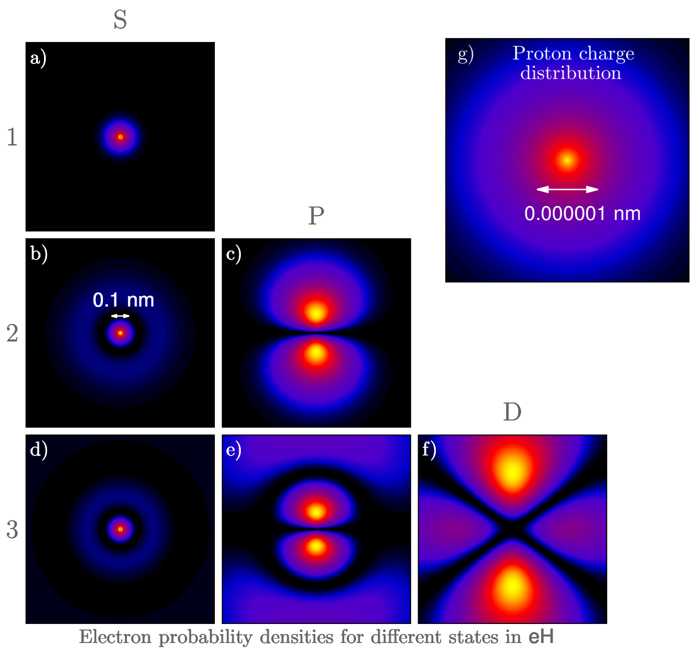

Figure 2. a-f) Projections of the probability densities for the electron in the Hydrogen atom to the xz-plane. The results are shown for electrons with principal quantum number $n = 1, 2, 3$ and orbital quantum numbers $l = S, P, D$. g) The proton charge distribution at the center of the hydrogen atom.

Moreover, one should also consider that the center of the atom is not empty, but is occupied by a proton with a finite size, which brings additional term to the Lamb shift that depends on the proton radius. This is usually a small correction to the total Lamb shift, but it is in fact the very term that is exploited to determine the proton radius. Disregarding the hyperfine coupling, new energy levels of the nS-states can be written as:

$$E(n, l = S) \cong -\frac{R_y}{n^2} + \frac{\Delta E^{1S}_{\text{Lamb}}}{n^3} \, , \quad \Delta E^{1S}_{\text{Lamb}} \cong \left(8.172 + 1.56 \left(\frac{r_E}{\text{fm}}\right)^2\right) \text{MHz} \, . \tag{1}$$

Equation (1) offers many possibilities how to perform the spectroscopic measurements. For example, when one is interested only in the Lamb shift and radius, a measurement of the energy difference $\Delta E_{2S\to 2P}$ for the $2S \to 2P$ transition could be performed, because then the first term in (1) dissapears and only Lamb terms survive. Alternatively, one could combine results of the $1S \to 2S$ and $1S \to 3S$ transitions and then exploit the different n-dependence of the two terms in (1) to simultaneously extract both the Rydberg constant and the proton charge radius. Using this philosophy many different transitions have been measured by many different groups [10]. When the obtained mean value for the proton charge radius is combined with the results of the selected scattering experiments, one obtains the recommended CODATA value for the radius, which equals to $r_E^{\text{CODATA}} = (0.8775 \pm 0.0051)$ fm.

4 Muonic hydrogen Lamb shift measurement

The precision of the hydrogen spectroscopy measurements is comparable to the precision of the findings of the nuclear scattering experiments. However, they are both inferior to the results of the muonic hydrogen Lamb shift measurement. A muon, being 200-times heavier than the electron, is confined to a much smaller orbit around the proton. This increases the overlap between the two and significantly amplifies the Lamb shift. The full QED calculation predicts the following energy difference for the $2S \rightarrow 2P$ transition [10]:

$$\Delta E^{\mu\mathrm{H}}_{\mathrm{Lamb}} = \left(209.9779 - 5.2262 \left(\frac{r_E}{\mathrm{fm}}\right)^2 + 0.0347 \left(\frac{r_E}{\mathrm{fm}}\right)^3\right) \mathrm{meV}\,. \tag{2}$$

According to (2) the contribution of the proton finite size represents only 1.8 % of the whole shift, which is much smaller than in the case of electronic (normal) hydrogen, where the proton part represents 15 % of the Lamb shift. However, the Lamb effect is now 100-times larger than in electronic hydrogen, which allows for much more precise determination of the proton charge radius. Furthermore, muonic and electronic hydrogen differ also in the sign of the Lamb shift, because different Feynman diagrams govern the corrections. In the electronic hydrogen the effect is dominated by the lepton vertex correction diagram, while in the muonic hydrogen, vacuum polarization represents the leading term.

The first successful measurement of the Lamb shift in muonic hydrogen was done at Paul Scherrer Institute (PSI) [2], using a moderated beam of muons incident on the hydrogen target. In the collisions highly excited muonic hydrogen atoms were created. Most of them decayed directly to the $1S$ state, while 1% of the muons de-excited to the $2S$ state. A tunable pulsed laser was then used to excite these states to the $2P$ level. If the frequency of the laser was right and the transition was successful, the $2P$ states de-excited to the $1S$ state, emitting 2 keV X-rays that were detected by photo-diodes. In the experiment a sharp peak at approximately 50 THz was observed, which corresponds to a Lamb shift of $\Delta E^{\mu\mathrm{H}}_{Lamb} = 206.2949(32)\,\mathrm{meV}$. This value, combined with the QED calculations given in (2), yields a very precise value for the proton charge radius ($r^{\mu\mathrm{H}}_{\mathrm{E}} = (0.84184 \pm 0.00067)\,\mathrm{fm}$) which is 4 % (or 7.9σ) smaller than the CODATA value.

