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On the precise determination of the differences of ρ-meson family parameters

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Abstract. Unlike previous approaches to determine the known ρ-meson family parameters, we have proposed new phenomenological approach based on the exploiting experimental data on $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\nu_\tau$ processes and on the Unitary and Analytic models of the corresponding electromagnetic and weak pion form factors.

1 Introduction

The rho-meson resonances, with quantum members $J^{PC} = 1^{--}$ to be specified by their mass and decay width, exist in three charged states ρ^0, ρ^+ and ρ^-, whereby the cleanest processes of their demonstrations are, for neutral rho-mesons the e^+e^- annihilation processes and for charged rho-mesons τ^--decay into two-pions. While masses of charged rho-mesons have to be identical due to the CPT theorem, one can naturally expect that the mass of the neutral rho-mesons will be different from previous ones.

A lot of experimental determinations of masses and decay widths by employing various strong, electromagnetic and weak processes of the rho-mesons have been carried out [1] and the current values of the difference of ground state $\rho(770)$ meson parameters have the following average values, $m_{\rho^0} - m_{\rho^\pm} = -0.7 \pm 0.8\,\mathrm{MeV}$ and $\Gamma_{\rho^0} - \Gamma_{\rho^\pm} = 0.3 \pm 1.3\,\mathrm{MeV}$, revealing no difference between charged and neutral $\rho(770)$ meson parameters. Similar estimates of a difference of excited $\rho(1450)$ and $\rho(1700)$ meson parameters do not exist up to now.

In this contribution, in connection with the updated $e^+e^- \to \pi^+\pi^-$ annihilation data [2, 3] we determine ρ^0 meson parameters for all three $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ resonance families applying the Unitary and Analytic (U&A) model of the pion electromagnetic form factor (FF). Moreover, such approach allows us to determine also $\rho^\pm$ parameters for all three $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ resonance families by using accurate weak pion FF data from the measurement of τ^--lepton decay into two pions [4] and the relation between pion electromagnetic FF and pion weak FF derived by conserved vector current (CVC) hypothesis in [5].

Table 1. U&A model values of fitting parameters for the masses and decay widths of ρ^0 (for e^+e^- data) and $\rho^\pm$ (for τ^--lepton decay data) mesons and their resonances.

Parameter	ρ^0 ($e^+ e^-$ data) [MeV]	$\rho^\pm$ (τ^- data) [MeV]	Δ (ρ^0-$\rho^\pm$) [MeV]
$m_{\rho(770)}$	758.23 ± 0.46	761.60 ± 0.95	$-\ 3.37 \pm 1.06$
$m_{\rho(1450)}$	1342.31 ± 46.62	1373.83 ± 11.37	-31.53 ± 47.99
$m_{\rho(1700)}$	1718.50 ± 65.44	1766.80 ± 52.36	-48.30 ± 83.81
$\Gamma_{\rho(770)}$	144.56 ± 0.80	139.90 ± 0.46	4.66 ± 0.85
$\Gamma_{\rho(1450)}$	492.17 ± 138.38	340.87 ± 23.84	151.30 ± 140.42
$\Gamma_{\rho(1700)}$	489.58 ± 16.95	414.71 ± 119.48	74.87 ± 120.67

2 U&A models of pion FF

A phenomenological approach, based on the synthesis of the experimental fact of a creation of ρ-meson family in e^+e^- annihilation into two pions, the asymptotic behavior

$$F_\pi(t)|_{t\to-\infty} \sim -\frac{16\pi f_\pi^2 \alpha_s(t)}{t}, \tag{1}$$

($f_\pi = 92.4 \pm 0.2$MeV – weak pion decay constant, $\alpha_s(t)$ – QCD running coupling constant) and the analytic properties, is still the most successful way of a reconstruction of the EM pion FF behavior. This approach leads to U&A model of EM pion FF represented by the expression

$$\begin{aligned}
F_\pi^{\mathrm{EM}}[W(t)] = &\left(\frac{1-W^2}{1-W_N^2}\right)^2 \frac{(W-W_Z)(W_N-W_P)}{(W_N-W_Z)(W-W_P)} \\
&\times\left\{\frac{(W_N-W_\rho)(W_N-W_\rho^*)}{(W-W_\rho)(W_-W_\rho^*)}\cdot\frac{(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-1/W_\rho)(W-1/W_\rho^*)}\left(\frac{f_{\rho\pi\pi}}{f_\rho}\right)\right. \\
&\left.+\sum_{v=\rho',\rho''}\left[\frac{(W_N-W_v)(W_N-W_v^*)}{(W-W_v)(W_-W_v^*)}\cdot\frac{(W_N+W_v)(W_N+W_v^*)}{(W+W_v)(W+W_v^*)}\right]\left(\frac{f_{v\pi\pi}}{f_v}\right)\right\}
\end{aligned} \tag{2}$$

