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$K^{\pm} \to \pi^{\pm}\pi^{0}e^{+}e^{-}$ decay width calculation in the lepton center-of-mass system

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Abstract. We consider the rare decay $K^{\pm} \to \pi^{\pm}\pi^{0}e^{+}e^{-}$ currently under analysis by the NA62 Collaboration. A new theoretical approach for the differential decay width has been performed. We compared the dependencies of the differential decay rate with respect to dilepton and dipion system masses in different theoretical descriptions.

1 Introduction

The radiative kaon decay $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$ is one of the most interesting decays for studying the low energy structure of QCD. The amplitude of this decay consists of two parts: a long distance contribution called Inner Bremsstrahlung (IB) and a direct emission (DE) part. IB contribution is associated with the $K^{\pm} \to \pi^{\pm}\pi^{0}$ decay according to Low's theorem [1], and DE can be calculated in the framework of the Chiral Perturbation Theory [2]. Despite the fact that the $K^{\pm} \to \pi^{\pm}\pi^{0}$ decay is suppressed by the $\Delta I = 1/2$ rule, the Bremsstrahlung contribution is still much larger than DE [3].

At present the NA62 Collaboration at CERN SPS is analyzing the experimental data on the radiative decay with a virtual photon that has not been observed up to now:

$$K^{\pm}(P_K) \to \pi^{\pm}\pi^{0}\gamma^{*}(q) \to \pi^{\pm}(p_1)\pi^{0}(p_2)e^{+}(k_1)e^{-}(k_2). \tag{1}$$

The advantage of this decay in comparison with the radiative decay with a real photon for the DE component extraction is obvious: the photon virtuality q^2 allows to analyze the additional kinematical region absent in the case of real photons. The solid theoretical base for this decay was developed in [4, 5]. We have recently revised a new approach [6] which simplifies greatly the calculations of this decay. Later on we consider shortly (for details see [6]) this approach and compare the computations of the decay width dependence on the invariant masses of the dilepton and dipion systems with the Monte-Carlo (MC) generator implemented by using the CERNLIB library [7], realized in the kaon rest frame formulation of the $K^{\pm} \to \pi^{\pm}\pi^{0}e^{+}e^{-}$ channel.

2 Covariant amplitude

As usual the invariant amplitude of the decay $K^{\pm} \to \pi^{\pm}\pi^{0}e^{+}e^{-}$ is parameterized as a product of leptonic and hadronic currents :

$$A = \frac{e}{q^2} j^{\mu}(k_1, k_2) J_{\mu}(p_1, p_2, q). \tag{2}$$

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where p_1, p_2 are the 4-momenta of charged and neutral pions, k_1, k_2 – the leptons 4-momenta and $q = k_1 + k_2$ is the momentum of the virtual photon.
The leptonic current:

$$j^\mu(k_1, k_2) = \bar{u}(k_2)\gamma^\mu v(k_1). \tag{3}$$

whereas the hadronic current is represented in terms of two electric form factors $F_{1,2}$ and the magnetic one F_3:

$$J_\mu(p_1, p_2, q) = F_1 p_{1\mu} + F_2 p_{2\mu} + F_3 \epsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\mu} q_\beta. \tag{4}$$

According to Low theorem, the Bremsstrahlung part can be written in terms of the matrix element for the kaon decay into two pions $M(K^+ \to \pi^+\pi^0)$ and the sum of amplitudes corresponding to radiation of the virtual photon by $K^\pm$–meson or charged pion:

$$M(K^+ \to \pi^+\pi^0\gamma^*)_B = eM(K^+ \to \pi^+\pi^0) \times \left(-\frac{\epsilon_\mu P_k^\mu}{(P_k \cdot q) - \frac{q^2}{2}} + \frac{\epsilon_\mu p_1^\mu}{(p_1 \cdot q) + \frac{q^2}{2}} \right). \tag{5}$$

