Abstract. The planned experiments at FAIR enable the study of medium modifications of D and B mesons in (dense) nuclear matter. Evaluating QCD sum rules as a theoretical prerequisite for such investigations encounters heavy-light four-quark condensates. We utilize an extended heavy-quark expansion to cope with the condensation of heavy quarks.

1 Introduction

The forthcoming experimental perspectives for in-medium heavy-light quark (i.e. D and B) meson spectroscopy, in particular at FAIR, are accompanied by the need for sophisticated theoretical analyses, e.g. [1–6]. When utilizing QCD sum rules [7–9], this requires a thorough discussion of heavy-quark condensates in general, and, in particular, in the nuclear medium [10]. Therefore, the heavy-quark expansion (HQE), originally developed for the heavy two-quark condensate $\langle \bar{Q}Q \rangle$ in vacuum, is extended here to four-quark condensates and to the in-medium case, thus going beyond previous approaches, e.g. [11]. Specific formulas are derived and presented which provide important pieces for the complete QCD sum rule analysis of D and B mesons in nuclear matter.

2 Recollection: HQE in vacuum

In [12], a general method is introduced for vacuum condensates involving heavy quarks $Q$ with mass $m_Q$. The heavy-quark condensate is considered as the one-point function

$$\langle 0 | \bar{Q}Q | 0 \rangle = -i \int \frac{d^4p}{(2\pi)^4} \langle 0 | Tr_{\epsilon,D} S_Q(p) | 0 \rangle$$

expressed by the heavy-quark propagator $S_Q$ in a weak classical gluonic background field in Fock-Schwinger gauge, $S_Q(p) = \sum_{k=0}^{\infty} S_Q^{(k)}(p)$ with $S_Q^{(k)}(p) = (-1)^k S_Q^{(0)}(p) \gamma^\mu \bar{A}_\mu S_Q^{(0)}(p) \cdots \gamma^\mu \bar{A}_\mu S_Q^{(0)}(p)$, incorporating the free heavy-quark propagator $S_Q^{(0)}(p) = (\gamma^\mu p_\mu + m_Q)/(p^2 - m_Q^2)$ and the derivative operator $\bar{A}$ emerging from a Fourier transform defined as $\bar{A}_\mu = \sum_{m=0}^{\infty} \bar{A}_\mu^{(m)}$ with $\bar{A}_\mu^{(m)} = -i \frac{1}{m(m+2)} \left( \frac{D_{\mu_1} \cdots D_{\mu_m} G_{\mu_1} \cdots \mu_m (x)}{x^0} \right)$ [13, 14]. In this way, the heavy-quark propagator interacts with the complex QCD ground state via soft gluons generating a series expansion in the inverse...
heavy-quark mass. The compact notation (1) differs from [12], but provides a comprehensive scheme easily extendable to in-medium condensates. The first HQE terms of the heavy two-quark condensate (1) reproduce [12]:

\[
\langle 0 | \bar{Q}Q | 0 \rangle = -\frac{g^2}{48\pi^2 m_Q} \langle G^2 \rangle - \frac{g^3}{1440\pi^2 m_Q^3} \langle G^3 \rangle - \frac{g^4}{120\pi^2 m_Q^3} \langle (DG)^2 \rangle + \ldots
\]

(2)

with the notation

\[
\langle G^2 \rangle = \langle 0 | G_{\mu\nu}^A G^{A\mu\nu} | 0 \rangle ,
\]

(3)

\[
\langle G^3 \rangle = \langle 0 | f^{ABC} G_{\mu\nu}^A G^{B\nu} G^{C\lambda} | 0 \rangle ,
\]

(4)

\[
\langle (DG)^2 \rangle = \langle 0 | \left( \sum_f \bar{q}_f \gamma_\mu t^A q_f \right)^2 | 0 \rangle .
\]

(5)

The graphic interpretation of the terms in (2) is depicted too: the solid lines denote the free heavy-quark propagators and the curly lines are for soft gluons whose condensation is symbolized by the crosses, whereas the heavy quark-condensate is symbolized by the crossed circles [15]. An analogous expression for the mixed heavy-quark gluon condensate can be obtained along those lines which contains, however, a term proportional to \( m_Q \). The leading-order term in (2) was employed already in [7] in evaluating the sum rule for charmonia.

