Analysis of the pion scalar form factor provides model independent values of $f_0(500)$ and $f_0(980)$ meson parameters

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Abstract. An existence of the scalar meson $f_0(500)$ is unambiguously confirmed by the pion scalar form factor analysis. The same is concerned also of the $f_0(980)$ scalar meson, though it is placed on the tail of the elastic region to be investigated in the analysis under consideration, therefore with less precise parameters values.

1 Introduction

In contrast to other $SU(3)$ known multiplets of hadrons, the identification of the scalar mesons nonet [1] is long-standing puzzle. It is even more concerned of the lowest of them, the sigma-meson [2] [3], now to be called $f_0(500)$ resonance.

Therefore in this presentation first of all we demonstrate an existence of the $f_0(500)$ by a pion scalar form factor (FF) analysis.

With this aim we construct an explicit form of the pion scalar FF by using its phase representation and the best description of the S-wave iso-scalar $\pi\pi$ phase shift data by the parametrization in the absolute valued of the pion c.m. three-momentum $q$ to be found starting from fully general considerations.

2 Pion scalar form factor phase representation

The pion scalar FF $\Gamma_\pi(t)$ is defined by the parametrization of the matrix element of the scalar quark density

$$<\pi^j(p_2) | \bar{m}(\bar{u}u + \bar{d}d) | \pi^j(p_1)> = \delta^{ij}\Gamma_\pi(t)$$

where $t = (p_2 - p_1)^2$ and $\bar{m} = \frac{1}{2}(m_u + m_d)$.

It posses all well known properties of the pion electromagnetic FF [4] [5].

The analyticity and the asymptotic behavior allow to derive dispersion relations without subtractions

$$\Gamma_\pi(t) = \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{Im\Gamma_\pi(t')}{t' - t} dt'$$

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or with one subtraction at $t = 0$

$$\Gamma_\pi(t) = 1 + \frac{t}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\Im \Gamma_\pi(t')}{t'(t' - t)} dt'$$

or two subtractions etc., which in combination with the elastic unitarity condition

$$\Im \Gamma_\pi(t) = \Gamma_\pi(t) e^{-i\delta_0^0(t)} \sin \delta_0^0$$

(it produces the identity $\delta_T = \delta_0^0$, where $\delta_0^0$ is the $S$-wave isoscalar $\pi \pi$ phase shift and $\delta_T$ is the phase of the pion scalar FF) give corresponding phase representations of $\Gamma_\pi(t)$

$$\Gamma_\pi(t) = P_n(t) \exp\left[ \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\delta_0^0(t')}{t' - t} dt' \right]$$

or

$$\Gamma_\pi(t) = P_n(t) \exp\left[ \frac{t}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\delta_0^0(t')}{t'(t' - t)} dt' \right]$$

etc., with arbitrary, however at $t = 0$ normalized, polynomial $P_n(t)$.

Nevertheless, only a best describing existing data parametrization of $\delta_0^0$, obtained further fully from general considerations, can decide which of previous phase representations will be used to calculate an explicit form of the pion scalar FF.

3 Analysis of $S_0^0 \pi \pi$ phase shift data

By means of the elastic unitarity condition one can carry out analytic continuation of $\Gamma_\pi(t)$ through upper and lower boundaries of the two-pion cut on the II. Riemann sheet and to come to the same functional expression. The latter reveals that the two-pion branch point is a square-root type. As a result by application of the conformal mapping

$$q = [(t - 4)/4]^{1/2}, \quad m_\pi = 1$$

two-sheeted Riemann surface of $\Gamma_\pi(t)$ in $t$-variable is mapped into one absolute valued pion c.m. three-momentum $q$-plane and the elastic cut disappears.

