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Heavy-quark expansion for D and B mesons in nuclear matter

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Abstract. The planned experiments at FAIR enable the study of medium modifications of D and B mesons in (dense) nuclear matter. Evaluating QCD sum rules as a theoretical prerequisite for such investigations encounters heavy-light four-quark condensates. We utilize an extended heavy-quark expansion to cope with the condensation of heavy quarks.

1 Introduction

The forthcoming experimental perspectives for in-medium heavy-light quark (i. e. D and B) meson spectroscopy, in particular at FAIR, are accompanied by the need for sophisticated theoretical analyses, e. g. [1–6]. When utilizing QCD sum rules [7–9], this requires a thorough discussion of heavy-quark condensates in general, and, in particular, in the nuclear medium [10]. Therefore, the heavy-quark expansion (HQE), originally developed for the heavy two-quark condensate $\langle \bar{Q}Q \rangle$ in vacuum, is extended here to four-quark condensates and to the in-medium case, thus going beyond previous approaches, e. g. [11]. Specific formulas are derived and presented which provide important pieces for the complete QCD sum rule analysis of D and B mesons in nuclear matter.

2 Recollection: HQE in vacuum

In [12], a general method is introduced for vacuum condensates involving heavy quarks Q with mass m_Q. The heavy-quark condensate is considered as the one-point function

$$\langle 0|\bar{Q}Q|0\rangle = -i \int \frac{d^4 p}{(2\pi)^4} \langle 0|\mathrm{Tr}_{\mathrm{c,D}}\, S_Q(p)|0\rangle \tag{1}$$

expressed by the heavy-quark propagator S_Q in a weak classical gluonic background field in Fock-Schwinger gauge, $S_Q(p) = \sum_{k=0}^{\infty} S_Q^{(k)}(p)$ with $S_Q^{(k)}(p) = (-1)^k S_Q^{(0)}(p)\gamma^{\mu_1}\tilde{A}_{\mu_1} S_Q^{(0)}(p) \dots \gamma^{\mu_k}\tilde{A}_{\mu_k} S_Q^{(0)}(p)$, incorporating the free heavy-quark propagator $S_Q^{(0)}(p) = (\gamma^\mu p_\mu + m_Q)/(p^2 - m_Q^2)$ and the derivative operator $\tilde{A}$ emerging from a Fourier transform defined as $\tilde{A}_\mu = \sum_{m=0}^{\infty} \tilde{A}_\mu^{(m)}$ with $\tilde{A}_\mu^{(m)} = -\frac{(-i)^{m+1} g}{m!(m+2)} \left(D_{\alpha_1} \dots D_{\alpha_m} G_{\mu\nu}(x) \right)_{x=0} \partial^\nu \partial^{\alpha_1} \dots \partial^{\alpha_m}$ [13, 14]. In this way, the heavy-quark propagator interacts with the complex QCD ground state via soft gluons generating a series expansion in the inverse

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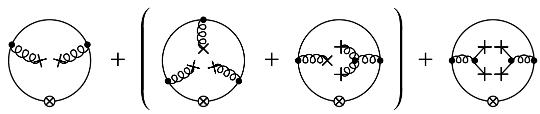
heavy-quark mass. The compact notation (1) differs from [12], but provides a comprehensive scheme easily extendable to in-medium condensates. The first HQE terms of the heavy two-quark condensate (1) reproduce [12]:

$$\langle 0|\bar{Q}Q|0\rangle = -\frac{g^2}{48\pi^2 m_Q}\langle G^2\rangle - \frac{g^3}{1440\pi^2 m_Q^3}\langle G^3\rangle - \frac{g^4}{120\pi^2 m_Q^3}\langle (DG)^2\rangle + \dots \tag{2}$$

$=$  $+ \dots$

with the notation

$$\langle G^2\rangle = \langle 0|G^A_{\mu\nu}G^{A\,\mu\nu}|0\rangle\,, \tag{3}$$

$$\langle G^3\rangle = \langle 0|f^{ABC}G^A_{\mu\nu}G^{B\,\nu}{}_{\lambda}G^{C\,\lambda\mu}|0\rangle\,, \tag{4}$$

$$\langle (DG)^2\rangle = \langle 0|\Big(\sum_f \bar{q}_f\gamma_\mu t^A q_f\Big)^2|0\rangle\,. \tag{5}$$

The graphic interpretation of the terms in (2) is depicted too: the solid lines denote the free heavy-quark propagators and the curly lines are for soft gluons whose condensation is symbolized by the crosses, whereas the heavy quark-condensate is symbolized by the crossed circles [15]. An analogous expression for the mixed heavy-quark gluon condensate can be obtained along those lines which contains, however, a term proportional to m_Q. The leading-order term in (2) was employed already in [7] in evaluating the sum rule for charmonia.