The vacuum HQE method was rendered free of UV divergent results for higher mass-dimension heavy-quark condensates by requiring at least one condensing gluon per condensed heavy-quark [15, 16], which prevents unphysical results, where the condensation probability of heavy-quark condensates rises for an increasing heavy-quark mass.

### 3 Application of HQE to in-medium heavy-light four-quark condensates

The above method can be extended to in-medium situations. Our approach contains two new aspects: (i) formulas analogous to equation (1) are to be derived for heavy-quark condensates, e.g. \( \langle \bar{Q}Q \rangle, \langle \bar{Q}Q \sigma GQ \rangle, \langle \bar{q}_f r^A q \bar{Q} r^A Q \rangle \), which additionally contribute to the in-medium operator product expansion (OPE) and (ii) medium-specific gluonic condensates, e.g. \( \langle G^2/4 - (vG)^2/v^2 \rangle, \langle G^3/4 - f^{ABC} G_{\mu\nu}^A G^{B\nu} G^{C\lambda} v_\mu v_\nu/v^2 \rangle \), enter the HQE of heavy-quark condensates for both, vacuum and additional medium condensates, where \( \langle \ldots \rangle \) denotes Gibbs averaging.

We are especially interested in heavy-light four-quark condensates entering the OPE of \( D \) and \( B \) mesons, inter alia, in terms corresponding to the next-to-leading-order perturbative diagrams with one light-quark \( (q) \) and one heavy-quark \( (Q) \) line cut. There are 24 two-flavour four-quark condensates in the nuclear medium [17] represented here in a compact notation by \( \langle \bar{q} T^A q \bar{Q} \Gamma T^A Q \rangle \), where \( \Gamma \) and \( \Gamma' \) denote Dirac structures and \( T^A \) with \( A = 0, \ldots, 8 \) are the generators of \( SU(3) \) supplemented by the unit element \( (A = 0) \). We obtain the analogous formula to (1) for heavy-light four-quark condensates:

\[
\langle \bar{q} T^A q \bar{Q} \Gamma T^A Q \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \langle \bar{q} T^A q \rangle Tr_{c,d} \left[ \Gamma T^A S_Q(p) \right] .
\]

(6)
The leading-order terms of this HQE are obtained for the heavy-quark propagators $S_Q^{(1)}$ containing $\tilde{A}_\mu^{(1)}$ and $S_Q^{(2)}$ with leading-order background fields $\tilde{A}_\mu^{(0)}$:

$$
\langle \bar{q} \Gamma A q \bar{Q} \Gamma' T A Q \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \langle \bar{q} \Gamma A q \text{ Tr}_{c,D} \big[ \Gamma' T A \left( S_Q^{(1)}(p) + S_Q^{(2)}(p) + \ldots \right) \big] \rangle 
$$

$$
= \langle \bar{q} \Gamma A q \bar{Q} \Gamma' T A Q \rangle^{(0)} + \langle \bar{q} \Gamma A q \bar{Q} \Gamma' T A Q \rangle^{(1)} + \ldots
$$

Evaluation of the first term of the expansion (8) for the complete list of two-flavour four-quark condensates in [17] gives three non-zero results:

$$
\langle \bar{q} \gamma^\nu t^A q \bar{Q} \gamma_5 t^A Q \rangle^{(0)} = -\frac{2}{3} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q^2} \log \frac{\mu^2}{m_Q^2} + \frac{1}{2} \left( \bar{q} \gamma^\nu t^A q \sum_f \bar{q}_f \gamma_5 t^A q_f \right),
$$

$$
\langle \bar{q} \gamma t^A q \bar{Q} \gamma^\mu t^A Q \rangle^{(0)} = -\frac{2}{3} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q^2} \log \frac{\mu^2}{m_Q^2} + \frac{2}{3} \left( \bar{q} \gamma t^A q \sum_f \bar{q}_f \gamma^\mu t^A q_f \right),
$$

