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# Analysis of the pion scalar form factor provides model independent values of $f_0(500)$ and $f_0(980)$ meson parameters

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**Abstract.** An existence of the scalar meson $f_0(500)$ is unambiguously confirmed by the pion scalar form factor analysis. The same is concerned also of the $f_0(980)$ scalar meson, though it is placed on the tail of the elastic region to be investigated in the analysis under consideration, therefore with less precise parameters values.

## 1 Introduction

In contrast to other $SU(3)$ known multiplets of hadrons, the identification of the scalar mesons nonet [1] is long-standing puzzle. It is even more concerned of the lowest of them, the sigma-meson [2] [3], now to be called $f_0(500)$ resonance.

Therefore in this presentation first of all we demonstrate an existence of the $f_0(500)$ by a pion scalar form factor (FF) analysis.

With this aim we construct an explicit form of the pion scalar FF by using its phase representation and the best description of the S-wave iso-scalar $\pi\pi$ phase shift data by the parametrization in the absolute valued of the pion c.m. three-momentum $q$ to be found starting from fully general considerations.

## 2 Pion scalar form factor phase representation

The pion scalar FF $\Gamma_\pi(t)$ is defined by the parametrization of the matrix element of the scalar quark density

$$< \pi^i(p_2) \mid \widehat{m}(\bar{u}u + \bar{d}d) \mid \pi^j(p_1) >= \delta^{ij}\Gamma_\pi(t)$$

where $\quad t = (p_2 - p_1)^2$ and $\quad \widehat{m} = \frac{1}{2}(m_u + m_d)$.

It posses all well known properties of the pion electromagnetic FF [4] [5].

The analyticity and the asymptotic behavior allow to derive dispersion relations without subtractions

$$\Gamma_\pi(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{Im\Gamma_\pi(t')}{t'-t}dt'$$

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or with one subtraction at $t = 0$

$$\Gamma_\pi(t) = 1 + \frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{Im\Gamma_\pi(t')}{t'(t'-t)}dt'$$

or two subtractions etc., which in combination with the elastic unitarity condition

$$Im\Gamma_\pi(t) = \Gamma_\pi(t)e^{-i\delta_0^0}sin\delta_0^0 \tag{1}$$

(it produces the identity $\delta_\Gamma \equiv \delta_0^0$, where $\delta_0^0$ is the $S$-wave isoscalar $\pi\pi$ phase shift and $\delta_\Gamma$ is the phase of the pion scalar FF) give corresponding phase representations of $\Gamma_\pi(t)$

$$\Gamma_\pi(t) = P_n(t)exp[\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_0^0(t')}{t'-t}dt'] \tag{2}$$

or

$$\Gamma_\pi(t) = P_n(t)exp[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_0^0(t')}{t'(t'-t)}dt'] \tag{3}$$

etc., with arbitrary, however at $t = 0$ normalized, polynomial $P_n(t)$.

Nevertheless, only a best describing existing data parametrization of $\delta_0^0$, obtained further fully from general considerations, can decide which of previous phase representations will be used to calculate an explicit form of the pion scalar FF.

## 3 Analysis of $S_0^0$ $\pi\pi$ phase shift data

By means of the elastic unitarity condition one can carry out analytic continuation of $\Gamma_\pi(t)$ through upper and lower boundaries of the two-pion cut on the II. Riemann sheet and to come to the same functional expression. The latter reveals that the two-pion branch point is a square-root type. As a result by application of the conformal mapping

$$q = [(t-4)/4]^{1/2}, \quad m_\pi = 1 \tag{4}$$

two-sheeted Riemann surface of $\Gamma_\pi(t)$ in $t$-variable is mapped into one absolute valued pion c.m. three-momentum $q$-plane and the elastic cut disappears.

Neglecting all higher branch points, there are only poles and zeros of $\Gamma_\pi(t)$ in $q$-plane and $\Gamma_\pi(t)$ can be represented in the form of a rational function

$$\Gamma_\pi(t) = \frac{\sum_{n=0}^{M} a_n q^n}{\prod_{i=1}^{N}(q-q_i)},$$

which leads to the parametrization

$$tan\ \delta_0^0(t) = \frac{A_1q + A_3q^3 + A_5q^5 + A_7q^7 + ...}{1 + A_2q^2 + A_4q^4 + A_6q^6 + ...} \tag{5}$$

or equivalently

$$\delta_0^0(t) = \frac{1}{2i}ln\frac{(1 + A_2q^2 + A_4q^4 + A_6q^6 + ..) + i(A_1q + A_3q^3 + A_5q^5 + A_7q^7 + ...)}{(1 + A_2q^2 + A_4q^4 + A_6q^6 + ..) - i(A_1q + A_3q^3 + A_5q^5 + A_7q^7 + ...)} \tag{6}$$

with all coefficients to be real.

The best description of existing data (see Fig.1) is achieved by 5 coefficients in (5) or (6) with $\chi^2/ndf = 1.41$, revealing the phase shift to approach asymptotically a constant $\pi/2$. As a result of the latter in the construction of the pion scalar FF one has to use its phase representation with one subtraction (3).