5 Possible explanations

The observed discrepancy created a great excitement in the physics community, because it puts QED and our understanding of the nuclear physics to a rigorous test. Since the observation of the inconsistency in 2010, various explanations for it have been offered.

The most trivial explanation would be an unidentified error in the existing measurements. However, both spectroscopic and nuclear scattering measurements have been reexamined and no problem has yet been found. Furthermore, all measurements except the muonic-hydrogen measurement were done by different groups and they all mutually agree.

A mistake in the existing calculations or an overlooked higher order term also do not appear to be a plausible scenario. The calculations were made by several theoretical groups using different approaches and they all get very similar results [11]. They invested a large effort to examine many different Feynman diagrams and the current consensus is that a missing higher order term is not responsible for the discrepancy, because they all contribute much less than the needed $300\,\mu\mathrm{V}$. Hence, this could suggest a hidden problem with a leading (vacuum polarization) term, or it could even be an indication of a fundamental problem with QED.

People have offered also many other explanations, ranging from the incomplete two-photon corrections to the contributions of the molecular ions. However, the most intriguing ideas are those claiming that the discrepancy can not be explained within the framework of Standard Model. The idea of a new force and corresponding mediator particle that breaks the electron-muon universality is interesting also because it could simultaneously explain the muonic $(g-2)_\mu$ puzzle. A promising candidate for a mediator particle is a U(1) gauge boson that moderates the interaction between the dark matter and the standard model particles. Unfortunately, no such particle has been found yet [12].

6 New experiments

To provide further constrains to the existing interpretations and to put new ideas to the test, several new experiment are underway or are foreseen for the near future.

High precision electron scattering experiments are scheduled at Thomas Jefferson National Accelerator Facility (TJNAF) and at Mainz Microtron (MAMI) at the Johannes Gutenberg University in Mainz. Both experiments aim to measure the proton charge form factor at Q^2 as low as $10^{-4}\,\mathrm{GeV}^2/c^2$, and with a sub-percent accuracy, which is necessary for precise determination of the proton charge radius. To reach very low momentum transfers, the MAMI experiment [6] uses a technique based on initial state radiation. The emission of a real photon by an electron prior to the interaction with a proton reduces the four-momentum transferred to the proton, which allows probing its structure at smaller Q^2. In the first approximation the radiative tail of the elastic peak represents a coherent sum of the initial and final state radiation [13]. Hence, precise measurements of the radiative tail at constant beam energy and scattering angle in combination with a sophisticated Monte-Carlo simulation to disentangle initial and final state contributions, open a doorway for determining the form-factors in the unexplored region. The experiment is already underway and its first results are expected at the end of 2014. On the other hand, the pRad experiment at TJNAF [14] uses a different approach to reach very small Q^2. They intend to measure the cross-section for elastic scattering of electrons off protons by using a windowless gaseous target and a highly segmented forward angle calorimeter. The calorimeter will also allow them to detect Møller electrons, which could then be used for precise luminosity determination. This magnetic-spectrometer-free approach will allow them to reach extremely low scattering angles (which could never be reached with magnetic spectrometers) and thus measuring the form-factor at extremely low Q^2. The experiment will run in 2015.

A very interesting upcoming experiment is also the MUon Scattering Experiment (MUSE) [15], which aims to measure the proton charge form-factor at $Q^2 \geq 2\times10^{-3}\,\mathrm{GeV}^2/c^2$ with a precision better than 1 %, by using both $\mu^\pm$ and $e^\pm$ beams. This will be the first experiment of its kind and will provide information on proton radius from a completely new perspective. The spectroscopic results exist for both electronic and muonic hydrogen, while the scattering data have been so far obtained only with the electron beams. Hence, with its findings MUSE will fill the remaining void and help to complete the proton radius puzzle picture. The experiment will be performed at PSI and is envisioned for 2016.

There are also many new spectroscopic experiments going on. New Lamb shift measurements in the electronic hydrogen are being made to improve the existing results. Investigations of leptonic states such as e^+e^- and $\mu^+\mu^-$, which provide stringent tests of the QED are also foreseen. Interesting insight into the matter can also be obtained by measuring the Lamb shift in heavier muonic atoms, such as $^2\mathrm{H}_\mu$, $^3\mathrm{H}_\mu$, $^3\mathrm{He}_\mu$ and $^4\mathrm{He}_\mu$. Of course, these measurements then need to be compared with the corresponding results from the nuclear scattering experiments of comparable accuracy, which are also not yet available. Therefore, a series of new electron scattering experiments of light nuclei are being scheduled. In particular, a high precision measurement of elastic scattering of electrons off deuterium has been performed at MAMI in March 2014 [16] that will provide a new scattering value for the deuteron charge radius. It will be five times better than the best existing value and will serve as a counterweight to the $^2\mathrm{H}_\mu$ results from PSI.

7 Conclusions

The proton radius puzzle is a conspicuous open question of the nuclear physics that, due to the possible profound implications on our understanding of physics world, deserves our full attention. The existing data, which were obtained with the state-of-the-art high precision experiments, show a consistent picture for the electronic hydrogen, while the spectroscopic measurement of the muonic hydrogen reports a significantly different value for the proton charge radius. Many different aspects of the problem have already been investigated, but the 4 % difference in radii or 0.15 % difference in the transition energy remains unexplained. Some claim that the puzzle can not be explained in terms of the Standard Model, while others assume that this could be an indication of a hidden fundamental problem of QED. In order to test any of the proposed hypotheses, further theoretical and experimental work is needed. Many new experiments are foreseen for the near future, which will reveal new insight into the puzzle.

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