The vacuum HQE method was rendered free of UV divergent results for higher mass-dimension heavy-quark condensates by requiring at least one condensing gluon per condensed heavy-quark [15, 16], which prevents unphysical results, where the condensation probability of heavy-quark condensates rises for an increasing heavy-quark mass.

3 Application of HQE to in-medium heavy-light four-quark condensates

The above method can be extended to in-medium situations. Our approach contains two new aspects: (*i*) formulas analogous to equation (1) are to be derived for heavy-quark condensates, e. g. $\langle\bar{Q}\not{v}Q\rangle$, $\langle\bar{Q}\not{v}\sigma GQ\rangle$, $\langle\bar{q}\not{v}t^A q\bar{Q}\not{v}t^A Q\rangle$, which additionally contribute to the in-medium operator product expansion (OPE) and (*ii*) medium-specific gluonic condensates, e. g. $\langle G^2/4 - (vG)^2/v^2\rangle$, $\langle G^3/4 - f^{ABC}G^A_{\mu\nu}G^{B\,\nu}{}_{\lambda}G^{C\,\lambda\kappa}v^\mu v_\kappa/v^2\rangle$, enter the HQE of heavy-quark condensates for both, vacuum and additional medium condensates, where $\langle\dots\rangle$ denotes Gibbs averaging.

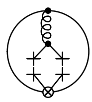
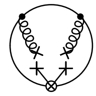
We are especially interested in heavy-light four-quark condensates entering the OPE of D and B mesons, inter alia, in terms corresponding to the next-to-leading-order perturbative diagrams with one light-quark (q) and one heavy-quark (Q) line cut. There are 24 two-flavour four-quark condensates in the nuclear medium [17] represented here in a compact notation by $\langle\bar{q}\Gamma T^A q\bar{Q}\Gamma' T^A Q\rangle$, where Γ and Γ' denote Dirac structures and T^A with $A = 0, \dots, 8$ are the generators of $SU(3)$ supplemented by the unit element ($A = 0$). We obtain the analogous formula to (1) for heavy-light four-quark condensates:

$$\langle\bar{q}\Gamma T^A q\bar{Q}\Gamma' T^A Q\rangle = -i\int\frac{d^4p}{(2\pi)^4}\langle\bar{q}\Gamma T^A q\ \mathrm{Tr}_{\mathrm{c,D}}\left[\Gamma' T^A S_Q(p)\right]\rangle\,. \tag{6}$$

The leading-order terms of this HQE are obtained for the heavy-quark propagators $S_Q^{(1)}$ containing $\tilde{A}_\mu^{(1)}$ and $S_Q^{(2)}$ with leading-order background fields $\tilde{A}_\mu^{(0)}$:

$$\langle \bar{q}\Gamma T^A q \bar{Q}\Gamma' T^A Q\rangle = -i \int \frac{d^4p}{(2\pi)^4} \langle \bar{q}\Gamma T^A q \; \mathrm{Tr}_{\mathrm{c,D}} \left[\Gamma' T^A \left(S_Q^{(1)}(p) + S_Q^{(2)}(p) + \ldots\right)\right]\rangle \tag{7}$$

$$= \langle \bar{q}\Gamma T^A q \bar{Q}\Gamma' T^A Q\rangle^{(0)} + \langle \bar{q}\Gamma T^A q \bar{Q}\Gamma' T^A Q\rangle^{(1)} + \ldots \tag{8}$$

=  +  + ...

Evaluation of the first term of the expansion (8) for the complete list of two-flavour four-quark condensates in [17] gives three non-zero results:

$$\langle \bar{q}\gamma^\nu t^A q \bar{Q}\gamma_\nu t^A Q\rangle^{(0)} = -\frac{2}{3}\frac{g^2}{(4\pi)^2}\left(\log\frac{\mu^2}{m_Q^2} + \frac{1}{2}\right)\langle \bar{q}\gamma^\nu t^A q \sum_f \bar{q}_f \gamma_\nu t^A q_f\rangle\,, \tag{9}$$

$$\langle \bar{q}\not{v} t^A q \bar{Q}\not{v} t^A Q\rangle^{(0)} = -\frac{2}{3}\frac{g^2}{(4\pi)^2}\left(\log\frac{\mu^2}{m_Q^2} + \frac{2}{3}\right)\langle \bar{q}\not{v} t^A q \sum_f \bar{q}_f \not{v} t^A q_f\rangle\,, \tag{10}$$

$$\langle \bar{q} t^A q \bar{Q}\not{v} t^A Q\rangle^{(0)} = -\frac{4}{3}\frac{g^2}{(4\pi)^2}\left(\log\frac{\mu^2}{m_Q^2} - \frac{1}{8}\right)\langle \bar{q} t^A q \sum_f \bar{q}_f \not{v} t^A q_f\rangle\,, \tag{11}$$

