Non-perturbative pion dynamics for the X(3872)

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Abstract. We discuss the role of non-perturbative pion dynamics on the near-threshold resonant X(3872) charmonium state, which is assumed to be an S-wave \(D \bar{D}^*\) bound system. We calculate the contribution to the width of the X(3872) from the \(D \bar{D} \pi\) intermediate state treated non-perturbatively and compare it with different approximate approaches. Further, we explore the quark-mass dependence of the pole position of the X(3872) state. We find that the trajectory of the X(3872) depends strongly on the assumed quark-mass dependence of the short-range interactions which can be determined in lattice QCD calculations.

1 Introduction

After more than a decade after the discovery of the X(3872) by the Belle collaboration its nature still remains an open question, see Ref. [1] for a review. The resonance has the mass \(M_X = (3871 \pm 0.17)\) MeV and thus resides very close to the neutral \(D \bar{D}^*\) threshold

\[ E_B = M_{D^0} + M_{\bar{D}^*0} - M_X = (0.12 \pm 0.26) \text{ MeV}. \]  

It is therefore natural to assume that it has a large molecular admixture [2], see also Refs. [3, 4].

Recently, the quantum numbers of this state were determined by the LHCb Collaboration to be \(1^{++}\) [5] which is consistent with its interpretation as an S-wave \(D^0 \bar{D}^{*0} / \bar{D}^0 D^{*0}\) bound state, see e.g. [6, 7]. The small binding energy relative to the \(D^0 \bar{D}^{*0}\) threshold allows for an effective field theory (EFT) formulation of the problem in analogy to the deuteron\(^1\). The pionless EFT framework based on pure contact \(D \bar{D}^*\) interactions was first applied to the X(3872) in Ref. [8]. Due to the relevance of other dynamical scales, such a treatment is expected to be valid only in very narrow region around the threshold. In particular, the three-body neutral channel \(n D^0 \bar{D}^0\) opens at the energy 7 MeV below the

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\(^1\text{Implications of heavy quark and heavy flavour symmetries were utilised in Ref. [10] to predict partner states of the X(3872).}\)
where $\lambda X(3872)$. The role of non-perturbative pions was investigated in many phenomenological studies, see e.g. Refs.[14–16], all of them however include one-pion-exchange (OPE) in the static limit, i.e. under the assumption that the D-mesons are infinitely heavy particles. Meanwhile, the pion in the $D\bar{D}^*$ potential can go on shell and thus the three-body $\pi D\bar{D}$ unitarity cuts should be taken into account.

In this Contribution we discuss effects induced by the non-perturbative pion dynamics on the $X(3872)$ state within the EFT framework, see Refs.[17, 18] for more details. In particular, we test the validity of the static OPE approximation for the partial decay width $X(3872) \to D\bar{D}\pi$ and study the dependence of the X binding energy on the light quark masses which is a precondition to extract a valuable information about the $DD^*$ interactions from upcoming and ongoing lattice simulations.

## 2 Formalism

We solve a system of coupled-channels Faddeev-type three-body equations for the $D\bar{D}\pi$ system in the $J^{PC} = 1^{++}$ channel

$$a^m_{00}(p, p', E) = \lambda_0 V^{m'0}_{00}(p, p') - \sum_{i=0,c} \lambda_i \int \frac{d^3 k}{\Delta_i(k)} V^{mn}_{0i}(k, p) a^{m'}_{i0}(k, p', E),$$

$$a^m_{c0}(p, p', E) = \lambda_c V^{m'n}_{c0}(p, p') - \sum_{i=0,c} \lambda_i \int \frac{d^3 k}{\Delta_i(k)} V^{mn}_{ci}(k, k) a^{m'}_{i0}(k, p', E),$$

where $\lambda_i$ stand for the known isospin coefficients and the OPE potential containing the three-body propagator at leading order reads

$$V^{mn}(p, p') = -g^2 \frac{p_p p_{p'}}{2m + m_\pi + p_p^2/2m + p_{p'}^2/2m_\pi - M - i0}.\quad (3)$$

Here, the indices $n, n'$ are contracted with the corresponding indices of the $D^*$ polarisation vectors, $m, m_\pi$ and $m_\pi$ stand for the $D, D^*$ and the pion mass, in order, and the energy $E$ is defined relative to the neutral two-body threshold $M = m_{\pi0} + m_0 + E$. Furthermore, the strength of the potential $g$ is extracted from the decay width $D^* \to D\pi$, see e.g. Ref. [18] for a more extended discussion of the input quantities. The OPE potential (3) connects the four $D$-meson channels defined as $|0\rangle = D^0\bar{D}^0$, $|0\rangle = D^0\bar{D}^0$, $|c\rangle = D^+ D^{*-}$, $|c\rangle = D^- D^{*+}$, and the amplitude $a_0 = (a_{00} - a_0)/2$ contains the relevant information about the X-pole. Note that the same three-body cut is also taken into account in the $D\bar{D}^*$ propagators $\Delta_i$ due to dressing $D^*$ by the self-energy ($\pi D$) loops.