with the conformal mapping $W(t) = \mathrm{i}\frac{\sqrt{q_{in}+q}-\sqrt{q_{in}-q}}{\sqrt{q_{in}+q}+\sqrt{q_{in}-q}}$, $q = \sqrt{\frac{t-t_0}{4}}$, $q_{in} = \sqrt{\frac{t_{in}-t_0}{4}}$ of four-sheeted Riemann surface into one W-plane, W_Z and W_P the zero and the pole, by means of which a contribution of the left-hand cut on the II. Riemann sheet is simulated, $f_{v\pi\pi}$ and f_v the vector-meson-pion and the universal vector-meson coupling constants, respectively, whereby

$$\left(\frac{f_{\rho'\pi\pi}}{f_{\rho'}}\right) = \frac{1}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}}\left\{1-\left(\frac{N_{\rho'}}{|W_{\rho'}|^4}-\left[1+2\frac{W_Z W_P}{W_Z-W_P}\mathrm{Re}W_\rho(1-|W_\rho|^{-2})\right]N_\rho\right)\left(\frac{f_{\rho\pi\pi}}{f_\rho}\right)\right\},$$

$$\left(\frac{f_{\rho''\pi\pi}}{f_{\rho''}}\right) = \frac{1}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}}\left\{-1+\left(\frac{N_{\rho''}}{|W_{\rho''}|^4}-\left[1+2\frac{W_Z W_P}{W_Z-W_P}\mathrm{Re}W_\rho(1-|W_\rho|^{-2})\right]N_\rho\right)\left(\frac{f_{\rho\pi\pi}}{f_\rho}\right)\right\},$$

$$N_\rho = (W_N-W_\rho)(W_N-W_\rho^*)\times(W_N-1/W_\rho)(W_N-1/W_\rho^*),$$

$$N_v = (W_N-W_v)(W_N-W_v^*)(W_N+W_v)(W_N+W_v^*); \quad v=\rho',\rho''.$$

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The model is defined on four-sheeted Riemann surface with complex conjugate poles (corresponding to unstable ρ-resonances) on unphysical sheets, and reflecting all known properties of EM pion FF. It depends on 10 physically interpretable free parameters, t_{in}, m_ρ, Γ_ρ, $f_{\rho\pi\pi}/f_\rho$, $m_{\rho'}$, $\Gamma_{\rho'}$, $m_{\rho''}$, $\Gamma_{\rho''}$, W_Z and W_P.

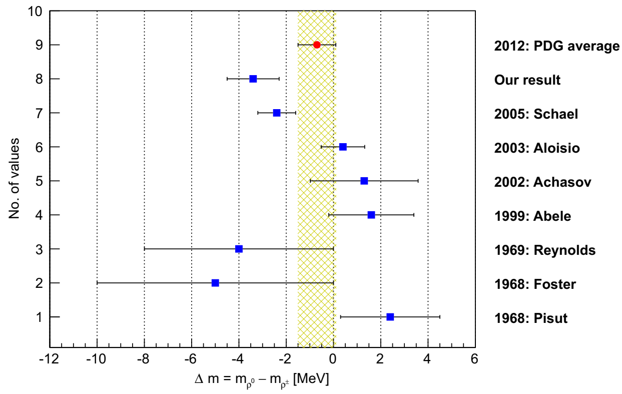

Figure 1. The comparison of published values for mass difference of neutral ρ^0 and charged $\rho^\pm$ mesons for the state $\rho(770)$ with our result and PDG average value.

Conserved vector current hypothesis allows one to relate the isovector part of the weak spectral function $v_0^{I=1}(s)$ characterizing the $\pi^+\pi^-$ system (s is the invariant mass squared of such system) with the spectral function $v_-(s)$ of $\pi^-\pi^0$ system. Because there also exists the dependence of the spectral function and pion form factor, one can relate $F_\pi^{\rm EM}$ – EM pion FF with $F_\pi^{\rm W}$ – the weak pion FF through the relation $F_\pi^{\rm W} = \sqrt{2}F_\pi^{\rm EM}$ (see [5]). In such a way it is possible to use the same U&A model of EM pion FF for the description of the weak pion FF data from τ^--lepton decay.

3 Conclusions

In contrast to previous approaches to determine the rho-meson family parameters, we have proposed new phenomenological approach based on $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\nu_\tau$ data analysis. The complex U&A models of EM and weak pion FF allow one to determine the masses and widths for neutral and charge rho-mesons and their excitations precisely, see Tab. 1. Our result for the difference $m_{\rho^0} - m_{\rho^\pm} = -3.37 \pm 1.06$ MeV is in the coincidence with the recently published values, see Fig. 1.

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