where logarithmic singularities are calculated in the $\overline{\mathrm{MS}}$ scheme, μ is the renormalization scale, and $t^A = T^A$ for $A = 1, \ldots, 8$. The non-zero contributions for the second term of (8) read

$$\langle \bar{q}q\bar{Q}Q\rangle^{(1)} = -\frac{1}{3}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle \bar{q}qG^A_{\mu\nu}G^{A\,\mu\nu}\rangle\,, \tag{12}$$

$$\langle \bar{q}t^A q\bar{Q}t^A Q\rangle^{(1)} = -\frac{1}{6}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle d^{ABC}\bar{q}t^A qG^B_{\mu\nu}G^{C\,\mu\nu}\rangle\,, \tag{13}$$

$$\langle \bar{q}\gamma_5 q\bar{Q}\gamma_5 Q\rangle^{(1)} = -\frac{1}{4}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle i\bar{q}\gamma_5 qG^A_{\mu\nu}G^A_{\alpha\beta}\varepsilon^{\mu\nu\alpha\beta}\rangle\,, \tag{14}$$

$$\langle \bar{q}\gamma_5 t^A q\bar{Q}\gamma_5 t^A Q\rangle^{(1)} = -\frac{1}{8}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle id^{ABC}\bar{q}\gamma_5 t^A qG^B_{\mu\nu}G^C_{\alpha\beta}\varepsilon^{\mu\nu\alpha\beta}\rangle\,, \tag{15}$$

$$\langle \bar{q}\not{v}q\bar{Q}Q\rangle^{(1)} = -\frac{1}{3}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle \bar{q}\not{v}qG^A_{\mu\nu}G^{A\,\mu\nu}\rangle\,, \tag{16}$$

$$\langle \bar{q}\not{v}t^A q\bar{Q}t^A Q\rangle^{(1)} = -\frac{1}{6}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle d^{ABC}\bar{q}\not{v}t^A qG^B_{\mu\nu}G^{C\,\mu\nu}\rangle\,, \tag{17}$$

$$\langle \bar{q}\sigma_{\mu\nu}t^A q\bar{Q}\sigma^{\mu\nu}t^A Q\rangle^{(1)} = -\frac{5}{6}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle f^{ABC}\bar{q}\sigma_{\mu\nu}t^A qG^{B\,\nu}{}_{\lambda}G^{C\,\lambda\mu}\rangle\,, \tag{18}$$

$$\langle \bar{q}\sigma_{\mu\nu}t^A q\bar{Q}\sigma_{\alpha\beta}t^A Qg^{\mu\alpha}v^\nu v^\beta\rangle^{(1)} = -\frac{5}{3}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle f^{ABC}\bar{q}\sigma_{\mu\nu}t^A qG^{B\,\nu}{}_{\alpha}G^{C\,\alpha\beta}v^\mu v_\beta\rangle\,, \tag{19}$$

$$\langle \bar{q}\gamma_5\gamma_\lambda t^A q\bar{Q}\sigma_{\mu\nu}t^A Q\varepsilon^{\mu\nu\lambda\tau}v_\tau\rangle^{(1)} = -\frac{5}{6}\frac{g^2}{(4\pi)^2}\frac{1}{m_Q}\langle f^{ABC}\bar{q}\gamma_5\gamma_\lambda t^A qG^B_{\alpha\beta}G^{C\,\beta}{}_{\gamma}\varepsilon^{\gamma\alpha\lambda\tau}v_\tau\rangle\,, \tag{20}$$

where f^{ABC} is the anti-symmetric structure constant of the color group and the corresponding symmetric object d^{ABC} is defined by the anti-commutator $\{t^A, t^B\} = \delta^{AB}/4 + d^{ABC} t^C$.

4 Summary and conclusions

The extension of the OPE for QCD sum rules of $\bar{q}Q$ and $\bar{Q}q$ mesons by four-quark condensates to mass dimension 6 yields heavy-light condensate contributions requiring HQE in a nuclear medium. The necessary steps to generalize the vacuum HQE [12] to cover in-medium situations are described and a general formula for the HQE of in-medium heavy-light four-quark condensates is presented. The two leading-order terms of this expansion for the complete list of two-flavour four-quark condensates [17] have been evaluated. In leading-order the results contain known condensate structures, thus, reducing the number of condensates entering the sum rule evaluation of mesons composed of a heavy and a light quark. It can be seen that the series does not exhibit a simple expansion in $1/m_Q$, not even in vacuum. Therefore, the lowest order terms are not suppressed by inverse powers of m_Q as for $\langle \bar{Q}Q \rangle$, challenging the omission of heavy-light four-quark condensates, as often done in previous sum rule analyses.

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