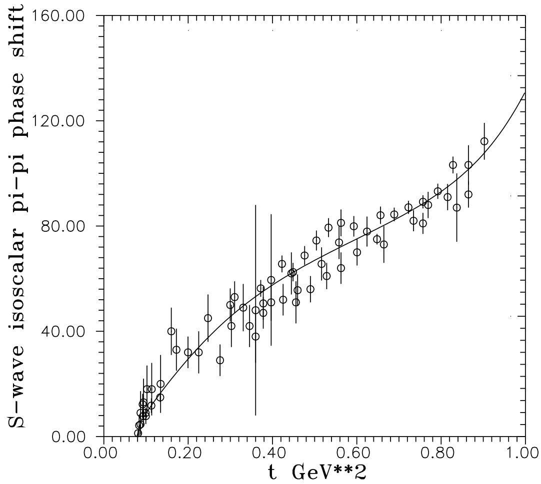


**Figure 1.** Description of the S-wave iso-scalar $\pi\pi$ phase shift by the [5/4] Pad‘e type approximation.

## 4 Explicit form of pion scalar form factor

Substituting the phase representation (6) with 5 nonzero coefficients into (3) one comes to $\Gamma_\pi(t)$

$$\Gamma_\pi(t) = P_n(t) exp\frac{(q^2+1)}{\pi i}\int_0^\infty \frac{q' \ln \frac{(1+A_2q'^2+A_4q'^4)+i(A_1q'+A_3q'^3+A_5q'^5)}{(1+A_2q'^2+A_4q'^4)-i(A_1q'+A_3q'^3+A_5q'^5)}}{(q'^2+1)(q'^2-q^2)}dq',$$

in which $m_\pi = 1$ is assumed.

As the integrand there is even function of its argument, i.e. it is invariant under the transformation $q' \to -q'$, the latter expression can be transformed into the following form

$$\Gamma_\pi(t) = P_n(t) exp\frac{(q^2+1)}{2\pi i}\int_{-\infty}^\infty \frac{q' \ln \frac{(1+A_2q'^2+A_4q'^4)+i(A_1q'+A_3q'^3+A_5q'^5)}{(1+A_2q'^2+A_4q'^4)-i(A_1q'+A_3q'^3+A_5q'^5)}}{(q'^2+1)(q'^2-q^2)}dq'$$

in which the integral is suitable for calculation by means of the theory of residua. The calculations lead to the algebraic expression of the pion scalar FF

$$\Gamma_\pi(t) = P_n(t)\frac{(q-q_1)}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)}\frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{(i-q_1)}$$

and its behavior at the elastic region is presented in Fig.2.

Now, investigating poles of the latter function, one finds that $-q_3$ and $-q_2$ on the second Riemann sheet in $t$-variable correspond to $f_0(500)$ and $f_0(980)$ scalar meson resonances with the masses and widths

$\mathbf{m_{f_0(500)} = (360 \pm 33)MeV}$, $\quad \mathbf{\Gamma_{f_0(500)} = (587 \pm 85)MeV}$,
$\mathbf{m_{f_0(980)} = (957 \pm 72)MeV}$, $\quad \mathbf{\Gamma_{f_0(980)} = (164 \pm 142)MeV}$,
where the errors correspond to the transferred errors of the coefficients $A_1, ...A_5$.

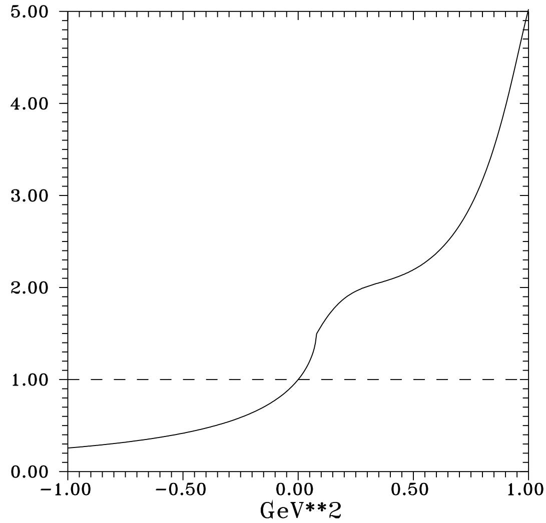


**Figure 2.** Behavior of the pion scalar form factor in $-1GeV^2 < t < 1GeV^2$ region.

## 5 Conclusions

By analysis of the pion scalar FF we have confirmed in a completely model independent way an existence of the $f_0(500)$ scalar meson resonance. With this aim we have started from the phase representation of the pion scalar FF with one subtraction and from a special parametrization of the S-wave iso-scalar $\pi\pi$ phase shift in the absolute value of the pion c.m. three-momentum $q$, found in general considerations. As a result pion scalar FF has been found in the form of a rational function and by investigation of its poles we have found $f_0(500)$, as well also $f_0(980)$ scalar meson resonance parameters.

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