$$
\langle \bar{q} \gamma^\mu t^A q \bar{Q} \gamma t^A Q \rangle^{(0)} = -\frac{4}{3} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q^2} \log \frac{\mu^2}{m_Q^2} - \frac{1}{8} \left( \bar{q} t^A q \sum_f \bar{q}_f \gamma^\mu t^A q_f \right),
$$

where logarithmic singularities are calculated in the $\overline{\text{MS}}$ scheme, $\mu$ is the renormalization scale, and $t^A = T^A$ for $A = 1, \ldots, 8$. The non-zero contributions for the second term of (8) read

$$
\langle \bar{q} q \bar{Q} Q \rangle^{(1)} = -\frac{1}{3} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} q G^A_{\mu\nu} G^{A\mu\nu} \rangle,
$$

$$
\langle \bar{q} t^A q \bar{Q} t^A Q \rangle^{(1)} = -\frac{1}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} t^A q G^{B\mu} G^{C\mu} \rangle,
$$

$$
\langle \bar{q} \gamma_5 t^A q \bar{Q} \gamma_5 t^A Q \rangle^{(1)} = -\frac{1}{4} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} \gamma_5 t^A q G^{A\mu} G^{A\mu} \rangle,
$$

$$
\langle \bar{q} \gamma t^A q \bar{Q} \gamma^\mu t^A Q \rangle^{(1)} = -\frac{1}{8} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} \gamma t^A q G^{B\mu} G^{C\mu} \rangle,
$$

$$
\langle \bar{q} t^A q \bar{Q} t^A Q \rangle^{(1)} = -\frac{1}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} t^A q G^{B\mu} G^{C\mu} \rangle,
$$

$$
\langle \bar{q} \gamma_5 t^A q \bar{Q} \gamma^\mu t^A Q \rangle^{(1)} = -\frac{5}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} \gamma_5 t^A q G^{B\mu} G^{C\mu} \rangle,
$$

$$
\langle \bar{q} \gamma_5 t^A q \bar{Q} \gamma t^A Q \rangle^{(1)} = -\frac{5}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} \gamma_5 t^A q G^{B\mu} G^{C\mu} \rangle,
$$

$$
\langle \bar{q} \gamma_5 t^A q \bar{Q} \gamma^\mu t^A Q \rangle^{(1)} = -\frac{5}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} \gamma_5 t^A q G^{B\mu} G^{C\mu} \rangle,
$$

$$
\langle \bar{q} \gamma_5 t^A q \bar{Q} \gamma t^A Q \rangle^{(1)} = -\frac{5}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} \gamma_5 t^A q G^{B\mu} G^{C\mu} \rangle.
where $f^{ABC}$ is the anti-symmetric structure constant of the color group and the corresponding symmetric object $d^{ABC}$ is defined by the anti-commutator $\{t^A, t^B\} = \delta^{AB}/4 + d^{ABC} t^C$.

4 Summary and conclusions

The extension of the OPE for QCD sum rules of $\bar{q}Q$ and $\bar{Q}q$ mesons by four-quark condensates to mass dimension 6 yields heavy-light condensate contributions requiring HQE in a nuclear medium. The necessary steps to generalize the vacuum HQE [12] to cover in-medium situations are described and a general formula for the HQE of in-medium heavy-light four-quark condensates is presented. The two leading-order terms of this expansion for the complete list of two-flavour four-quark condensates [17] have been evaluated. In leading-order the results contain known condensate structures, thus, reducing the number of condensates entering the sum rule evaluation of mesons composed of a heavy and a light quark. It can be seen that the series does not exhibit a simple expansion in $1/m_Q$, not even in vacuum. Therefore, the lowest order terms are not suppressed by inverse powers of $m_Q$ as for $\langle \bar{Q}Q \rangle$, challenging the omission of heavy-light four-quark condensates, as often done in previous sum rule analyses.

Acknowledgements

This work is supported by BMBF 05P12CRGHE and the Austrian Science Fund (FWF) under project number P25121-N27.

References