Neglecting all higher branch points, there are only poles and zeros of $\Gamma_\pi(t)$ in $q$-plane and $\Gamma_\pi(t)$ can be represented in the form of a rational function

$$\Gamma_\pi(t) = \frac{\sum_{n=0}^{M} a_n q^n}{\prod_{i=1}^{N} (q - q_i)},$$

which leads to the parametrization

$$\tan \delta_0^0(t) = \frac{A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \ldots}{1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \ldots}$$

or equivalently

$$\delta_0^0(t) = \frac{1}{2i} \ln \frac{(1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \ldots) + i(A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \ldots)}{(1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \ldots) - i(A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \ldots)}$$

with all coefficients to be real.

The best description of existing data (see Fig.1) is achieved by 5 coefficients in (5) or (6) with $\chi^2/ndf = 1.41$, revealing the phase shift to approach asymptotically a constant $\pi/2$. As a result of the latter in the construction of the pion scalar FF one has to use its phase representation with one subtraction (3).
4 Explicit form of pion scalar form factor

Substituting the phase representation (6) with 5 nonzero coefficients into (3) one comes to $\Gamma_\pi(t)$

$$
\Gamma_\pi(t) = P_n(t) \exp \left( \frac{q^2 + 1}{\pi i} \right) \int_{-\infty}^{\infty} \frac{q'}{(q^2 + 1)(q'^2 - q^2)} \ln \left( \frac{1 + A_2 q'^2 + A_4 q'^4 + i(A_1 q' + A_3 q'^3 + A_5 q'^5)}{1 + A_2 q^2 + A_4 q^4 - i(A_1 q + A_3 q^3 + A_5 q^5)} \right) dq',
$$

in which $m_\pi = 1$ is assumed.

As the integrand there is even function of its argument, i.e. it is invariant under the transformation $q' \rightarrow -q'$, the latter expression can be transformed into the following form

$$
\Gamma_\pi(t) = P_n(t) \exp \left( \frac{q^2 + 1}{2\pi i} \right) \int_{-\infty}^{\infty} \frac{q'}{(q^2 + 1)(q'^2 - q^2)} \ln \left( \frac{1 + A_2 q'^2 + A_4 q'^4 + i(A_1 q' + A_3 q'^3 + A_5 q'^5)}{1 + A_2 q^2 + A_4 q^4 - i(A_1 q + A_3 q^3 + A_5 q^5)} \right) dq'
$$

in which the integral is suitable for calculation by means of the theory of residua. The calculations lead to the algebraic expression of the pion scalar FF

$$
\Gamma_\pi(t) = P_n(t) \frac{(q - q_1)}{(q + q_2)(q + q_3)(q + q_4)(q + q_5)} \frac{(i + q_2)(i + q_3)(i + q_4)(i + q_5)}{(i - q_1)}
$$

and its behavior at the elastic region is presented in Fig.2.

Now, investigating poles of the latter function, one finds that $-q_3$ and $-q_2$ on the second Riemann sheet in $t$-variable correspond to $f_0(500)$ and $f_0(980)$ scalar meson resonances with the masses and widths

$m_{f_0(500)} = (360 \pm 33)\text{MeV}, \quad \Gamma_{f_0(500)} = (587 \pm 85)\text{MeV},$

$m_{f_0(980)} = (957 \pm 72)\text{MeV}, \quad \Gamma_{f_0(980)} = (164 \pm 142)\text{MeV},$

where the errors correspond to the transferred errors of the coefficients $A_1, \ldots A_5$.  

Figure 1. Description of the S-wave iso-scalar $\pi\pi$ phase shift by the $[5/4]$ Pad’e type approximation.
5 Conclusions

By analysis of the pion scalar FF we have confirmed in a completely model independent way an existence of the $f_0(500)$ scalar meson resonance. With this aim we have started from the phase representation of the pion scalar FF with one subtraction and from a special parametrization of the S-wave iso-scalar $\pi\pi$ phase shift in the absolute value of the pion c.m. three-momentum $q$, found in general considerations. As a result pion scalar FF has been found in the form of a rational function and by investigation of its poles we have found $f_0(500)$, as well also $f_0(980)$ scalar meson resonance parameters.

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References