Comparing this expression with Eq.4, one obtains relations between the electric form factors F_1^B, F_2^B relevant to the bremsstrahlung and decay amplitude $M(K^+ \to \pi^+\pi^0)$ [5]:

$$\begin{aligned} F_1^B &= \frac{2ie(qP - qQ)}{(q^2 + qQ + qP)(q^2 + 2qP)} M(K^+ \to \pi^+\pi^0), \\ F_2^B &= \frac{-2ie}{q^2 + 2qP} M(K^+ \to \pi^+\pi^0). \end{aligned} \tag{6}$$

3 Dilepton center-of-mass system

In the dilepton c.m.s. ($\vec{q} = \vec{k}_1 + \vec{k}_2 = 0$) the virtual photon 4-momentum $q = (\omega, 0, 0, 0)$ and $k = \omega(0, v\vec{n})$; ω is the virtual photon energy, $\vec{n}$ is the unit vector and $v = \sqrt{1 - \frac{4m_e^2}{\omega^2}}$ is the lepton velocity. In this system the lepton currents product summed over spins $t_{\mu\nu} = \sum j^\mu j^\nu$ has the property $t_{00} = t_{0k}$ ($k = 1, 2, 3$) which essentially simplifies the expression for the product of the lepton and hadron currents:

$$J_\mu J_\nu t_{\mu\nu} = s_e \left(|\vec{J}|^2 - (\vec{J}\vec{v})^2 \right) = s_e \left(|\vec{J}|^2 - v^2 (\vec{J}\vec{n})^2 \right). \tag{7}$$

The square of the matrix element reads:

$$\sum_\lambda |A|^2 = \frac{2e^2}{s_e} \left(|\vec{J}|^2 - \frac{(\vec{J}\vec{q})(\vec{J}\vec{q})}{s_e} \right). \tag{8}$$

Integrating this expression over the solid angle from the phase space, one obtains:

$$\sum_\lambda \int |A|^2 d\Omega_q = \frac{2\pi e^2}{s_e} |\vec{J}|^2 \left(1 - \frac{k^2}{3q^2} \right) = \frac{8\pi e^2}{s_e} |\vec{J}|^2 \left(1 - \frac{v^2}{3} \right). \tag{9}$$

The square of the hadron current is a function of pions three-dimensional momenta $\vec{p}_1, \vec{p}_2$ in the dilepton c.m.s:

$$|\vec{J}|^2 = \vec{p}_1^{\,2} |F_1|^2 + \vec{p}_2^{\,2} |F_2|^2 + 2(\vec{p}_1 \vec{p}_2) Re F_1 F_2^* + s_e (\vec{p}_1^{\,2} \vec{p}_2^{\,2} - (\vec{p}_1 \vec{p}_2)^2) |F_3|^2. \tag{10}$$

Dividing the pions momenta into longitudinal and transverse parts and making use the Lorentz transformations, we express them in terms of the pion momentum p^* in the dipion c.m.s:

$$p_{1L} = \gamma p^* \cos\theta + \beta E_1^*; \quad p_{2L} = -\gamma p^* \cos\theta + \beta E_2^*; \quad |\vec{p_{1\perp}}| = |\vec{p_{2\perp}}| = p^* \sin\theta. \tag{11}$$

where θ is the angle between the charged pion in the dipion c.m.s and the dipion flight direction, $\gamma = \frac{M_K^2 - s_\pi - s_e}{2\sqrt{s_\pi s_e}}$ is the relevant Lorentz factor and $\beta = \sqrt{\gamma^2 - 1}$. Gathering the appropriate expressions, we obtain [6]:

$$\begin{aligned}
d\Gamma &= \frac{\alpha^2}{4(4\pi)^3 M_K s_e}(|F_1|^2\vec{p_1}^2 + |F_2|^2\vec{p_2}^2 + 2(\vec{p_1}\cdot\vec{p_2})Re(F_1F_2^*) \\
&+ s_e[\vec{p_1}^2\vec{p_2}^2 - (\vec{p_1}\cdot\vec{p_2})^2]|F_3|^2)(1 - \frac{v^3}{3})ds_\pi ds_e dcos\theta; \\
F_1^B &= \frac{2i(\gamma E_2^* - \beta p^* cos\theta}{(\gamma E_1^* + \beta p^* cos\theta + \omega/2)(M_K^2 - s_\pi)}|M(K^\pm \to \pi^\pm\pi^0)|e^{i\delta_0^2}; \\
F_2^B &= \frac{2i}{(M_K^2 - s_\pi)}|M(K^\pm \to \pi^\pm\pi^0)|e^{i\delta_0^2}; \\
F_1^{DE} &= \frac{2iG_8}{f_\pi}e^{i\delta_1^1}\left(N_E^0\omega(\gamma E_2^* - \beta p^* cos\theta) + \frac{2}{3}\omega^2 N_E^1 + 2q^2 L_9\right); \\
F_2^{DE} &= \frac{2iG_8}{f_\pi}e^{i\delta_1^1}\left(N_E^0\omega(\gamma E_1^* + \beta p^* cos\theta) + \frac{1}{3}\omega^2 N_E^2\right); \\
F_3^{DE} &= \frac{2eG_8}{f_\pi}e^{i\delta_1^1}N_M^0.
\end{aligned} \tag{12}$$

These formulae allows to calculate the differential decay width using the minimum set of variables (s_e, s_π, θ).

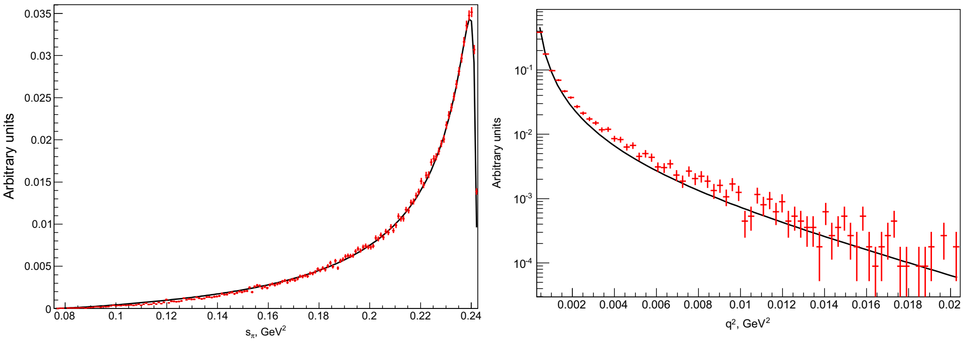

Figure 1. *Comparison between the full differential decay width with respect to invariants q^2 and s_π calculated by our approach (solid line) and with the Monte Carlo generator (dots are given with their statistical errors).*

4 Summary

A short description of the $K^\pm \to \pi^\pm\pi^0 e^+e^-$ differential decay width in the dilepton center-of-mass has been presented. The validity of the obtained computations in this new theoretical approach was checked by using MC generator Fig.1. As it is seen from the Fig.2, the s_π behavior is completely different for the DE contribution compared to the IB which dominates in the full decay width.

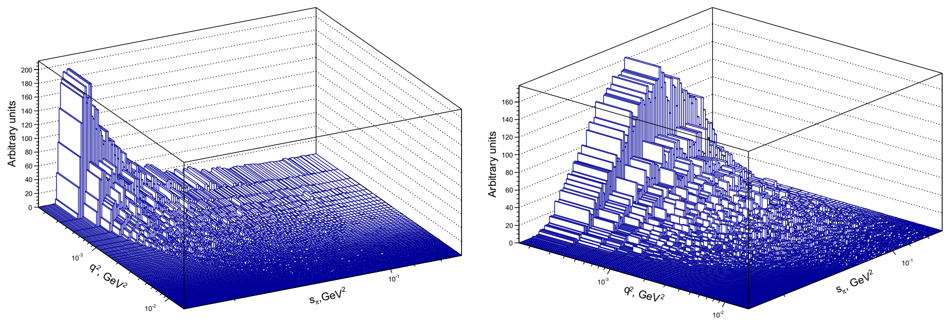

Figure 2. *On the left: the full differential decay width with respect to invariants q^2 and S_π. On the right: the contribution of DE in the differential decay width.*

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