It should be stressed that in full analogy to the NN problem [19, 20], the OPE does not fall off at large momenta and thus requires renormalisation. The $D\bar{D}^*$ potential in an S-wave ($V^{SS}$) is to be modified to include the contact interaction $C_0$

$$V^{SS}(p, p') \to C_0 + V^{SS}(p, p').$$

It is shown in Ref. [13] that the 3-body unitary cuts play the crucial role in the $D_\pi D\bar{D}_\pi$ system, if the $D\pi$ width is dominated by the $S$-wave $D_\pi \to D_\pi\pi$ decay.

In Ref. [21] the role of relativistic corrections in the non-perturbative approach including 3-body effects was addressed.
For the sharp cut-off regularisation scheme used in our calculation $C_0(\Lambda)$ is adjusted to produce a bound state of the $X(3872)$ for any given cut-off $\Lambda$.

In order to analyse the light quark-mass dependence we allow all quantities such as the $D$ and $D^*$-meson masses, the coupling constant and the pion decay constant to vary with $m_\pi$, i.e. we perform an expansion of all such quantities in terms of the parameter $\delta m_\pi/M$ [18], where the small scale is the difference of the running and physical pion masses $\delta m_\pi = m_\pi - m_\pi^{\text{phys}}$, while the large scale $M$ is given by a typical hadronic scale $\sim 1$ GeV. In addition to OPE, also the contact term has to vary with $m_\pi$ to ensure that the binding energy $E_B(m_\pi)$ is approximately $\Lambda$-independent for the running pion mass. Assuming that the leading correction to the the physical-limit quantity $C_0^{\text{ph}}(\Lambda)$ is analytic with the quark masses, we may write

$$C_0(\Lambda, m_\pi) = C_0^{\text{ph}} + \delta C_0 = C_0^{\text{ph}}(\Lambda) \left( 1 + f(\Lambda) \frac{m_\pi^2 - m_\pi^{\text{phys}}^2}{M^2} \right).$$  

The leading $\Lambda$-dependence of the contact interaction is captured by $C_0^{\text{ph}}(\Lambda)$, while the dimensionless function $f(\Lambda)$ absorbs the extra $\Lambda$-dependence which appears for values of the pion mass away from the physical point. Therefore, we fix the $\Lambda$-dependence of the contact interaction requiring that both the binding energy $E_B$ as well as its slope at the physical point, $(\partial E_B/\partial m_\pi)|_{m_\pi = m_\pi^{\text{phys}}}$, are $\Lambda$-independent.

### 3 Discussion and conclusions

First, we discuss the impact of non-perturbative pions on the decay width $X \to D\bar{D}\pi$ [17], as shown in Fig. 1. We find that the perturbative inclusion of pions is justified, while the static approximation with non-perturbative pions leads to a significant overestimation of this observable. Thus, we conclude that the appropriate treatment of the three-body dynamics is mandatory.

The pion mass dependence of the binding energy of the $X(3872)$ is illustrated in Fig. 2. The trajectory of the $X(3872)$ depends strongly on the assumed quark-mass dependence of the short-range interactions which can be parametrized by the slope $(\partial E_B/\partial m_\pi)|_{m_\pi = m_\pi^{\text{phys}}}$ and is, in principle, measurable in lattice QCD. This is demonstrated in Fig. 2 where the resulting pion mass dependence is shown for two different values of the slope of a natural size. The sizeable difference between the pion-full and pion-less approaches at higher values of the pion mass for positive values of the slope indicates the important role of pion dynamics in this scenario, see also Ref. [11] for an analogous study within the X-EFT\textsuperscript{4}. These findings will be useful for chiral extrapolations of the future lattice-QCD

\textsuperscript{4}Our results are in conflict with those of Ref. [22], the differences however can be traced back to conceptual problems of the approach used there, see Ref. [23] for a detailed discussion.
\[ \xi = \frac{m_{\pi}}{m_{\pi}} \]

Figure 2. Pion mass dependence of the X(3872) binding energy. The red filled band corresponds to the positive slope \( \frac{\partial E_B}{\partial m_{\pi}}\big|_{m_{\pi} = m_{\pi}^D} = 0.7 \times 10^{-2} \) while the black filled band corresponds to the negative slope \( \frac{\partial E_B}{\partial m_{\pi}}\big|_{m_{\pi} = m_{\pi}^D} = -1.5 \times 10^{-2} \). In both cases, the ultraviolet cutoff in the integral equations is varied in the range \( \Lambda \in [400, 700] \) MeV. The dashed and dash-dotted curves represent the corresponding results of the pionless approach. The blue point depicts the first lattice calculation of the X(3872) [24].

results for the X(3872) binding energy (see Ref. [24] for the first results) and will provide insights into its binding mechanism once the value of the slope parameter is determined.

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